

Multivariable Calculus

Course Objectives:

The primary objective of this course is to introduce students, the extension of the studies of single variable differential and integral calculus to functions of two or more independent variables with the geometry and visualization of curves and surfaces. To aware the students about the techniques multiple integrations and higher order derivatives.

Learning Outcomes:

After completing the course the student will be able to

- Learn the concept of limit, continuity and differentiations of functions of more than one.
- Understand the maximization and minimization of multivariable functions with the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double, and triple integral formulations.
- Familiarize with the Green's, Stokes' and Gauss divergence theorems and their applications.

Unit I

Functions of several variables, limit and continuity of functions of two variables: partial differentiation, total differentiability, sufficient condition for differentiability, chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes.

Unit II

Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, double integration over rectangular region, double integration over non rectangular region, double integrals in polar co-ordinates.

Unit III

Triple integrals, triple integral over a parallelepiped and solid regions, volume by triple integrals, cylindrical and spherical co-ordinates, change of variables in double integrals and triple integrals.

Unit IV

Definition of vector field, divergence and curl, line integrals, applications of line integrals: mass and work, fundamental theorem for line integrals, conservative vector fields, independence of path, Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stokes' theorem, the divergence theorem.

Books Recommended:

- ✓ *M. J. Strauss, G. L. Bradley and K. J. Smith: Calculus, 3rd Edition, Dorling Kindersley (India) Pvt. Ltd. Pearson Education, Delhi, 2007.*
- ✓ *E. Marsden, A. J. Tromba and A. Weinstein: Basic Multivariable Calculus, Springer Student International Edition, Indian reprint, 2005.*

Book for References:

- ✓ *S. C. Mallik and S. Arora: Mathematical Analysis, New Age International Publications, New Delhi, 2005.*
- ✓ *Tom Apostol: Mathematical Analysis, Narosa Publishing House, 2002.*
- ✓ *G. B. Thomas and R. L. Finney: Calculus, 9th Ed., Pearson Education, Delhi, 2005.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source <http://ndl.iitkgp.ac.in> ; <http://ocw.mit.edu> ; <http://mathforum.org>*

Differential Geometry

Course Objective:

The objective of this course is to explore geometry of curves and surfaces in \mathbb{R}^2 and \mathbb{R}^3 with their intrinsic properties and curvatures.

Learning Outcomes:

After completing the course the student will be able to

- Understand the notion of plane curves, space curves, curvature, torsion and the existence of space curves.
- Learn the theory of surfaces and learn to calculate first fundamental forms.
- Learns on geodesics on a surface and learns to calculate curvatures.
- Learns calculating second fundamental forms, curvatures and discovers minimal surfaces.

Unit I

Theory of Space Curves: space curves, arc length, tangent, normal and binormal, osculating plane, curvature, torsion, Serret-Frenet formulae, contact between curves and surfaces, osculating circles and spheres, involute and evolutes, existence of space curves, Helices.

Unit II

Theory of surfaces: parametric curves on surfaces, surfaces of revolution, helicoids, metric, direction coefficients. First Fundamental forms.

Unit III

Geodesics, canonical geodesic equations, nature of geodesics on a surface of revolution, normal property of geodesics, Torsion of a geodesic: geodesic curvature, Gauss-Bonnet theorem, Gaussian curvature, surfaces of constant curvature.

Unit IV

Second Fundamental forms, principal curvatures, lines of curvature, Rodrigue's formula, conjugate and asymptotic lines. Developables, developable associated with space curves and curves on surfaces, minimal surfaces. Fundamental Theory of surfaces.

Books Recommended:

- ✓ *T.J. Willmore, An Introduction to Differential Geometry, Dover Publications, 2012.*
- ✓ *Pressley, Elementary Differential Geometry, Springer International Edition, 2014*

Books for References:

- ✓ *O'Neill, Elementary Differential Geometry, 2nd Ed., Academic Press, 2006.*
- ✓ *C.E. Weatherburn, Differential Geometry of Three Dimensions, Cambridge University Press 2003.*
- ✓ *D.J. Struik, Lectures on Classical Differential Geometry, Dover Publications, 1988.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source <http://ndl.iitkgp.ac.in> ; <http://ocw.mit.edu> ; <http://mathforum.org>*