Core XI **Semester V**

Real Analysis-III

Course Objective:

After a first course in real analysis in undergraduate program, the ideas of uniform continuity, uniform convergence and approximation by polynomials are crucial in analysis. In addition to the functions of bounded variation and their integrators, the student has to learn differentiating functions from R^n to R^m. The techniques of integration of a function with respect to another function and the basic ideas of finding a Fourier series are also included.

Learning Outcomes:

After completing the course the student will be able to

- Find the Fourier series of a function.
- Calculate Riemann Stieltjes integrals and know whether a function is of bounded variation or not.
- Learn how to define derivatives on R^n including the existence of partial derivatives, inverse function theorem and implicit function theorem.
- Learn about metric spaces and their topological properties.

Unit I

Basic concepts of Fourier series, Fourier series of even and odd functions, half range series, Fourier series on other intervals, orthogonal systems of functions, theorem on best approximation, properties of Fourier coefficients, Riesz-Fisher theorem, Riemann-Lebesgue lemma, Dirichlet integral, Integral representation for the partial sum of a Fourier series, convergence of Fourier series.

Unit II

Function of bounded variation, examples, total variation, function of bounded variation expressed as difference of increasing functions, rectifiable paths, Riemann-Stieltjes integrals, properties and techniques, necessary and sufficient condition for existence of the integral, mean value theorem for Riemann-Stieltjes integrals, reduction to Riemann integrals.

Unit III

Differentiation in R^n, partial derivatives, directional derivatives, sufficient condition for differentiability, chain rule, , mean value theorem, Jacobians, contraction mapping principle, inverse function theorem, implicit function theorem, rank theorem, differentiation of integrals, Taylor theorem in many variables.

Unit IV

Metric spaces, definitions and examples, open and closed sets, interior and exterior points, convergence and completeness, continuity and uniform continuity, compactness, connectedness.

Books Recommended:

- ✓ *W. Rudin, PrinciplesofMathematicalAnalysis, McGrawHill,3rd edition.*
- ✓ *T. Apostol, MathematicalAnalysis, Pearson,2ndedition.*
- ✓ *S C Malik and SavitaArora, Mathematical Analysis- New Age International, 5th edition*

Books for Reference:

- ✓ *Terrence Tao, Analysis-I, Hindustan book agency.*
- ✓ *Terrence Tao, Analysis-II, Hindustan book agency*
- ✓ *K. A. Ross, Elementary Analysis: The theory of Calculus, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.*
- ✓ *Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett, Student Edition, 2011.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Core XII

Differential Equations-II

Course Objective:

The objective of this course is to understand basic methods for solving nonlinear first order ordinary differential equations and existence of solutions along with some special type of second order ordinary differential equations of mathematical physics. Also, students will be exposed to second order partial differential equations arising in thermal physics and thermodynamics.

Learning Outcomes:

After completing the course the student will be able to

- Understandfirst order nonlinear ordinary differential equations and existence of solutions
- Learn the methods to find solutions of second order linear ordinary differential equations with constant coefficients and variable coefficients.
- The different methods for solving first and second order partial differential equations and can take more courses on wave equation, heat equation, diffusion equation, gas dynamics, nonlinear evolution equations etc. All these courses are important in engineering and industrial applications for solving boundary value problems.
- Get idea to solve various mathematical models of ODE and PDE which may be helpful for simulation process.

Unit I

Existence and Uniqueness of Solutions: Lipschitz condition, Gronwall type inequality,successive approximations, Picard's theorem, non-uniqueness of solutions, continuation and dependence on initial conditions, existence of solutions in the large.

Unit II

Solution of second order ODE with constant coefficients, power series solutions of ordinary and singular points, and special functions of Legendre's differential equations, Bessel's differential equations and their properties.

Unit III

Charpit's method, special types of first order PDE, Jacobi's method, Linear second order PDE, canonical forms of second order PDE and characteristics curves, one dimensional wave equation, its origin and elementary solutions, vibration of an infinite string, vibration of a semi finite string, vibration of a string of finite length, existence of unique solution.

Unit IV (Practical)

Laboratory work for the following problems using *MATLAB / Mathematica / Maple* etc.

1) Plot the Fourier series of the following functions:
   i. $f(x) = x^2, x \in [-1,1]$
   ii. $f(x) = \begin{cases} 1, & 0 < x < \pi \\ -1, & -\pi < x < 0 \end{cases}$
   iii. $f(x) = \sin x,\ \ 0 < x < \frac{\pi}{2}$
2) Solution of wave equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ for the following associated conditions:
   (i) $u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), x \in \mathbb{R}, t > 0$
   (ii) $u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), u(0,t) = 0\ x \in (0,\infty), t > 0$
   (iii)$u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), u_x(0,t) = 0\ x \in (0,\infty), t > 0$
   (iv)$u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), u(0,t) = 0,\ u(l,t) = 0,\ 0 < x < l, t > 0$
3) Solution of one dimensional heat equation $\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = 0$ for the following conditions
   (i) $u(x,0) = \varphi(x), u(0,t) = a, u(l,t) = b, 0 < x < l, t > 0$
   (ii) $u(x,0) = \varphi(x), x \in \mathbb{R}, 0 < t < T$
   (iii) $u(x,0) = \varphi(x), u(0,t) = a, x \in (0,\infty), t \geq 0.$

Books Recommended:

- ✓ *Deo and Raghavendra, Text Book of Ordinary Differential Equations, Tata McGraw-Hill Pub. Company Ltd, New Delhi,2017.*
- ✓ *Simmons G F, Differential equation, Tata McGrawHill,1991.(Ch-5. 28-30, Ch-8. 44-47, Ch-6, 33-36)*
- ✓ *I. Sneddon, Elements of Partial Differential Equations,McGraw-Hill, International Students Edition.(Ch-2. 10,11,13; Ch-3. 4-7; Ch-5. 1,2), 2006.*
- ✓ *TynMyint-U and LokenathDebnath, Linear Partial Differential Equations for Scientists and Engineers, 4th edition, Birkhauser, Indian reprint,2014.*

Books for Reference:

- ✓ *J. N. Sharma and Kehar Singh, PDE for Engineers and Scientists, Narosa, New Delhi,2009.*
- ✓ *T Amarnath, An Elementary Course in Partial Differential Equations, Narosa Publications, 2003.*
- ✓ *Martin Braun, Differential Equations and their Applications, Springer International Student Ed. 1978.*
- ✓ *S. L. Ross, Differential Equations, 3rd Edition, John Wiley and Sons, India, 2014.*
- ✓ *C.Y. Lin, Theory and Examples of Ordinary Differential Equations, World Scientific, 2011.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ;* http://mathforum.org
- *Suggested digital platform: NPTEL/SWAYAM/MOOCs*

Core XIII

Numerical Analysis & Scientific Computing

Course Objectives:

The objective of this course is to acquaint the students with a wide range of numerical methods to solve algebraic and transcendental equations, linear system of equations, interpolation and curve fitting problems, numerical integration, initial and boundary value problems, etc. Develop adequate skills to apply those methods in real world problems.

Learning Outcomes:

After completing the course the student will be able to

- Understand the errors in computation, find the roots of algebraic and transcendental equations, familiarize with convergence, advantages and limitations of those numerical techniques, learn to apply Gauss−Jacobi, Gauss−Seidel methods to solve system of linear equations.
- Get aware of using interpolation techniques to solve polynomials.
- Learn numerical differentiation and integrations by using different techniques.
- Understand the techniques to find approximate solutions of ODE and PDE.

Unit I

Errors in approximation, absolute, relative and percentage errors, round-off error, solution of algebraic and transcendental equations: bisection method, Regula-Falsi method, secant method, method of iteration, Newton Raphson method, order of convergence, systems of simultaneous equations: Gauss elimination method, Gauss Jordon method, LU decomposition method, Iterative methods: Jacobi method and Gauss-Seidel method.

Unit II

Finite differences, interpolation techniques for equal intervals-Newton forward and backward, Gauss forward, Gauss backward, interpolation, interpolation with unequal intervals-Newton's divided difference method, Lagrange method, Hermite interpolation,

Numerical differentiation using Newton forward and backward formulae, numerical integration using Newton-Cotes formulas, trapezoidal rule, Simpson rules, Gauss-Legendre, Gauss-Chebyshev formulas.

Unit III

Solution of ordinary differential equations: Taylor series method, Picard's method, Euler method, Euler modified method, Runge–Kutta methods.

Unit IV (Practical)

Practical / Lab work to be perform in Computer Lab:

Use of computer algebra system (CAS) software: Python/ Sage Math / Mathematica/ MATLAB/ Maple/ Maxima/ Scilab/ R or any other (open source) software etc., for developing at least the following numerical programs:

1. Bisection method, Newton-Raphson method and Secant method.
2. LU decomposition method.
3. Gauss−Jacobi method and Gauss−Seidel method.
4. Lagrange interpolation and Newton interpolation.
5. Trapezoidal rule and Simpson's rules.
6. Taylor series method, Picard's method, Euler method, Euler modified method and Runge−Kutta Methods.

Note: ***Non-programmable scientific calculator is allowed in the examination.***

Books Recommended:

- ✓ *M. K. Jain, S. R. K. Iyengar& R. K. Jain: Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, India, 2016.*
- ✓ *R. K. Gupta: Numerical Methods: Fundamentals and Applications, Cambridge University Press, 2019.*

Books for Reference:

- ✓ *Brian Bradie: A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third impression, 2011.*
- ✓ *Curtis F. Gerald & Patrick O. Wheatley: Applied Numerical Analysis, Pearson Education. India, 2007.*
- ✓ *S. D. Conte & S. de Boor: Elementary Numerical Analysis: An Algorithmic Approach, 1980.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Core XIV **Semester VI**