# Oscillations (Simple Harmonic Motion)

## BY AP Sir, Sakaar Classes

| Formula / Topic Name | Formula(e) | Conditions / Notes |
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| **1. Standard Equation of SHM** | $\frac{d^2x}{dt^2} + \omega^2 x = 0$ | Differential equation condition for any particle executing SHM. |
| **2. Displacement** | $x = A\sin(\omega t + \phi)$<br>or<br>$x = A\cos(\omega t + \phi)$ | General displacement from mean position.<br>$\phi$: Initial phase (epoch).<br>Use $\sin$ if starts from mean, $\cos$ if from extreme. |
| **3. Angular Frequency** | $\omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$ | $\omega$: Angular frequency (rad/s).<br>Depends on system properties ( $k, m$), not amplitude. |
| **4. Velocity ($v$)** | $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$<br>$v = \pm\omega\sqrt{A^2 - x^2}$ | $v_{max} = A\omega$ (at mean position, $x = 0$).<br>$v_{min} = 0$ (at extreme position, $x = \pm A$). |
| **5. Acceleration ($a$)** | $a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t + \phi)$<br>$a = -\omega^2 x$ | Direction is always towards mean position.<br>$a_{max} = \omega^2 A$ (at extreme).<br>$a_{min} = 0$ (at mean). |

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| **6. Restoring Force** | $F = -kx$ | Linear SHM condition. Force is proportional to displacement and opposite in direction. |
| | $F = -m\omega^2 x$ | |
| **7. Phase Difference** | $\Delta\phi = \phi_2 - \phi_1$ | Time diff $\Delta t = \frac{T}{2\pi}\Delta\phi$. |
| | | Path diff $\Delta x = \frac{\lambda}{2\pi}\Delta\phi$ (Wave context). |
| **8. Kinetic Energy (KE)** | $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$ | Max at mean position ( $K_{max} = \frac{1}{2}kA^2$). |
| | $K = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$ | Zero at extreme position. |
| **9. Potential Energy (PE)** | $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$ | Assuming $U = 0$ at mean position. |
| | $U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$ | Max at extreme ($U_{max} = \frac{1}{2}kA^2$). |
| **10. Total Energy (TE)** | $E = K + U = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$ | TE is constant (conserved) in undamped SHM. |
| | | $E \propto A^2$ and $E \propto f^2$. |
| **11. Average Energies** | $< K >_{cycle} = < U >_{cycle} = \frac{1}{4}kA^2$ | Over one complete cycle of oscillation. |
| | $< E >_{cycle} = \frac{1}{2}kA^2$ | |
| **12. Spring-Mass System (Horizontal/Vertical)** | $T = 2\pi\sqrt{\frac{m}{k}}$ | Period is independent of $g$ and amplitude. |
| | | $k$: Spring constant. |
| **13. Springs in Series** | $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$ | End-to-end connection. Force is same, extension adds up. |
| **14. Springs in Parallel** | $k_{eq} = k_1 + k_2 + \ldots$ | Side-by-side or mass between two fixed springs. Extensions are same. |

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| **15. Cutting a Spring** | $k \cdot l = \text{constant} \implies k \propto \frac{1}{l}$ | If spring of length $l$ is cut into $n$ equal parts, stiffness of each part becomes $nk$. |
| **16. Two Block System (Reduced Mass)** | $T = 2\pi\sqrt{\frac{\mu}{k}}$ | Where reduced mass $\mu = \frac{m_1 m_2}{m_1+m_2}$. |
| | | Blocks oscillate relative to center of mass. |
| **17. Simple Pendulum** | $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$ | For small angular amplitude ( $\theta < 5°$). |
| | | $l$: Length from pivot to CM of bob. |
| **18. Pendulum in Lift** | **Accelerating Up:** $g_{eff} = g + a$ | If lift falls freely ($a = g$), $g_{eff} = 0$, $T \to \infty$ (No oscillation). |
| | $T = 2\pi\sqrt{\frac{l}{g+a}}$ | |
| | **Accelerating Down:** $g_{eff} = g - a$ | |
| | $T = 2\pi\sqrt{\frac{l}{g-a}}$ | |
| **19. Pendulum in Truck/Car** | $g_{eff} = \sqrt{g^2 + a^2}$ | Truck moving horizontally with acceleration $a$. Mean position shifts by $\tan\theta = a/g$. |
| | $T = 2\pi\sqrt{\frac{l}{(g^2+a^2)^{1/2}}}$ | |
| **20. Pendulum with Charged Bob** | $g_{eff} = g + \frac{qE}{m}$ (E field down) | Electric field $E$ applied vertically. |
| | $g_{eff} = g - \frac{qE}{m}$ (E field up) | |
| **21. Pendulum of Infinite Length** | $T = 2\pi\sqrt{\frac{1}{g(\frac{1}{l}+\frac{1}{R_e})}}$ | If $l \approx R_e$ (Earth's radius). |
| | | If $l \to \infty$, $T = 2\pi\sqrt{\frac{R_e}{g}} \approx 84.6 \text{ min.}$ |

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| **22. Second's Pendulum** | $T = 2 \text{ seconds}$ | Length $l \approx 0.993 \text{ m}$ (on Earth). |
| **23. Physical Pendulum** | $T = 2\pi\sqrt{\frac{I}{mgd}}$ | $I$: Moment of Inertia about pivot. |
| | | $d$: Distance between pivot and Center of Mass. |
| **24. Torsional Pendulum** | $T = 2\pi\sqrt{\frac{I}{C}}$ | $I$: MOI of disc/body. |
| | | $C$: Torsional constant ($Nm/rad$) of the wire. |
| **25. Superposition (Same freq)** | $x_{res} = A\sin(\omega t + \theta)$ | $\delta$: Phase difference between two waves. |
| | $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$ | $\tan\theta = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$ |
| **26. Liquid in U-Tube** | $T = 2\pi\sqrt{\frac{h}{g}}$ or $T = 2\pi\sqrt{\frac{L}{2g}}$ | $h$: Height of liquid column in one arm at equilibrium. |
| | | $L$: Total length of liquid column. |
| **27. Body Floating in Liquid** | $T = 2\pi\sqrt{\frac{m}{A\rho g}} = 2\pi\sqrt{\frac{h_{submerged}}{g}}$ | $A$: Cross-sectional area. $\rho$: Density of liquid. |
| | | Slightly depressed and released. |
| **28. Tunnel through Earth** | $T = 2\pi\sqrt{\frac{R_e}{g}} \approx 84.6 \text{ min}$ | Particle dropped in a tunnel along diameter or chord. (Assuming uniform density). |
| **29. Ball in Concave Dish** | $T = 2\pi\sqrt{\frac{R-r}{g}}$ | $R$: Radius of curvature of dish. $r$: Radius of ball. |
| | | For small oscillations ($r \ll R$, $T \approx 2\pi\sqrt{R/g}$). |
| **30. Piston in Cylinder** | $T = 2\pi\sqrt{\frac{mV}{PA^2}}$ | $V$: Volume, $P$: Pressure, $A$: Area of piston. |

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| | | (Adiabatic process usually considered, add factor $\gamma$ in denominator for adiabatic). |
| **31. Amplitude with** | $A = A\, f^n$ where $f < 1$ | If amplitude decays by a constant |