Calculation of Neutral current in a four-wire three phase system

A. De Francesco and A. Pendharkar

Abstract: A process for calculating the neutral current is described and from which a set of equations for calculating the magnitude and phase angle of the neutral current are stated under various conditions and compared. A small error in the phase angle can significantly impact the magnitude of the neutral current. Using power quality data, the equations were utilised to calculate the neutral current at the fundamental and compared against the measured neutral current. Better agreement between the calculated and measured neutral currents occurred when the phase angles of the phase currents were utilised.

Practical power quality monitoring

Consider three phase equipment connected to a 4 wire 'Y' three phase system as shown in Figure 1. A Power Quality Analyser (PQA) is connected to measure phase voltage v_{an}, v_{bn}, v_{cn}, phase currents i_a, i_b, i_c, and neutral current i_n. The current through the Earth wire i_g (also commonly referred to as the ground wire) is a result of leakage and is usually insignificant under no fault condition and therefore not monitored. In any case the net currents entering and exiting the equipment must sum to zero and includes the ground current.

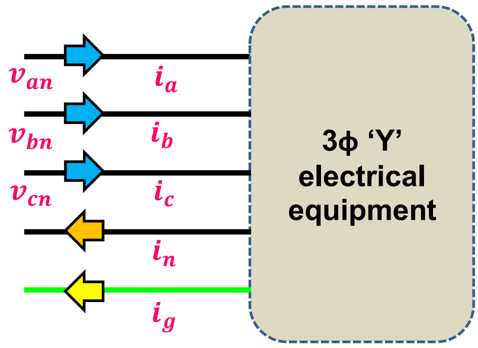

Figure 1: Connection of three phase equipment

Clients who have approached us for PQ data analysis have often unintentionally skipped configuring the neutral current measurement, leading to no neutral data. The neutral current can provide useful PQ information both at the fundamental and harmonic frequencies. In this work the scope it limited only to the fundamental frequency.

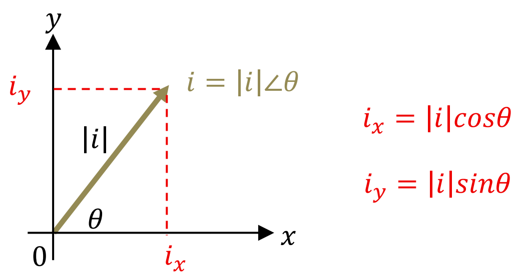

Figure 2: Decomposition of a phasor

Calculation of neutral current using vectors

A current phasor can be decomposed into its x and y components as shown in Figure 2. Now consider three current phasors given as $i_a = |i_a|\angle\theta_a$, $i_b = |i_b|\angle\theta_b$, and $i_c = |i_c|\angle\theta_c$. The neutral current can be calculated according to the process summarised in Table 1.

Phasor	i_x	i_y
i_a	$i_{ax} = \|i_a\|cos\theta_a$	$i_{ay} = \|i_a\|sin\theta_a$
i_b	$i_{bx} = \|i_b\|cos\theta_b$	$i_{by} = \|i_b\|sin\theta_b$
i_c	$i_{cx} = \|i_c\|cos\theta_c$	$i_{cy} = \|i_c\|sin\theta_c$
i_n	$i_{nx} = i_{ax} + i_{bx} + i_{cx}$	$i_{ny} = i_{ay} + i_{by} + i_{cy}$
$\|i_n\| = \sqrt{{i_{nx}}^2 + {i_{ny}}^2}, \quad \theta_n = tan^{-1}\left(\frac{i_{ny}}{i_{nx}}\right)$		

Table 1: Process for calculating the neutral current

Step 1: Decompose each phasor into its x and y components.

Step 2: Sum all x and y components of each phasor to give the resultant x and y vector of the neutral current.

Step 3: Reconstruct the magnitude and angle of the neutral phasor.

Calculation of neutral current using equations

Case A: General equations

Substituting expressions provided in Step 1 into Step 2 and then further substituting into Step 3 and simplifying yields equations (1a) and (1b) which respectively describe the magnitude and phase angle of the neutral current in terms of the magnitude and phase angle of the current phasors i_a, i_b, and i_c.

$$|i_n| = \{|i_a|^2 + |i_b|^2 + |i_c|^2 + 2|i_a||i_b|\cos(\theta_a - \theta_b) + 2|i_b||i_c|\cos(\theta_b - \theta_c) + 2|i_c||i_a|\cos(\theta_c - \theta_a)\}^{\frac{1}{2}} \ldots (1a)$$

$$\theta_n = tan^{-1}\left(\frac{|i_a|\sin\theta_a + |i_b|\sin\theta_b + |i_c|\sin\theta_c}{|i_a|\cos\theta_a + |i_b|\cos\theta_b + |i_c|\cos\theta_c}\right) \ldots (1b)$$

Equations (1a) and (1b) provide the exact result and assumes steady state sinusoidal conditions and no measurement errors (perfect data), in reality current sensors introduce errors. The equations are referred to as the general equations but can be simplified further into Case B and C.

Case B: Unbalanced magnitudes with balanced complex loads

With balanced complex loads, the angles of the phase currents i_a, i_b, and i_c are displaced from each other by 120° and imply a constant displacement angle, θ_d such that $\theta_a = 0^\circ + \theta_d$, $\theta_b = -120^\circ + \theta_d$ and $\theta_c = 120^\circ + \theta_d$. Equation (1a) simplifies to equation (2a). Equation (1b) simplifies to equation (2b) only in the case where θ_d is zero, otherwise use equation (1b).

$$|i_n| = \sqrt{|i_a|^2 + |i_b|^2 + |i_c|^2 - |i_a||i_b| - |i_b||i_c| - |i_c||i_a|} \ldots (2a)$$

$$\theta_n = tan^{-1}\left(\frac{\sqrt{3}\{-|i_b| + |i_c|\}}{2|i_a| - |i_b| - |i_c|}\right), \theta_d = 0^\circ \ldots (2b)$$

Case C: Balanced magnitudes with balanced complex loads

By inspection of equation (2a) the neutral current vanishes ($|i_n|$=0) when the magnitudes of all phase currents are equal, this outcome is widely documented in many textbooks.

Comparing neutral current magnitudes of Cases A and B

Table 2 shows the error in the neutral current magnitude between (1a) and (2a) with a 1-degree change in the reference angle $\Delta\theta$, over a variation of one fundamental cycle and according to the prescription given. Note that the phase angle prescription intentionally violates the rule given for Case B and invalidates the use of (2a). Scenarios [s1-s4] show the impact with different levels of current unbalance of 1%, 2%, 5% and 10%. The key takeaway is that the error is highly sensitive to changes in phase angle when the magnitudes are closely balanced. The more unbalanced a system is, the less the phase error impacts the total measurement.

	ia	ib	ic	Error @ Δθ=1
ΔMag. (%) [s1]	0	1	-1	50%
ΔMag. (%) [s2]	0	2	-2	24%
ΔMag. (%) [s3]	0	5	-5	4.70%
ΔMag. (%) [s4]	0	10	-10	0.49%
phase	0	-(120+Δθ)	(120+Δθ)	
Error=100x(IN (Case A)-IN (Case B))/IN (Case A)				

Reference magnitude set to unity.

Table 2: Comparing errors between Case A and Case B

Application to Power Quality data

Figure 3 shows magnitudes and angles of phase currents at the fundamental frequency, measured with 500A CT probes using a PQA. The phase angles for each phase are reasonably constant and approximately displaced by 120° with average deviation from the reference values of $(-15^{\circ}, 13.2^{\circ}, 22.6^{\circ})$. Since the phase deviation for each phase is different equation (1a) should be utilised.

Equations (1a), (1b) and (2a) are applied to the phase currents in Figure 3 with the resulting neutral currents graphed in Figure 4 together with the measured neutral current. All Neutral currents are rms averaged over the logging period and changes compared with respect to |IN|. Magnitude profiles of the measured and calculated neutral current, |IN| and |IN (Case A)| are in close agreement with an error of -1.27% whereas the magnitude profile of the calculated neutral current |IN (Case B)| is notably different from that of the measured neutral current with an error of -36.97%. The negative errors indicate that the measured neutral current is higher in magnitude. The large disparity in the errors highlights the sensitivity to the phase angle and therefore the importance of including the phase angle. The calculated neutral current phase INϕ (Case A) is in reasonable agreement with the measured neutral current phase.

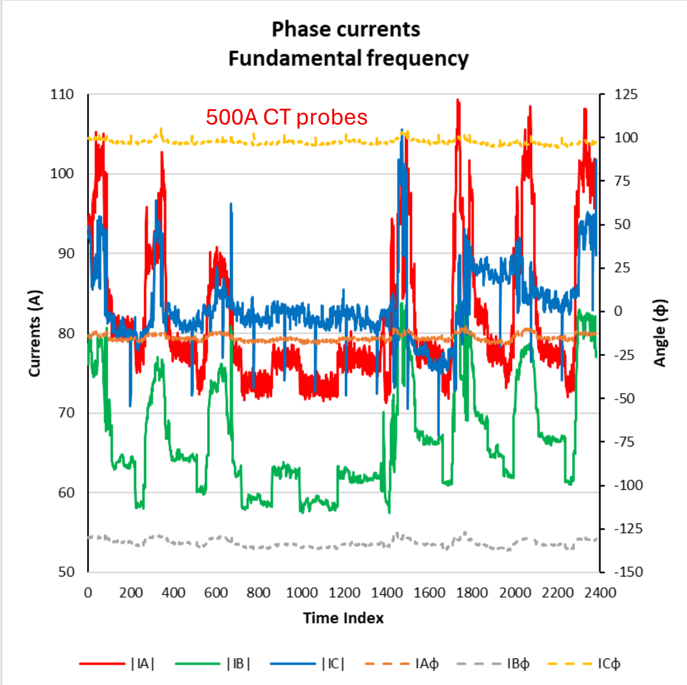

Figure 3: Phase currents – fundamental frequency

Current measurement errors

CT probes and Rogowski Coils (RC) are two current sensors that are commonly utilised with PQAs. They both exhibit ratio and phase errors, but the latter is (i) very sensitive to the location of the current flowing through it and hence yields a positional error that can be higher than the rated accuracy of the RC; and (ii) the voltage developed from the RC at low currents can be in μV resulting in a low signal-to-noise ratio, introducing further errors in the measurement.

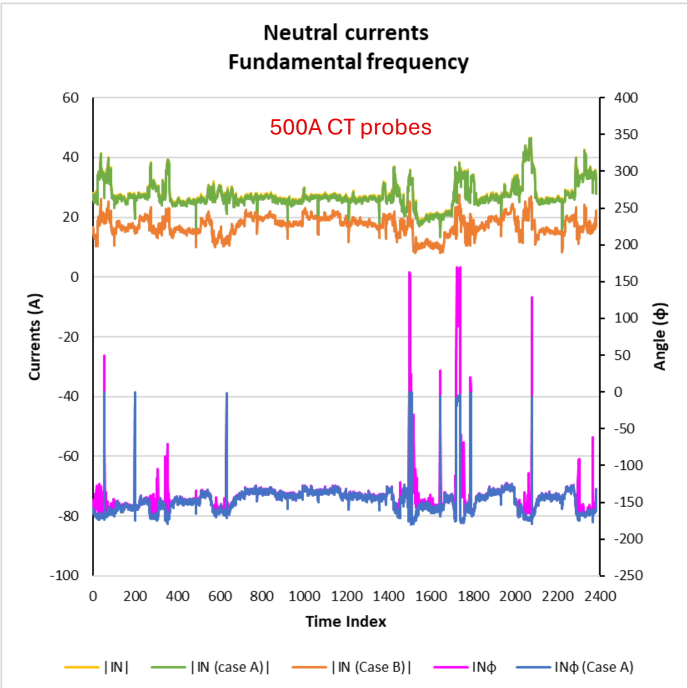

Figure 4: Neutral currents – fundamental frequency

In many PQ investigations, and for the data shown in Figures 3 and 4, for convenience the same current sensors are utilised for all current channels. Note, that the maximum neutral current is approximately 9% of the rated current of the current sensor and may yield higher than expected errors.