Course Objective:

The objective of this course is to introduce students to L^pspaces,Banach Spaces, Hilbert Spaces etc. Students will also be exposed to bounded linear operators on Hilbert spaces which is required to study quantum mechanics, scattering theory and spectral theory, etc. Knowledge of real Analysis, measure theory and linear algebra is pre requisite for this course

Learning Outcomes:

After completing the course the student will be able to

- Handle inequalities in L^pspaces, and normed linear spaces.
- Learn all basic results on Hilbert spaces for further application.
- Know on Fourier series with respect to an orthonormal basis and related results and basic results of Banach space
- Know on more results on Banach spaces andbounded linear operators with spectrum on Banach spaces.

Unit I

Review of Metric spaces (not a part of examination), L^p andl^pspaces, inequalities (Holder, Minkowski, Jensen), completeness of Lp, denseness and separability, normed linear spaces, properties of normed linear spaces, continuity of linear maps.

Unit –II

Inner product spaces, examples, Hilbert spaces, examples, closed subspaces, existence of a unique element of smallest norm, orthogonal complements and properties, projection theorem, Riesz representation theorem, orthonormal sets, Gram-Schimdtorthonormalization.

Unit –III

Orthonormal basis, Fourier expansion, Bessel's inequality, Riesz-Fischer theorem, Parseval's formula, Banach spaces, examples, Hahn Banach theorem, Baire's category theorem.

Unit IV

Open mapping theorem, closed graph theorem, uniform boundedness principle,

duals of $L^p[a, b]$, bounded linear operators on Banach spaces, spectrum ofaboundedoperators and properties,resolventset and examples.

Books Recommended:

- ✓ *B. V. Limaye-FunctionalAnalysis,3rdEd,2014.*
- ✓ *E. Kreyszig-Functional Analysis –Wiley-India, 2007.*

Books for Reference

- ✓ Goffman and Pedrick A first Course in Functional Analysis, AMS, 2017.
- ✓ J. B. Conway, A course in Functional Analysis, 2nd Ed., Springer, 2006
- ✓ P. K. Jain and O. P. Ahuja, Functional Analysis, 2nd Ed., New Age International Publication, New Delhi, 2004
- ✓ Markus Haase, Functional Analysis: An Elementary Introduction, American Mathematical Society, 2014.
- ✓ Suggested digital platform: NPTEL/SWAYAM/MOOCs.
- ✓ e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org

Core XXI

Analytic Number Theory

Course Objectives:

The aim of this course is to study number theory by using analytic tools such as inequalities, limits, calculus, etc.

Learning Outcomes:

After completing the course the student will be able to

- Understand the arithmetical functions and their relations.
- Find the average order of multiplicative functions and know the distribution of prime numbers.
- Know the prime number theorem and Ramanujan's sum
- Know the basic theory of Riemann zeta function and related L-function.

Unit I

The arithmetical functions and their relations, Mobius function, Euler totient function, Mangolt function, Liouville's function, The, divisor function, The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, multiplicative functions. The Bell series of an arithmetical function and Dirichlet multiplications, Derivatives of arithmetical functions, The Selberg identity.

Unit II

The big oh notation, Euler's summation formula, some elementary asymptotic formulas, averages of arithmetical functions, The average order of divisor functions, The average order of Euler totient function, The average order of Mobious and Mangoldt functions, The partial sums of a Dirichlet product, applications to the Mobius and Mangoldt functions, some elementary theorems on distribution of prime numbers, Chebyshev's functions and their relations with $\pi(x)$.

Unit III

Some equivalent forms of the prime number theorem, Shapiro Tauberian theorem, The partial sums of the Mobius function, brief sketch of an elementary proof of the prime number theorem, Ramanujan's sum and generalizations, quadratic residues, Legendre's symbol and its properties, Gauss's lemma, The quadratic reciprocity law, The Jacobi symbol**.**

Unit IV

The half-plane of absolute convergence of a Dirichlet series, Euler products, analytic

properties of Dirichlet series, mean value formulas for Dirichlet series, an integral formulas for the coefficients and the partial sums of a Dirichlet series, The Riemann zeta function and the L-function, properties of the gamma function, integral representation for the Hurwitz zeta function, analytic continuation of the Hurwitz zeta function, analytic continuation of the Riemann zeta function and the L-function.

Books Recommended:

- ✓ *T. M. Apostol, Introduction to Analytic Number Theory, Springer International Edition, 2010.*
- ✓ *Analytic Number Theory: Exploring the Anatomy of Integers, Jean-Marie De Koninck, Florian Luca, American Mathematical Society, 2012.*

Books For Reference:

- ✓ *A Primer of Analytic Number Theory: From Pythagoras to Riemann, Jeffrey Stopple, Cambridge University Press, 2003.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Core XXII

Complex Analysis-II

Course Objectives:
This course introduces the basic concepts of conformal mappings, entire functions, Weierstrass infinite products, Hadamard's factorization theorem, Gamma function, Zeta function and normal family.

Learning Outcomes: On the completion of this course, students will be able to

- Solve problems involving conformal mappings.
- Understand the applications of Cauchy integrals and properties of harmonic functions.
- Handle Gamma function, Riemann zeta function and familiarize with analytic continuations.
- Solve problems involving infinite products, equicontinuity and normal family.

Unit I

Mappings of elementary functions and cross ratio, bilinear transformations and its properties, mapping of some elementary functions, mappings of z^2, e^z, $\sin z, \log z$, z+1/z, etc., conformal mappings.

Unit II

Maximum modulus theorems, Schwartz lemma, Argument principle, Rouche's theorem, applications to fundamental theorem of calculus, uniqueness and identity theorems, Hurwitz's theorem, Harmonic functions, mean value theorem, Poisson integral formula, Harnack's inequality and theorem, Hadamard three circle theorem.

Unit III

Weierstrass' factorization theorem, Gamma function and its properties, Riemann zeta function, Riemann's functional equation, Mittag-Leffler's expansion theorem and its applications, analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation.

Unit IV

Canonical products, Jensen's formula, Poisson-Jensen formula, Hadamard's three circles theorem, order of an entire function, exponent of convergence, Borel's theorem, Hadamard's factorization theorem. equicontinuity, normal family, families of analytic functions.

Books Recommended:

- ✓ *L. V. Ahlfors: Complex Analysis: McGraw Hill, 3rd Edition (2017).*
- ✓ *S. Ponnusamy and Herb Silverman: Complex variables with Applications: Birkhauser, (2006) (Indian Edition 2012).*

Books for References:

- ✓ *J. Bak and D. J. Newman: Complex analysis (3rd Edition), Undergraduate Texts in Mathematics, Springer-Verlag, NewYork, 1997.*
- ✓ *H. A. Priestly: Introduction to complex analysis, Oxford University Press, 2008.*
- ✓ *D. Sarason: Complex Function Theory: AMS, Second Edition, 2007.*
- ✓ *E. M. Stein and R. Shakarchi: Complex analysis: Princeton University Press, 41 William Street, Princeton, New Jersey, 2003.*
- ✓ *John B. Conway: Function of one complex variable: Springer International Student Edition, Narosa Publishing House, Second Edition, 2002.*
- ✓ *R. V. Churchill, J. W. Brown and R. F. Verhey: Complex variables and applications, McGraw Hill, 9th Edition, 2013.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Core XXIII

Differential Equations-III

Course Objective:

The objective of this course is to

- Understand the basic methods for qualitative behavior of solutions of ordinary differential equations and boundary value problems,
- Understand the basic methods to solve system of differential equations,
- Expose students about some of the real life problems using the system of differential equations,
- Expose the students to canonical forms of hyperbolic, elliptic and parabolic PDEs.

Learning Outcomes:

On the completion of this course, students will be able to

- Handle oscillation properties of ordinary differential equations and sturmLiouville differential equations
- Know existence and uniqueness theorems and application to mathematical modelling.
- Learn solutions of heat equation and various boundary value problems for Laplace equation
- Solve various PDEs using Green's function

Unit I

- **Oscillation of Second Order Linear Differential Equations:** Fundamental results, Sturm's Comparison Theorem and Hille-Wintner type oscillation.
- **Second Order Boundary Value Problem:** Sturm-Liouville differential equation, eigen value problems, Green's function and Picard's Theorem.

Unit II

System of first order equations, existence and uniqueness theorems, fundamental matrix, homogeneous and nonhomogeneous linear systems with constant coefficient, mathematical formulation of Predatory-pray model, epidemic model of influenza, battle model and their solutions.

Unit III

One dimensional heat equation and its origin, Heat conduction problem for an infinite rod and finite rod, existence and uniqueness of solution, two dimensional heat equation and Laplace equation, boundary value problems, maximum and minimum principles, uniqueness and continuity theorems, Dirichlet problem for a circle, Dirichlet problem for annulus, Neumann problem for a circle.

Unit IV

Solution of heat equation, wave equation, Laplace equation and Helmholzequation

by Green's function method and examples.

Books Recommended:

- ✓ *Deo and Raghavendra; Text Book of Ordinary Differential Equations, Tata McGraw-Hill Pub. Company Ltd, New Delhi, 2017.*
- ✓ *Belinda Barnes and Glenn R. Fulford; Mathematical Modeling with Case Studies, A Differential Equation Approaching Maple and Matlab, 2nd Ed., Taylor and Francis group, London and NewYork,2009.*
- ✓ *TynMyint-U and LokenathDebnath; Linear Partial Differential Equations for Scientists and Engineers, 4th edition, Birkhauser, Indian reprint,2014.*

Books for References:

- ✓ *K. Shankar Rao, Introduction to partial differential equations, PHI learning private Ltd., 2011.*
- ✓ *J. N. Sharma, K. Singh, Partial Differential Equations for Engineers and Scienists, Narosa, 2nd Edition, 2009.*
- ✓ *Robert C.McOwen: Partial Differential Equations, Pearson Education Inc., 2002.*
- ✓ *Martin Braun, Differential Equations and their Applications, Springer International Student Ed., 1978.*
- ✓ *S.L. Ross, Differential equations,3rd Ed., John Wiley and Sons, India, 2014.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

THRUST AREAS FOR DISSERTATION/PROJECT WORK

The student should work for his dissertation in a topic in one of these areas or any area