CIRCULAR MOTION

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Formula / Topic Name	Formula	Conditions / Usage
1. Kinematics of Circular Motion		
Angular Velocity (ω)	$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$ $$\omega_{inst} = \frac{d\theta}{dt}$$	Rate of change of angular displacement.
Linear Velocity (v)	$$v = r\omega$$	Relation between linear speed (v) and angular speed (ω) for a particle at radius r.
Angular Acceleration (α)	$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$	Rate of change of angular velocity.
Tangential Acceleration (a_t)	$$a_t = r\alpha = \frac{dv}{dt}$$	Responsible for changing the **magnitude** of velocity (speed). Zero in U.C.M.
Centripetal (Radial) Acceleration (a_c or a_r)	$$a_c = \frac{v^2}{r} = r\omega^2 = v\omega$$	Responsible for changing the **direction** of velocity. Always directs towards center.
Net Acceleration (a_{net})	$$a_{net} = \sqrt{a_c^2 + a_t^2}$$	Vector sum of radial and tangential acceleration.
Angle of Net Acceleration (ϕ)	$$\tan\phi = \frac{a_c}{a_t}$$	ϕ is the angle made by net acceleration with the tangential direction.
Equations of Circular Motion	$$\omega = \omega_0 + \alpha t$$	**Condition:** Only valid when Angular Acceleration (α) is **constant**.

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

2. Dynamics of Circular Motion		
Centripetal Force (F_c)	$F_c = \frac{mv^2}{r} = mr\omega^2$	Real force required to keep a body in circular motion (provided by Tension, Friction, Normal, etc.).
Centrifugal Force (Pseudo Force)	$F_{cf} = \frac{mv^2}{r}$	Acts radially outward. **Condition:** Only applicable in a **rotating (non-inertial) reference frame**.
3. Banking of Roads & Turning		
Bending of Cyclist	$\tan\theta = \frac{v^2}{rg}$	θ with vertical. Condition for no skidding while turning.
Car on Level Circular Road (No Banking)	$v_{max} = \sqrt{\mu_s rg}$	Maximum safe speed to avoid skidding. μ_s = coefficient of static friction.
Banked Road (Frictionless)	$\tan\theta = \frac{v^2}{rg}$	Optimum speed $v_{opt} = \sqrt{rg\tan\theta}$. No wear and tear on tires.
Banked Road (With Friction) - Max Speed	$v_{max} = \sqrt{rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)}$	Speed limit to avoid slipping **outwards** (up the incline).
Banked Road (With Friction) - Min Speed	$v_{min} = \sqrt{rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta}\right)}$	Speed limit to avoid slipping **inwards** (down the incline).
4. Conical Pendulum		

Angular Velocity	$\omega = \sqrt{\frac{g}{L\cos\theta}} = \sqrt{\frac{g}{h}}$	L = length of string, h = vertical height of point of suspension from circle center.
Time Period (T)	$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$	Time for one complete revolution.
Tension in String	$T_{tension} = \frac{mg}{\cos\theta} = mL\omega^2$	
5. Vertical Circular Motion (String)		
Velocity at any point	$v = \sqrt{u^2 - 2gh}$	u = speed at bottom, h = height from bottom.
Tension at any point	$T = \frac{mv^2}{r} + mg\cos\theta$	θ is angle with vertical downward direction.
Critical Velocity (Top)	$v_{top} = \sqrt{gR}$	Minimum speed at top to keep string taut ($T_{top} \geq 0$).
Critical Velocity (Bottom)	$v_{bottom} = \sqrt{5gR}$	Minimum speed at bottom to complete the full circle.
Critical Velocity (Horizontal Point)	$v_{mid} = \sqrt{3gR}$	Speed at the point where string is horizontal.
Tension Difference	$T_{bottom} - T_{top} = 6mg$	Valid for any vertical circular motion under gravity.
Condition for Oscillation	$0 < v_{bottom} \leq \sqrt{2gR}$	Particle oscillates like a pendulum (doesn't reach horizontal level).
Condition for Leaving Circle	$\sqrt{2gR} < v_{bottom} < \sqrt{5gR}$	Particle leaves the circular path in the upper half (T becomes 0 before v).
6. Specific Formulae for Questions		

"Death Well" (Rotor)	$$v_{min} = \sqrt{\frac{gR}{\mu}}$$	Min speed to prevent falling. Friction acts upwards balancing weight.
Vehicle on Convex Bridge	$$v_{max} = \sqrt{gR}$$	Max speed to maintain contact with the bridge (Normal reaction $N = 0$).
Radius of Curvature (Projectile)	$$R_{curv} = \frac{v^2}{a_\perp}$$	At top of trajectory: $$R = \frac{u^2 \cos^2 \theta}{g}$$
Toppling of Car on Turn	$$v_{max} = \sqrt{\frac{gra}{h}}$$	$2a$ = distance between wheels (track width), h = height of Center of Mass. Condition: Topples if $v > v_{max}$.