Core V **Semester III**

Probability

Course Objective:

The objective of the course is to make the student understand basics of probability which is of use in everyday life.

Learning Outcomes:

After completing the course the student will be able to

- Learn the basics of probability and random variables with axioms of probability.
- Know the discrete and continuous distributions and learn how to calculate mean, variance and moments of them.
- Learn on limit theorems with their applications and know about the conditional expectations.
- Learn on Markov chains and their applications.

Unit I

Sample space and events, probability axioms, probability defined on events, conditional probabilities, Independent events, Bayes formula, real random variables, discrete and continuous random variables, probability distribution function, probability mass/density functions, mathematical expectation, and properties, variance and standard deviation.

Unit II

Discrete distributions: uniform, binomial, Poisson, geometric, negative binomial, continuous distributions: uniform, normal, exponential, their expectations and variance, moments, moment generating function, characteristic function and computation of these for the distributions, joint distribution function and its properties, joint probability density functions, marginal and conditional distributions, independent random variables.

Unit III

Limit theorems: Markov inequality, Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers, application to problems, conditional probability and conditional expectation, discrete case, continuous case, applications, expectation of function of two random variables, conditional expectations,

bivariate normal distribution, correlation coefficient, joint moment generating function and calculation of covariance, linear regression for two variables.

Unit IV

Central limit theorem for independent and identically distributed random variables with finite variance, Markov chains, Chapman-Kolmogorov equations, classification of states, Gambler Ruin problem.

Book Recommended

1. *Sheldon Ross, Introduction ToProbability Models (9th Edition), Academic Press, Indian Reprint, 2007.*
2. *Robert V. Hogg, Joseph W. MckeanAndAllen T. Craig, Introduction To Mathematical Statistics, Pearson Education, Asia, 2007.*
3. *Kai Lai Chung, Elementary Probability Theory WithStochastics Process, Springer International Students Edition, (Narosa Publ.)*

Book for Reference

- ✓ *Alexander M. Mood, Franklin A. Graybill and Duane C. Boes, Introduction to the Theory of Statistics, (3rd Edition), Tata McGraw- Hill, Reprint 2007.*
- ✓ *Chow Y S , Teicher H Probability theory Springer International edition*
- ✓ *Irwin Miller and Marylees Miller, John E. Freund's Mathematical Statistics with Applications (7th Edition), Pearson Education, Asia, 2006.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

Core VI

Differential Equations-I

Course Objective:

Differential Equations introduced by Leibnitz in 1676 models almost all Physical, Biological, Chemical systems in nature. The objective of this course is to familiarize the students to various methods of solving differential equations, partial differential equations and to have a qualitative applications through models. The students have to solve problems to understand the methods.

Learning Outcomes:

After completing the course the student will be able to

- Get the idea to solve first order linear ordinary differential equations of different types those are arising in physical problems.
- Get the idea to solve second order linear ordinary differential equations of different types those are arising in physical problems.
- Get basic ideas of first order partial differential equations, its formulation in two, three variables and variable separable method for identify the solutions.
- Get idea to solve various mathematical models of ODEs and PDEs which may be helpful for simulation process.

Unit I

Differential equations and mathematical models, general, particular, explicit, implicit and singular solutions of a differential equation, exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equations and Bernoulli's equation, compartmental model, population model for single species.

Unit II

General solution of homogeneous equation of second order, principle of superposition, Wronskian, its properties and applications, method of undetermined coefficients, method of variation of parameters, linear homogeneous and non-homogeneous equations of higher

order with constant coefficients, Euler's equations.

Unit III

Partial Differential Equations - Basic concepts and definitions, origin of first order PDEs, Classification of first order PDEs, Pfaffian differential forms and equations, solution of Pfaffian differential equations in three variables, Cauchy's problem for first order PDEs, linear equations of first order, integral surfaces passing through a given curve, Cauchy's method of characteristics, compatible systems, method of separation of variables for solving first order and second order partial differential equations.

Unit IV (Practical)

The students will implement the following problems in the computer Lab using *Matlab / Mathematica / Maple* etc.

1.Plotting of second order solution family of differential equations.
2.Plotting of third order solution family of differentialequations.
3.Population growth model (exponential caseonly).
4.Population decay model (exponential caseonly).
5.Solution of Cauchy problem for first order PDEs.
6.Finding the characteristics for the first order PDEs.
7.Plot the integral surfaces of a given first order PDE with initial data.

Books Recommended:

- ✓ *J. Sinha Roy and S. Padhy: A course of Ordinary and Partial differential equations, Kalyani Publishers, New Delhi,2018.*
- ✓ *Belinda Barnes and Glenn R. Fulford, Mathematical Modeling with Case Studies, A Differential Equation Approaching Maple and Matlab, 2nd Edn. Taylor and Francis group, London and New York, 2009.*
- ✓ *Sneddon; Elements of Partial Differential Equations, McGraw-Hill, International Students Edition, 1957.*

Books for Reference:

- ✓ *G. F. Simmons, Differential equation, Tata McGraw Hill, 1991.*
- ✓ *J. N. Sharma and Kehar Singh, PDE for Engineers and Scientists, Narosa, New Delhi, 2009.*
- ✓ *Martin Braun, Differential Equations and their Applications, Springer International Student Ed.1978.*
- ✓ *S. L. Ross, Differential Equations, 3rd Edition, John Wiley and Sons, India, 2014.*
- ✓ *C.Y. Lin, Theory and Examples of Ordinary Differential Equations, World Scientific, 2011.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

1. e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org

Core VII

Linear Algebra

Course Objective:

The objective of this course is to acquaint students with matrix operations, solution of system of equations, vector spaces and linear transformations. In addition, the student will learn about eigenvalues, diagonalization, canonical forms, etc., which has many applications in almost all areas of science and engineering.

Learning Outcomes: After completing the course the student will be able to

- Determine basis and the dimension of a finite-dimensional vector space, know the relation between rank and nullity of a linear transformation.
- Therelation between matrix and linear transformation.
- To find solution of system of linear equations, compute eigenvalues, eigenvectors of a matrix and linear transformation.
- About orthogonality of vectors and application of it to different form of matrix, introduced to different operators.

Unit I

Vector spaces, subspaces, span of a set, more about subspaces, linear dependence, independence, product and quotient space, dimension and basis, linear transformations, range and kernel of a linear map, rank and nullity of linear map.

Unit II

Inverse of linear transformation, consequences of rank – nullity theorem, the space L(U, V), composition of linear maps, matrix associated with linear map, linear map associated with matrix, rank and nullity of a matrix, determinant minors and rank of a matrix, transpose of a matrix and special type of matrices, elementary row operations

Unit III

System of linear equations, matrix inversion, application of determinant to linear equations, eigenvalues and eigenvectors, similarity of matrices, invariant subspaces, minimal polynomial (eigenvalues and the minimal polynomial), upper triangular matrices, diagonalizable operators (diagonal matrices, conditions for diagonalizability).

Unit IV

Inner product space: inner products and norms, orthonormal bases, orthogonal complements, self-adjoint and normal operators, spectral theorems, isometries, unitary operators, characteristic polynomial, Cayley – Hamilton theorem, Jordan form, trace, quadratic form, application to reduction of quadrics.

Books Recommended:

- ✓ *V. Krishnamurthy. V.P. Mainra, J. L. Arora, An introduction to linear algebra, Affiliated East – West press Pvt. Ltd., New Delhi, 1976.*
- ✓ *Sheldon Axler, Linear algebra done right (Fourth edition),Springer, 2024.*

Books for References:

- ✓ *Seymour Lipschutz and Marc Lars Lipson, Linear Algebra (Schaum's outlines, Fourth Edition), McGraw Hill, New York, 2009.*
- ✓ *A. RamachandraRao and P. Bhimsankaram, Linear Algebra (Second Edition), Hindustan Book Agency, 2000.*
- ✓ *Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra (Fourth Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.*
- ✓ *Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007.*
- ✓ *S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org ; https://linear.axler.net; and https://library.oapen.org/handle/20.500.12657/85067*