

Kinetic Theory of Gases and Thermodynamics

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1. Kinetic Theory of Gases (KTG) & Gas Laws

Formula Name / Topic	Formula(e)	Conditions / specific usage
Ideal Gas Equation	$PV = nRT$	General Ideal Gas Law.
	$PV = \frac{m}{M}RT$	n : moles, m : mass, M : Molar mass, ρ : density, N : number of molecules, k_B : Boltzmann constant ($1.38 \times 10^{-23} J/K$).
	$P = \rho \frac{RT}{M}$	
	$PV = Nk_B T$	
Boyle's Law	$P \propto \frac{1}{V} \implies P_1 V_1 = P_2 V_2$	Constant Temperature (T) (Isothermal). Graph P vs $1/V$ is a straight line through origin.
Charles's Law	$V \propto T \implies \frac{V_1}{T_1} = \frac{V_2}{T_2}$	Constant Pressure (P) (Isobaric). T must be in Kelvin.
Gay-Lussac's Law	$P \propto T \implies \frac{P_1}{T_1} = \frac{P_2}{T_2}$	Constant Volume (V) (Isochoric). Pressure Law. T must be in Kelvin.
Avogadro's Law	$V \propto n \implies \frac{V_1}{n_1} = \frac{V_2}{n_2}$	Constant P and T. Equal volumes of gases contain equal number of molecules.

Dalton's Law of Partial Pressure

$$P_{total} = P_1 + P_2 + \dots + P_n$$

For non-reacting gas mixture.

$$P_i = x_i P_{total}$$

$$x_i = \frac{n_i}{n_{total}} \text{ (Mole fraction).}$$

Graham's Law of Diffusion

$$r \propto \frac{1}{\sqrt{M}} \implies \frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{\rho_2}{\rho_1}}$$

r : Rate of diffusion/effusion.

Lighter gases diffuse faster.

At const P and T .

Pressure of an Ideal Gas

$$P = \frac{1}{3} \rho v_{rms}^2$$

v_{rms} : Root Mean Square speed.

$$P = \frac{1}{3} \frac{mN}{V} v_{rms}^2$$

Pressure depends on density and square of RMS speed.

$$P = \frac{2}{3} E \text{ (where } E \text{ is K.E. per unit volume)}$$

E : Total translational K.E. per unit volume.

Root Mean Square Speed (v_{rms})

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{m_{molecule}}} = \sqrt{\frac{3P}{\rho}}$$

Speed effective in calculating kinetic energy.

Note: T must be in Kelvin, M in kg/mol.

For mixture:

$$v_{rms}(mix) = \sqrt{\frac{n_1 M_1 v_1^2 + n_2 M_2 v_2^2}{n_1 M_1 + n_2 M_2}}$$

Average Speed (v_{avg})

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8k_B T}{\pi m}}$$

Arithmetic mean of speeds.

$$v_{avg} \approx 0.92 v_{rms}$$

Most Probable Speed (v_{mp})

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2k_B T}{m}}$$

Speed possessed by maximum fraction of molecules.

Ratio

$$v_{mp} : v_{avg} : v_{rms} = \sqrt{2} : \sqrt{8/\pi} : \sqrt{3}$$

Kinetic Energy (Translational)	Per molecule: $K.E. = \frac{3}{2}k_B T$	depends ONLY on Temperature (T).
	Per mole: $K.E. = \frac{3}{2}RT$	Independent of nature of gas.
Degrees of Freedom (f)	Monoatomic: $f = 3$ (3 trans)	Used to calculate internal energy and C_v .
	Diatomic (rigid): $f = 5$ (3 trans + 2 rot)	Note: For NEET/JEE, unless specified "high temp", take Diatomic $f = 5$.
	Diatomic (vib at high T): $f = 7$	
	Triatomic (linear): $f = 5$ (rigid) / 7 (vib)	
	Triatomic (non-linear): $f = 6$	
Law of Equipartition of Energy	Energy associated with each D.O.F = $\frac{1}{2}k_B T$ (per molecule)	Total Internal Energy (U) depends on f .
Internal Energy (U)	$U = \frac{f}{2}nRT$	For an ideal gas, U is a function of T only.
		Change: $\Delta U = \frac{f}{2}nR\Delta T = nC_v\Delta T$
Mean Free Path (λ)	$\lambda = \frac{1}{\sqrt{2}\pi d^2 n_v}$	d : diameter of molecule, n_v : number density (N/V).
		$\lambda \propto \frac{T}{P}$ (since $n_v = P/k_B T$).
Mixture of Gases	$M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$	Used when non-reacting gases are mixed.
	$C_{v(mix)} = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$	
	$\gamma_{mix} = \frac{C_{p(mix)}}{C_{v(mix)}}$	

2. Thermodynamics

Formula Name / Topic	Formula(e)	Conditions / specific usage
First Law of Thermodynamics (FLOT)	$dQ = dU + dW$	Conservation of Energy.
	$Q = \Delta U + W$	Sign Convention (Chemistry opposite for W):
		Heat added to system: $Q > 0$
		Work done BY gas (expansion): $W > 0$
		Internal Energy increases: $\Delta U > 0$
Work Done (General)	$W = \int_{V_1}^{V_2} P dV$	Area under P-V curve on Volume axis gives Work Done.
Mayer's Relation	$C_P - C_V = R$	Valid for Ideal Gas (per mole).
		$C_P > C_V$ because work is done in isobaric expansion.
Specific Heat Ratio (γ)	$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$	Monoatomic $\gamma = 5/3 = 1.67$
		Diatomic $\gamma = 7/5 = 1.4$
		Triatomic $\gamma = 4/3 = 1.33$
Bulk Modulus of Gas (B)	General: $B = -V \frac{dP}{dV}$	Resistance to compression.
	Isothermal (B_T): $B_T = P$	Adiabatic elasticity is γ times Isothermal elasticity ($B_S = \gamma B_T$).
	Adiabatic (B_S): $B_S = \gamma P$	
		Isobaric ($P=\text{const}$): $B = 0$.

		Isochoric ($V=\text{const}$): $B = \infty$.
Isochoric Process (V const)	$W = 0$	Volume constant ($\Delta V = 0$).
	$Q = \Delta U = nC_V \Delta T$	Gay-Lussac's Law holds.
	$\frac{P_1}{T_1} = \frac{P_2}{T_2}$	
Isobaric Process (P const)	$W = P(V_2 - V_1) = nR(T_2 - T_1)$	Pressure constant.
	$Q = nC_P \Delta T$	Charles's Law holds.
	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$	Fraction of heat into internal energy: $1/\gamma$.
		Fraction of heat into work: $1 - 1/\gamma$.
Isothermal Process (T const)	$W = nRT \ln(\frac{V_2}{V_1}) = 2.303nRT \log(\frac{V_2}{V_1})$	Temperature constant ($\Delta T = 0$).
	$W = nRT \ln(\frac{P_1}{P_2})$	Internal energy change is zero for ideal gas.
	$\Delta U = 0 \implies Q = W$	Boyle's Law ($P_1 V_1 = P_2 V_2$).
Adiabatic Process ($Q = 0$)	Equation: $PV^\gamma = \text{const}$	No heat exchange ($dQ = 0$).
	$TV^{\gamma-1} = \text{const}$	Occurs suddenly/quickly or in insulated containers.
	$P^{1-\gamma} T^\gamma = \text{const}$	
Work in Adiabatic Process	$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1}$	Since $Q = 0$, $W = -\Delta U$.

Expansion causes cooling ($T_2 < T_1$).

Compression causes heating.

Slope of P-V Graph

Isothermal Slope: $\frac{dP}{dV} = -\frac{P}{V}$

Adiabatic curve is steeper than Isothermal by a factor of γ .

Adiabatic Slope: $\frac{dP}{dV} = -\gamma \frac{P}{V}$

Polytropic Process

$$PV^x = \text{const}$$

General process.

Molar Heat Capacity:
 $C = C_V + \frac{R}{1-x}$

Work: $W = \frac{nR(T_1 - T_2)}{x-1}$

Cyclic Process

$$\Delta U_{net} = 0$$

Work = Area enclosed by the loop.

$$Q_{net} = W_{net}$$

Clockwise = +ve Work (Engine).

Anticlockwise = -ve Work (Refrigerator).

Efficiency of Heat Engine (η)

$$\eta = \frac{\text{Work Output}}{\text{Heat Input}} = \frac{W}{Q_{in}}$$

Q_{in} : Heat absorbed from source.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}}$$

Q_{out} : Heat rejected to sink.

Carnot Engine

$$\eta = 1 - \frac{T_{sink}}{T_{source}}$$

Maximum theoretical efficiency.

T must be in Kelvin.

Valid for reversible cycle only.

Refrigerator (COP - β)

$$\beta = \frac{\text{Heat Extracted}}{\text{Work Input}} = \frac{Q_{\text{cold}}}{W}$$

Coefficient of Performance.

$$\beta = \frac{Q_{\text{cold}}}{Q_{\text{hot}} - Q_{\text{cold}}} = \frac{T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}}$$

Relationship with efficiency:
 $\beta = \frac{1-\eta}{\eta}$

Entropy (ΔS)

$$\Delta S = \int \frac{dQ_{\text{rev}}}{T}$$

Measure of disorder.

Ideal Gas:

$$\Delta S = nC_v \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right)$$

Adiabatic Reversible: $\Delta S = 0$
(Isentropic).

$$\text{Phase Change: } \Delta S = \frac{mL}{T}$$

Free Expansion

$$W = 0, Q = 0, \Delta U = 0, \Delta T = 0$$

Expansion against vacuum ($P_{\text{ext}} = 0$).

Neither isothermal nor
adiabatic in strict sense, but
 $\Delta T = 0$ for ideal gas.

Note from AP Sir:

- Always check units: Pressure in Pa (N/m^2), Volume in m^3 , Temperature in Kelvin (K).
- $R = 8.314 J/(mol \cdot K)$ when using SI units.
- $R = 0.0821 (L \cdot atm)/(mol \cdot K)$ when Pressure is atm and Volume is Liters.