Wave Motion (Mechanical Waves)

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1. Basics of Wave Motion & Progressive Waves

Formula / Topic Name	Formula	Conditions / Usage Notes
General Plane Progressive Wave Equation	$y = A\sin(\omega t \pm kx + \phi)$	y : Displacement, A : Amplitude, ω : Angular freq, k : Propagation constant. **(-) sign**: Wave moving in +x direction. **(+) sign**: Wave moving in -x direction.
Angular Frequency (ω)	$\omega = 2\pi f = \frac{2\pi}{T}$	f : Frequency (Hz), T : Time period.

Propagation Constant (k)	$$k = \frac{2\pi}{\lambda}$$	$$\lambda$$: Wavelength. Represents phase change per unit length.
Wave Velocity (v)	$$v = f\lambda = \frac{\omega}{k}$$	Speed at which the disturbance travels through the medium.
Particle Velocity (v_p)	$$v_p = \frac{\partial y}{\partial t} = \omega A \cos(\omega t \pm kx)$$	Velocity of a particle oscillating about its mean position.
Relation: Particle vs Wave Velocity	$$v_p = -v \times (\text{slope of y-x graph})$$ $$v_p = -v \left(\frac{\partial y}{\partial x}\right)$$	Used to find the direction of particle motion if the wave shape is known.
Particle Acceleration (a_p)	$$a_p = \frac{\partial^2 y}{\partial t^2} = -\omega^2 y$$	Maximum acceleration $$a_{max} = \omega^2 A$$ occurs at extreme positions ($$y = \pm A$$).
Phase Difference ($\Delta\phi$)	$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$ $$\Delta\phi = \frac{2\pi}{T}\Delta t$$	Relation between Phase difference and Path difference (Δx) or Time difference (Δt).

2. Speed of Waves in Media

Formula / Topic Name	Formula	Conditions / Usage Notes

Speed of Transverse Wave on String	$v = \sqrt{\frac{T}{\mu}}$	T : Tension in string, μ : Linear mass density (m/L). **Imp:** Ensure μ is mass per unit length, not volume density.
Speed of Sound (General)	$v = \sqrt{\frac{E}{\rho}}$	E : Modulus of Elasticity, ρ : Density of medium.
Speed in Solids (Rod)	$v = \sqrt{\frac{Y}{\rho}}$	Y : Young's Modulus.
Speed in Fluids (Liquids/Gases)	$v = \sqrt{\frac{B}{\rho}}$	B : Bulk Modulus.
Newton's Formula (Gases)	$v = \sqrt{\frac{P}{\rho}}$	Assumed Isothermal process. (Incorrect historically, gives lower value).
Laplace Correction (Gases)	$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$	Assumed Adiabatic process.

		$\gamma = C_p/C_v$ (Adiabatic index). T: Temp in Kelvin, M: Molar mass.
Effect of Temperature on Sound Speed	$v \propto \sqrt{T}$ $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$	Speed increases with temperature.
Effect of Humidity	$v_{moist} > v_{dry}$	Moist air is less dense than dry air ($\rho_{moist} < \rho_{dry}$), so speed increases.

3. Superposition & Interference

Formula / Topic Name	Formula	Conditions / Usage Notes
Resultant Amplitude (A_{res})	$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$	ϕ is the phase difference between two interfering waves.
Resultant Intensity (I_{res})	$I_{res} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$	Since $I \propto A^2$.

Constructive Interference (Maxima)	Condition: $\phi = 2n\pi$ Path diff $\Delta x = n\lambda$ $A_{max} = A_1 + A_2$ $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$	Where $n = 0, 1, 2, ...$
Destructive Interference (Minima)	Condition: $\phi = (2n - 1)\pi$ Path diff $\Delta x = (2n - 1)\frac{\lambda}{2}$ $A_{min} = \|\|A_1 - A_2$ $I_{min} = (\sqrt{I_1} - \sqrt{I_2})$	Where $n = 1, 2, 3, ...$
Ratio of Intensities	$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$	Useful for questions given amplitude ratio $r = A_1/A_2$.

4. Stationary (Standing) Waves

Formula / Topic Name	Formula	Conditions / Usage Notes
General Equation	$$y = 2A\sin(kx)\cos(\omega t)$$	Or $$y = 2A\cos(kx)\sin(\omega t)$$. Amplitude of particle at $$x$$ is $$A(x) = 2A\sin(kx)$$.
Nodes & Antinodes	**Nodes:** Zero amplitude points. **Antinodes:** Max amplitude points.	Distance between consecutive Node & Node or Antinode & Antinode = $$\lambda/2$$. Distance between Node & Antinode = $$\lambda/4$$.
String Fixed at Both Ends	$$f_n = \frac{nv}{2L} = n\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right)$$	$$n = 1, 2, 3...$$ (Number of loops). $$n = 1$$: Fundamental/1st Harmonic.

		$$n = 2$$: 2nd Harmonic/1st Overtone. All harmonics are present.
String Fixed at One End	$$f_n = \frac{(2n-1)v}{4L}$$	$$n = 1, 2, 3...$$ Only odd harmonics are present ($$f_1, 3f_1, 5f_1...$$).
Sonometer Law	$$f \propto \frac{1}{L}$$, $$f \propto \sqrt{T}$$, $$f \propto \frac{1}{\sqrt{\mu}}$$	Used for comparing frequencies when length or tension changes.

5. Organ Pipes (Sound Columns)

Formula / Topic Name	Formula	Conditions / Usage Notes
Open Organ Pipe	$$f_n = \frac{nv}{2L}$$	Open at both ends. Similar to string fixed at both ends. All harmonics present ($$1 : 2 : 3...$$

		).
Closed Organ Pipe	$$f_n = \frac{(2n-1)v}{4L}$$	Closed at one end. Similar to string fixed at one end. Only odd harmonics present ($$1:3:5...$$).
End Correction (e)	$$e \approx 0.6r$$	$$r$$: Radius of pipe. Antinode forms slightly outside the open end.
Corrected Lengths	Open Pipe: $$L_{eff} = L + 2e$$ Closed Pipe: $$L_{eff} = L + e$$	Use these lengths in frequency formulas for precise calculation.
Resonance Tube	$$v = 2f(L_2 - L_1)$$	$$L_1$$: First resonance length ($\lambda/4$). $$L_2$$: Second resonance length ($3\lambda/4$). Eliminates end correction error.

6. Beats & Doppler Effect

Formula / Topic Name	Formula	Conditions / Usage Notes
Beat Frequency (f_b)	$$f_b = \|f_1 - f_2$$	Number of beats per second. Requires $$f_1 \approx f_2$$.
Tuning Fork Loading/Filing	**Waxing (Loading):** Mass $$\uparrow$$, Freq $$\downarrow$$. **Filing:** Mass $$\downarrow$$, Freq $$\uparrow$$.	Used to determine unknown frequency based on change in beat frequency.
Doppler Effect (General)	$$f' = f_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$$	$$f'$$: Apparent freq, $$f_0$$: Source freq. $$v$$

		: Speed of sound. v_o : Observer velocity. v_s : Source velocity.
Doppler Sign Convention	**Numerator (v_o):** (+) if Observer moves TOWARDS source.	"Towards" tends to increase frequency.
	Denominator (v_s): (-) if Source moves TOWARDS observer.	"Away" tends to decrease frequency.
Effect of Wind (v_w)	$$f' = f_0 \left(\frac{(v \pm v_w) \pm v_o}{(v \pm v_w) \mp v_s} \right)$$	Add v_w to v if wind blows Source $\rightarrow$ Observer. Subtract if wind blows Observer $\rightarrow$ Source.

7. Intensity & Energy Density

Formula / Topic Name	Formula	Conditions / Usage Notes
Intensity (I)	$$I = 2\pi^2 f^2 A^2 \rho v$$	Power per unit area. Depends on square of frequency and amplitude.
Intensity vs Distance	**Point Source:**	Spherical wavefronts vs Cylindrical wavefronts.