How are Power Factor and Efficiency Connected?

A. De Francesco and A. Pendharkar

Abstract: Two passive grid-to-customer networks representing lagging and leading Power Factor (PF) are presented to explore the relationship between PF and efficiency. Equations for PF and efficiency are stated for each network. The PF and efficiency metrics are graphed versus frequency and examined for various scenarios. Efficiency, although limited by transmission resistance and due to the resulting i^2R loss, is maximised when the reactive components vanish, but not necessarily unity. The inclusion of the transmission resistance introduces a dip in the PF frequency response but also improves the system PF at the desired frequency.

Power Factor (PF)

$$PF = \frac{P}{S} = \frac{P}{V_{rms}I_{rms}} = \frac{P}{\sqrt{P^2 + Q^2}} \dots (1)$$

Power Factor at the fundamental frequency is normally referred to as Power Factor (PF) or Displacement Power Factor (DPF). True Power Factor (TPF) includes harmonic frequencies. In the general case PF can be calculated using equation (1) where:

- P is the average power, also referred to as real/active power;
- Q is the reactive power, also referred to as imaginary power; and
- S is the apparent power and given by the product of the RMS voltage and RMS current.

Alternatively, the apparent power can be expressed as the square root of the square of P and Q since they are 90 degrees apart.

What does PF physically represent?

The equation itself indicates that any system requires active and reactive power, but only active power generates useful work. So, in effect the PF is a measure of how much useful power can be extracted from the system. Power that is not extracted for useful work is being stored within the system and therefore not made available. No power is wasted, just reallocated. The PF can range between -1 and 1 since average power can be negative, a scenario that is becoming more prominent with renewables and islanded micro-grids as power is fed back upstream to the grid.

Displacement Power Factor (DPF)

$$DPF = \cos(\theta) \dots (2)$$

In passive circuits the DPF can be expressed as the cosine of the angle between the voltage and current as stated in equation (2) [1]. The cosine of a positive or negative angle in the first quadrant always generates a positive PF. PF is said to be 'lagging' if the voltage phasor leads the current phasor (inductive circuit) and 'leading' if the current phasor leads the voltage phasor (capacitive circuit).

Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \dots (3)$$

Efficiency is the ratio of output average power to input average power and is given by equation (3). The efficiency metric provides an indication of how much of the extracted average power is put to work or consumed.

Application example: grid connected to passive equipment

Figure 1 shows two passive networks connected to the grid. The network on the left-hand side shows a parallel L-R load with lagging PF. The network on the right-hand side shows a parallel C-R load with a leading PF. In both networks R_1 represents the active load whilst R_2 represents the transmission resistance of the distribution network connecting the grid to the customer equipment. The inductive and capacitive elements of the transmission line are omitted. The PF is measured at the generator and the efficiency is the ratio of the power dissipated by R_1 to the total input power from the generator and includes R_2. The two networks are analysed and the generalised equations for PF and efficiency are shown in Table 1, explicitly in terms of circuit components. As expected, the equations are frequency dependent and provide insight into how the PF and efficiency are connected.

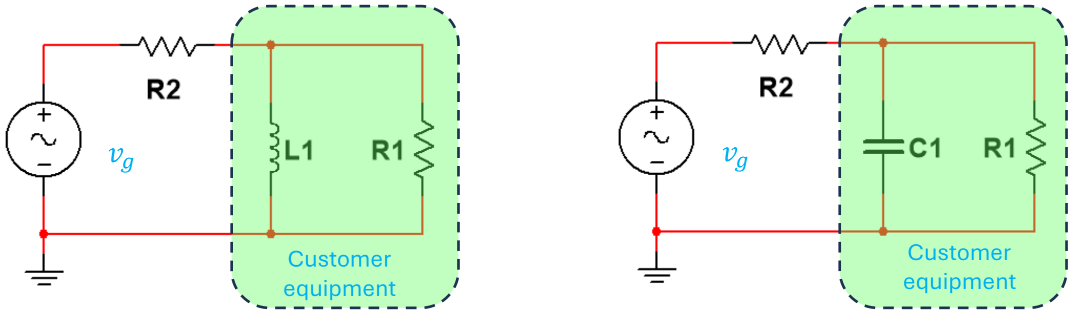

Figure 1: Grid connected to passive equipment, (left) lagging PF, (right) leading PF

PF lagging	PF leading
$PF = \dfrac{1}{\sqrt{1 + \dfrac{(\omega L_1)^2}{\left\{R_2 + \frac{1}{R_1}\left(1 + \frac{R_2}{R_1}\right)(\omega L_1)^2\right\}^2}}} \ldots (4a)$	$PF = \dfrac{1}{\sqrt{1 + \dfrac{(\omega C_1)^2}{\left\{\left(1 + \frac{R_2}{R_1}\right)\frac{1}{R_1} + R_2(\omega C_1)^2\right\}^2}}} \ldots (5a)$
$\eta = \dfrac{1}{\frac{R_1 R_2}{(\omega L_1)^2} + \left(1 + \frac{R_2}{R_1}\right)} \ldots (4b)$	$\eta = \dfrac{R_1}{R_1 + R_2\{1 + (\omega C_1 R_1)^2\}} \ldots (5b)$

Table 1: PF and efficiency equations – grid connected to passive equipment

Case 1: No transmission resistance, $R_2 = 0$

This is an ideal and unrealistic condition but merely serves as a reference. Table 2 shows the simplified equations. Note the efficiency tends to unity in both networks and is frequency independent. The PF is now solely dependent upon R_1 and the reactive element in the circuit.

Case 1	PF lagging	PF leading
$R_2 = 0,$ $\eta = 1$	$PF = \dfrac{1}{\sqrt{1 + \left(\frac{R_1}{\omega L_1}\right)^2}} = \dfrac{1}{\sqrt{1 + \left(\frac{R_1}{X_L}\right)^2}}$	$PF = \dfrac{1}{\sqrt{1 + (\omega C_1 R_1)^2}} = \dfrac{1}{\sqrt{1 + \left(\frac{R_1}{X_C}\right)^2}}$

Table 2: PF and efficiency equations – grid connected to passive equipment with $R_2 = 0$

Equations in Table 2 indicate that the PF is approximately unity if the ratio of R_1 to the reactive impedance is much lower than one. Practically, this condition implies that the power stored in the reactive element is much less than the power consumed by the load. Equations in Table 2 are utilised to calculate the values of the reactive elements at the desired frequency, 50Hz, at the required PF, and graphed in Figure 2, refer to scenarios 1 to 3.

As expected, the efficiency is unity at all frequencies. The PF in the lagging network is initially zero and increases with frequency, tending to unity as less grid current flows into the inductor. Alternatively, the PF in the leading network is initially unity and tends towards zero as more grid current is shunt by the capacitor. The scenarios show that increasing the inductance and decreasing the capacitance in the respective networks not only flattens the PF curves but are desirable in achieving unity PF.

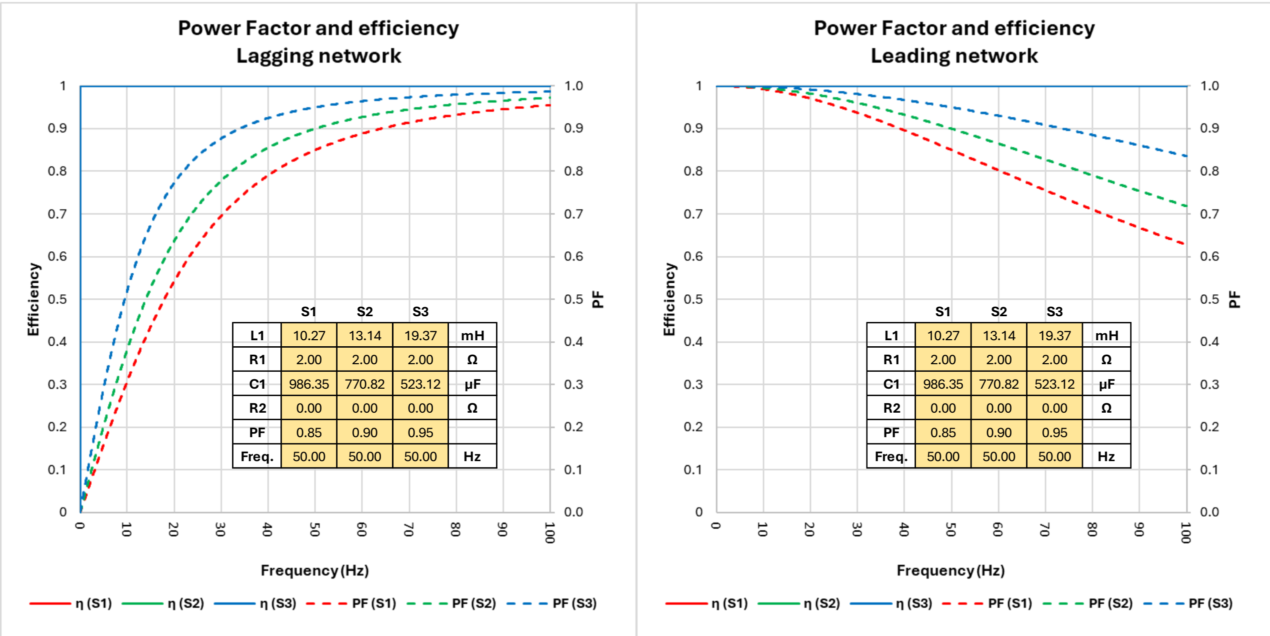

Figure 2: PF and efficiency curves for lagging (left) and leading (right) networks with R_2 set to zero

Figure 2 also highlights that there appears to be a frequency above (for lagging) or below (for leading) where the PF is essentially unity, giving rise to the concept of cut-off PF. This feature would be useful in the context of harmonics.

Case 2: Perfect reactive compensation, $L_1 = \infty$ and $C_1 = 0$

Table 3 shows the simplified equations. In both networks the PF tends to unity and the efficiency is now dependent solely on R_1 and R_2 and independent of frequency. Therefore, efficiency is not necessarily unity with unity PF.

Case 2	PF lagging and PF leading
$L_1 = \infty,\ C_1 = 0,\ PF = 1$	$\eta = \frac{R_1}{R_1 + R_2}$

Table 3: PF and efficiency equations – grid connected to passive equipment with $L_1 = \infty$ and $C_1 = 0$

Case 3: Assume a finite transmission resistance, $R_2 \neq 0$

The equations in Table 1 are graphed in Figure 3 for various scenarios with finite values of R_2.

The efficiency and PF curves for the lagging and leading networks are depicted with solid and dashed traces respectively. At DC (zero frequency) the inductor is a short circuit and no power is delivered to R_1 whilst the capacitor is an open circuit allowing the most power to be delivered to R_1, resulting in zero efficiency for the lagging network and the highest efficiency for the leading network. The PF in the lagging network is unity since all the power is dissipated through R_2. The PF in the leading network is also unity because all the power is delivered to R_1 and R_2.

At frequencies higher than 50Hz, the opposite occurs, the inductor becomes an open circuit allowing the most power to be delivered to R_1 whilst the capacitor becomes a short circuit and no power delivered to R_1, resulting in the highest efficiency for the lagging network and zero efficiency for the leading network.

The PF in the lagging network approaches unity as all the power is delivered to R_1 and R_2. The PF in the leading network also approaches unity as all the power is delivered to R_2.

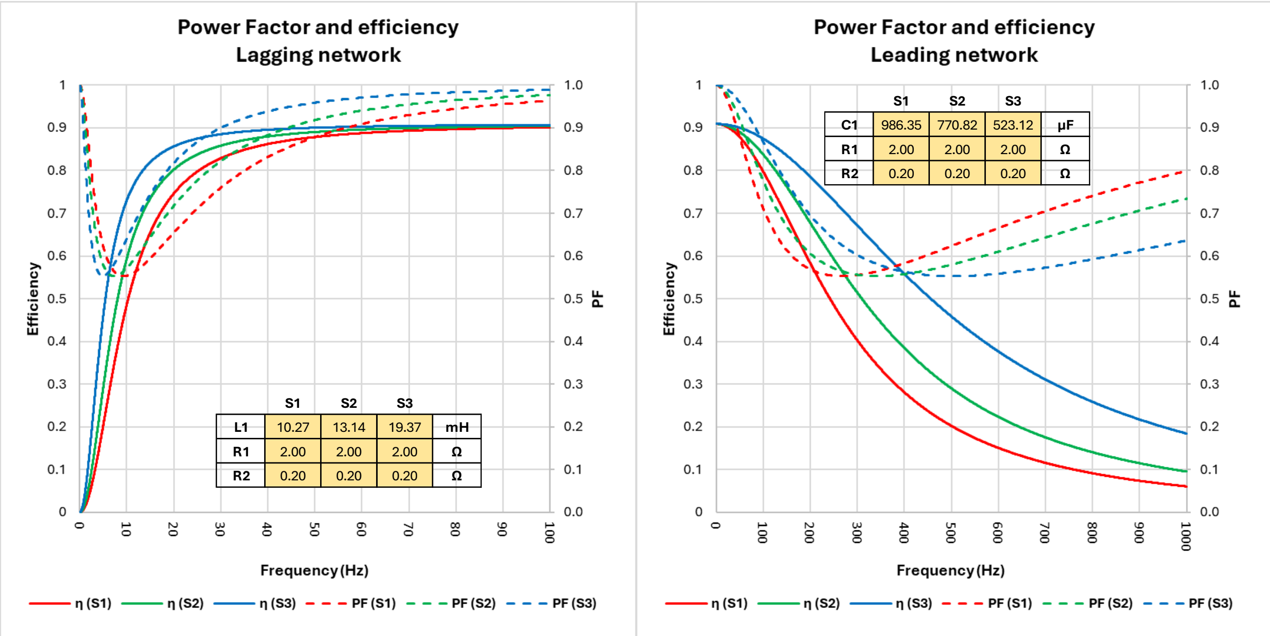

Figure 3: PF and efficiency curves for lagging (left) and leading (right)networks

The PF and efficiency at 50Hz are summarised in Table 4.

Item	PF lagging			PF leading		
Scenario	S1	S2	S3	S1	S2	S3
Efficiency	0.878	0.890	0.900	0.879	0.890	0.900
PF (set values)	0.850	0.900	0.950	0.850	0.900	0.950
PF	0.878	0.918	0.959	0.879	0.919	0.959

Table 4: PF and efficiency values at 50Hz [Figure 3]

It is noteworthy that the efficiency has dropped as expected, but the PF has increased from the set values in Figure 2. This is because the inclusion of R_2 generates additional dissipated power due to the grid current [${i_g}^2 R_2$] and improves the resulting PF. Note, the grid current is composed of real and imaginary components and both contribute to the heating of R_2. Also, the inclusion of R_2 introduces a dip in the PF frequency response for both networks and all at the same depth of 0.553 but occurring at different locations, [9.34, 7.30, 4.96] Hz and [267.6, 342.4, 504.5] Hz of the lagging and leading networks respectively. The depth of the dip is proportional to the ratio of R_1 to R_2. As the ratio increases the dip drops to resemble the PF curves given in Figure 2.

To further investigate the dip, the phase angle of the grid current was graphed (not shown) and found that it too exhibited a minima or maxima angle of -/+56.443 degrees at the location of the dip for both lagging and leading networks. Using equation (2) the cosine of either angle yields the minimum PF of 0.553.

Concluding remarks

While PF and efficiency are related, they are not the same metric. A high PF does not always indicate high efficiency.

References

[1] H. Goodman, "Circuit theory and techniques, Volume 1", page 232, John Wiley and Sons, Brisbane, New York, Chichester, Toronto, Singapore, 1985.