Core XVI **Semester VII**

Measure Theory & Integration

Course Objectives:

The aim of this course is to provide a foundation for student to many branches of mathematics such as functional analysis, harmonic analysis, ergodic theory, and probability theory, etc.. In this course, the students will be introduced to,Lebesgue measure and integration, signed measures, Hahn-Jordan decomposition, Radon-Nikodym derivative and product measures.

Learning Outcomes:

After completing the course the student will be able to

- Calculate Riemann-Stieltjes integrals and Lebesgue integrals of simple measurable functions.
- Know how to calculate Lebesgue Integral of any measurable functions and learn how to apply monotone and dominated convergence theorems.
- Learn the concept of measure on abstract spaces and work on various modes of convergence of a sequence of measurable functions.
- Learn on complex measures, Radon Nikodym derivatives and related results.

Unit I

Lebesgue outer measure, measurable sets, Borel sets, regularity, measurable functions, Borel and Lebesgue measurability, non-measurable sets, integration of nonnegative functions, simple functions, Lebesgue integration of simple function.

Unit II

Approximation of measurable functions by simple functions, Lebesgue integral of measurable functions and properties, Fatou's lemma, monotone convergence theorem, Lebesgue dominated convergence theorem, integration of series, Riemann and Lebesgue integrals, differentiation, Dini derivatives, Lebesgue differentiation theorem.

Unit III

Abstract measure spaces, measure and outer measure, extension of a measure, uniqueness of the extension, completion of a measure, integration with respect to a

measure, Modes of convergence, convergence in measure, almost uniform convergence, fundamental in measure convergence, Egorov's theorem.

Unit IV

Signed measure, absolute continuity, Hahn decompositions, Jordan decomposition, Lebesgue decomposition, Radon-Nikodym theorem, applications of Radon Nikodym Theorem, product measure, Fubini theorem.

Books Recommended:

- ✓ *G. De Barra: Measure Theory and Integration, New age International, 1981.*
- ✓ *H. L. Royden: Real Analysis, Pearson, Fourth Edition, 2010.*

Books for Reference:

- ✓ *H. L. Royden and P. M. Fitzpatrick: Real Analysis, Fourth Edition, Pearson Asia Education Ltd and China Machine Press, 2010.*
- ✓ *C. D. Aliprantis, O. Burkinshaw: Principles of Real Analysis, Elsevier, 2011.*
- ✓ *J .Yeh: Real Analysis (Theory of Measure and Integration), 3rd Edition, World Scientific Publication, 2024.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Core XVII

Algebra-III

Course Objectives:

To present a systematic study of field theory and Galois theory.

Learning Outcomes:

After completing the course the student will be able to

- Understand the basic concept of field extension, and splitting fields.
- Understand the significance of separable extension, cyclotomic polynomials, Galois group.
- Understand the structures and properties of finite fields, composite extensions, simple extensions.
- Determine the Galois group of a polynomial and understand the conditions under which polynomial equations can be solved using radicals.

Unit I

Basic theory of field extension, algebraic extension, classical straightedge and compass construction, splitting fields and algebraic closures.

Unit II

Separable and inseparable extension, Cyclotomic polynomials and extensions, Galois theory, basic definitions, The fundamental theorem of Galois theory.

Unit III

Finite fields, Composite extensions and simple extensions, cyclotomic extensions and abelian extensions over Q, Galois groups of polynomials over Q.

Unit IV

Solvability and radical extension, Insolvability of quintic, computations of Galois group over Q.

Books Recommended:

- ✓ *D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley-India edition, 2013.*
- ✓ *Joseph A. Gallian, Contemporary Abstract Algebra (4th Edition), Narosa Publishing House, New Delhi, 1999.(IX Edition 2010).*

Books for Reference:

- ✓ *John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.*
- ✓ *I.N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.*

- ✓ *e-Learning Source http://ndl.iitkgp.ac.in; http://ocw.mit.edu; http://mathforum.org*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

Core XVIII

Topology

Course Objective:

This is an introductory course in Topology. The objective of this course is to have knowledge on topological spaces, continuity, connectedness, compactness and separation axioms. Topology on quotient spaces, product spaces and metric spaces are also discussed. The student will also learn on basic ideas of algebraic topology such as homotopy, fundamental groups and covering spaces.

Learning Outcomes:

After taking the course the student will be able to

- Know on basics of topological spaces with examples and is able to construct new topologies using idea of product topology, quotient topology, etc .
- Solve problems involving continuous maps, homeomorphisms between two spaces , connectedness and compactness.
- Learn examples and properties of Hausdorff, regular, normal, separable, first and second countable spaces.
- Understand more results in separation axioms and learn on basic concepts of algebraic topology like homotopy, fundamental groups, and covering spaces.

Unit I

Cartesian product of a family of sets, Axiom of choice and its equivalents(without proof),Topological spaces, examples, open sets, closed sets, basis and subbasis for a topology, closure and interior of sets, subspace topology, order topology, continuous functions, homeomorphisms, product topology, quotient topology.

Unit II

Metric topology, standard topology, uniform topology, lower limit topology, connectedness, examples, local connectedness, Path-connectedness, connected subsets of real line, compact spaces, examples, locally compacts spaces, sequential compactness, limit point compactness, compact subsets of real line.

Unit III

Countability axioms, first and second countable spaces, separable and Lindolf spaces, separation axioms, regular & completely regular space, normal spaces, Urysohn Lemma.

Unit IV

Urysohnmetrization theorem, Tychonoff theorem, compactness in metric spaces, compact open topology homotopy of paths, fundamental group, covering space.

Books Recommended:

- ✓ *J R Munkres, Topology: A First Course,Pearson,2ndedition,2000.*
- ✓ *M A Armstrong, Basic Topology. Springer, 1983.*

Books for Reference:

- ✓ *K. D. Joshi, Introduction to General Topology, Wiley Eastern Limited.*
- ✓ *T. S. Singh, Elements of Topology, CRC press (special Indian Edition) 2015.*
- ✓ *O. Viro, O Ivanov, V Kharlamov and N Netsvetaev, Elementary Topology, problem Text book, American Mathematical society, 2008.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Core XIX

Mathematical Methods

Course Objective:

The objective of this course is to prepare a student in basics of Integral transforms, Integral equations and calculus of variations. These tools have applications in other science and engineering fields and are necessary to understand.

Learning Outcomes:

After completing the course the student will be able to

- Calculate Laplace transform, Fourier transform and apply them in areas of differential equations immediately.
- **F**ind solutions ofVolterra integral equations.
- Find solutions ofFredholmintegral equations.
- Use methods in calculus of variations to solve extremal problems in Differential equations and physics.

Unit I

Laplace transforms: definitions, properties, Laplace transforms of some elementary functions, convolution theorem, inverse Laplace transformation and applications. Fourier transforms: definitions, properties, Fourier transforms of some elementary functions, convolution, Fourier transform of derivatives, Fourier transforms as a limit of Fourier series.

Unit II

VolterraIntegral Equations: Basic concepts, relationship between linear differential equations and Volterraintegral equations, resolvent kernel of Volterraintegral equations, solution of integral equations by resolvent kernel, The method of successive approximations, convolution type equations, solutions of integral differential equations with the aid of Laplacetransforms.

Unit III

Fredholmintegral equations: Fredholmequations of the second kind, fundamentals, iterated kernel, constructing the resolvent kernel with the aid of iterated kernels, integral equations with degenerate kernels, characteristic numbers and eigen functions, solution of homogeneous integral equations with degenerate kernel, non-homogeneous symmetric equations, Fredholmalternatives.

Unit IV

Calculus of variations: extremal of functional, The variation of a functional and its properties, Euler's equations, field of extremals, sufficient conditions for the extremum of a functional, conditional extremum moving boundary problem, discontinuous problems, one sided variations, Ritzmethod.

Books Recommended:

- ✓ *A. J. Jerri; Introduction to Integral Equations with Applications, John-Wiley & SONS, INC., 1999.*
- ✓ *LokenathDebnath; Integral Transforms and Their Applications, CRC Press, New York.*
- ✓ *A. S. Gupta; Calculus of Variations with Applications, PHI, Pvt. Ltd., New Delhi.*

Books for Reference:

- ✓ *I. Sneddon, The use of Integral Transformations (Tata McGraw Hill),1972.*
- ✓ *MurrayRSpiegel,LaplaceTransforms, Schaum'sSeries,1965.*
- ✓ *Gelfand and Fomin, Calculus of Variations, Dover Pub,2003.*
- ✓ *Krasnov, Problems and Exercises in Calculus of Variations, Mir Publ.,1970.*
- ✓ *Ram P Kanwa, Linear Integral Equations (Academic Press),2013.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Core XX **Semester VIII**

Functional Analysis