

# ELASTICITY

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Formula Name / Topic	Formula	Condition / Context / Use Case
Normal Stress ( $\sigma$ )	$\sigma = \frac{F_{\perp}}{A}$	Restoring force ( $F_{\perp}$ ) acting per unit area perpendicular to the cross-section.
Tangential / Shear Stress ( $\sigma_t$ )	$\sigma_t = \frac{F_{\parallel}}{A}$	Force ( $F_{\parallel}$ ) acting parallel to the surface area. Causes shape change without volume change.
Longitudinal Strain ( $\varepsilon_l$ )	$\varepsilon_l = \frac{\Delta L}{L}$	Change in length per unit original length (Tensile or Compressive).
Shearing Strain ( $\phi$ )	$\phi \approx \tan \phi = \frac{x}{L}$	Relative displacement ( $x$ ) between parallel layers separated by distance $L$ .
Volumetric Strain ( $\varepsilon_v$ )	$\varepsilon_v = -\frac{\Delta V}{V}$	Change in volume per unit original volume. Negative sign indicates decrease in volume with pressure increase.
Hooke's Law	<p>Stress <math>\propto</math> Strain</p> <p>Stress = <math>E \times</math> Strain</p>	Valid only within the <b>Proportional Limit</b> . $E$ is the Modulus of Elasticity.
Young's Modulus ( $Y$ )	<p><math>Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}</math></p> <p><math>Y = \frac{FL}{A\Delta l} = \frac{mgL}{\pi r^2 \Delta l}</math></p>	Used for solids (wires, rods) undergoing length change. Specific for a material.
Bulk Modulus ( $B$ or $K$ )	$B = \frac{-P}{\Delta V/V} = -V \frac{\Delta P}{\Delta V}$	Relates volume change to pressure change. Applicable to solids, liquids, and gases.
Compressibility ( $K$ )	$K = \frac{1}{B}$	Reciprocal of Bulk Modulus.

**Modulus of Rigidity / Shear Modulus ( $\eta$  or  $G$ )**

$$\eta = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F}{A\phi}$$

Resistance to change in shape.  
Only for solids.

**Poisson's Ratio ( $\sigma$ )**

$$\sigma = -\frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

Theoretical limits:  $-1$  to  $0.5$ .

$$\sigma = \frac{\Delta D/D}{\Delta L/L}$$

Practical limits:  $0$  to  $0.5$ .

**Work Done in Stretching (Strain Energy  $U$ )**

$$U = \frac{1}{2} \times F \times \Delta l$$

Total potential energy stored in a stretched wire.

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Volume}$$

**Energy Density ( $u$ )**

$$u = \frac{1}{2} \times \text{stress} \times \text{strain}$$

Energy stored per unit volume.

$$u = \frac{1}{2} Y (\text{strain})^2 = \frac{(\text{stress})^2}{2Y}$$

**Elongation due to Self Weight**

$$\Delta l = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

Extension of a hanging rod/wire due to its own gravity.  $M$ =mass,  $\rho$ =density. Note the factor **2** in denominator (acts at Center of Mass).

**Thermal Stress**

$$\sigma_{\text{thermal}} = Y\alpha\Delta T$$

Rod fixed between rigid supports.

$$\text{Force } F = Y A \alpha \Delta T$$

$\alpha$  = coeff. of linear expansion,  
 $\Delta T$  = temp change.

**Analogy with Spring Constant ( $k$ )**

$$k = \frac{YA}{L}$$

Treating a wire as a spring ( $F = kx$ ). Useful for series/parallel combination of wires.

**Wires in Series**

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Composite wire with same Force/Tension acting on both segments.

$$\Delta l_{net} = \Delta l_1 + \Delta l_2$$

<b>Wires in Parallel</b>	$k_{eq} = k_1 + k_2$	Composite wire where extensions are forced to be equal ( $\Delta l_1 = \Delta l_2$ ).
	$F_{net} = F_1 + F_2$	
<b>Interatomic Force Constant (<math>k_a</math>)</b>	$k_a = Y \times r_0$	$r_0$ is the equilibrium interatomic distance.
<b>Depression of a Beam (Cantilever)</b>	$\delta = \frac{WL^3}{3YI_g}$	Beam fixed at one end, loaded ( $W$ ) at the other. $I_g$ is Geometrical Moment of Inertia.
<b>Depression of Beam (Supported at ends)</b>	$\delta = \frac{WL^3}{48YI_g}$	Beam supported at both ends, load $W$ in the center.
<b>Torsion of a Cylinder</b>	$C = \frac{\pi\eta r^4}{2L}$	Restoring couple per unit twist (Torsional rigidity).
<b>Breaking Stress</b>	Breaking Force = Breaking Stress $\times A$	Breaking stress depends on material, not dimensions. Breaking Force depends on area.
<b>Relation: <math>Y, B, \sigma</math></b>	$Y = 3B(1 - 2\sigma)$	Relates Young's, Bulk Modulus and Poisson's ratio.
<b>Relation: <math>Y, \eta, \sigma</math></b>	$Y = 2\eta(1 + \sigma)$	Relates Young's, Rigidity Modulus and Poisson's ratio.
<b>Relation: <math>Y, B, \eta</math></b>	$\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$	Useful when $\sigma$ is not given.
<b>Relation: <math>\sigma</math> in terms of <math>B, \eta</math></b>	$\sigma = \frac{3B - 2\eta}{6B + 2\eta}$	Calculation of Poisson's ratio from moduli.

### Quick Tips for Numerical Questions (NEET/JEE)

- Wire Cut into n parts:** If a wire of Young's Modulus  $Y$  is cut into  $n$  equal parts, the Young's Modulus of each part remains  $Y$  (Material property), but the spring constant becomes  $nk$ .
- % Change questions:** If strain is small ( $< 5\%$ ), use  $\frac{\Delta R}{R} \times 100$ . For volume of wire  $V = A \times L$  (constant),  $\frac{\Delta A}{A} = -\frac{\Delta L}{L}$  (ignoring  $\sigma$  effects for simple resistance type q's) or use conservation of volume  $A_1 L_1 = A_2 L_2$ .
- Adiabatic vs Isothermal Modulus:**