Motion in 2D and Projectile Motion

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Formula Name / Topic	Formula(e)	Conditions / Usage
1. General Motion in 2D		
Position Vector	$\vec{r} = x\hat{i} + y\hat{j}$	Cartesian coordinates at time t
Velocity Vector	$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j}$	Instantaneous velocity
Acceleration Vector	$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j}$	Instantaneous acceleration
2. Projectile Motion (Ground to Ground)		**Assumptions:** Air resistance neglected, g is constant downwards.
Components of Initial Velocity	$u_x = u\cos\theta$	θ is angle with horizontal
	$u_y = u\sin\theta$	
Motion Parameters (x & y)	$a_x = 0$, $v_x = u\cos\theta$ (constant)	x-motion is uniform; y-motion is under gravity
	$a_y = -g$	
Time of Flight (T)	$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$	Projectile lands at same vertical level as launch
Maximum Height (H)	$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$$	Vertical component of velocity becomes zero at max height
Horizontal Range (R)	$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u_xu_y}{g}$$	Distance between launch and landing point on same level
Condition for Max Range	$\theta = 45^\circ \Rightarrow R_{max} = \frac{u^2}{g}$	For a given initial speed u

Relation between H and R	$$R \tan\theta = 4H$$	Useful for direct relation problems
Equation of Trajectory	$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$ OR $$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$	To find y given x (eliminating t)
Instantaneous Velocity	$\vec{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$ $	\vec{v}
Angle with Horizontal (α)	$\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$	Direction of motion at time t
3. Horizontal Projectile (From Tower)		**Condition:** Thrown horizontally ($u_y = 0$) from height h
Time of Flight	$$T = \sqrt{\frac{2h}{g}}$$	Time to reach ground
Horizontal Range	$$R = u \times T = u\sqrt{\frac{2h}{g}}$$	Horizontal distance covered
Velocity at Ground	$$v = \sqrt{u^2 + 2gh}$$	Conservation of energy or vector sum
4. Projectile on Inclined Plane		**Incline angle α, Projection angle θ (w.r.t incline)**
Time of Flight (Incline)	$$T = \frac{2u\sin\theta}{g\cos\alpha}$$	Component of g perpendicular to plane is $g\cos\alpha$

Range on Incline (R_{inc})	$$R_{inc} = \frac{2u^2 \sin\theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$	Projecting UP the incline
Range on Incline (R_{inc})	$$R_{inc} = \frac{2u^2 \sin\theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$$	Projecting DOWN the incline

5. Relative Motion in 2D

Relative Velocity	$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$	Velocity of A with respect to B
Rain-Man Problem	$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$	$\vec{v}_{rm}$ is how rain appears to the man
River-Boat: Crossing River	$\vec{v}_{b,g} = \vec{v}_{b,r} + \vec{v}_{r,g}$	$\vec{v}_{b,r}$ = velocity of boat in still water
Condition: Shortest Path	$\sin\theta = \frac{v_r}{v_{br}}$ (upstream)	Drift = 0 (Requires $v_{br} > v_r$)