

## **Core III**

### **Semester II Real Analysis-I**

#### **Course Objective:**

The objective of the course is to introduce the basics of real number system and the properties of sequence and series of real numbers. The ideas of completeness, least upper bound property, denseness, limit, continuity and uniform continuity will also be introduced. This is one of the core courses essential to start doing mathematics.

#### **Learning Outcomes:**

On successful completion of this course, students will be able to

- Learn basics of real number system and test countability of a set.
- Know on sequence of real numbers and their basic properties.
- Test convergence of an infinite series.
- Find limit and continuity of functions and test uniform continuity of functions.

#### **Unit I**

Finite and infinite sets, countable and uncountable sets, examples, algebraic and order Properties of  $\mathbf{R}$ , uncountability of  $\mathbf{R}$ , completeness property of  $\mathbf{R}$ , applications of the supremum property, Intervals, nested interval property, denseness of rationals in  $\mathbf{R}$ .

#### **Unit II**

Sequence and their limits, limit theorems, monotone sequences, monotone Convergence theorem, subsequences, divergence criteria, monotone subsequence theorem, Bolzano Weierstrass theorem for sequences, Cauchy sequence, Cauchy's convergence criterion.

#### **Unit III**

Infinite series, convergence and divergence of infinite series, Cauchy criterion, Tests for convergence: comparison test, limit comparison test, ratio test, Cauchy's nth root test, Raabe's test, integral test, alternating series, Leibniz test, absolute and conditional convergence.

#### **Unit IV**

Limits of functions, limit theorems, some extensions of limit concept, continuous functions and their combinations, continuous functions on intervals, boundedness theorem, maximum

minimum theorem, intermediate value theorem, uniform continuity, examples, uniform continuity theorem.

### **Books Recommended:**

- ✓ *R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 3<sup>rd</sup> Edn., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.*
- ✓ *G. Das and S. Pattanayak, Fundamentals of Mathematical Analysis, TMH Publishing Co., 30<sup>th</sup> reprints, 2021.*

### **Books for Reference:**

- ✓ *S. C. Mallik and S. Arora, Mathematical Analysis, New Age International Publications.*
- ✓ *A. Kumar, S. Kumaresan, A basic course in Real Analysis, CRC Press, 2014.*
- ✓ *Brian S. Thomson, Andrew M. Bruckner, and Judith B. Bruckner, Elementary Real Analysis, Prentice Hall, 2001.*
- ✓ *Gerald G. Bilodeau, Paul R. Thie, G. E. Keough, An Introduction to Analysis, Jones & Bartlett, Second Edition, 2010.*
- ✓ *e-Learning Source <http://ndl.iitkgp.ac.in> ; <http://ocw.mit.edu> ; <http://mathforum.org>*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

## **Core IV**

### **Algebra-I**

#### **Course Objectives:**

To present a systematic and rigorous study on algebraic structures like groups, rings and some important results with their applications. After pursuing this course, one can opt for advanced topics in groups, rings and their applications to problems in physics, computer science and engineering.

#### **Learning Outcomes:**

After completing this course, students will be able to

- Understand permutation groups with some results and application in Rubik's cube.
- Understand the concept of homomorphisms, isomorphisms, normal subgroups and factor groups.
- Explore more properties of rings and ideals rigorously.
- Get introduced to the concept of reducibility and irreducibility of polynomials and concept of unique factorization domain.

#### **Unit I**

Permutation groups, definition and notations, cyclic notation, properties of permutations, isomorphisms, definition and examples, Cayley's theorem, properties of isomorphisms, automorphisms, cosets, properties of cosets, Lagrange's theorem and consequences, an application of cosets to permutation groups, an application of cosets to Rubik's cube.

#### **Unit II**

External direct products, definition and examples, properties of external direct products, the group of units modulo  $n$  as an external direct product, applications, normal subgroups, factor groups, application of factor groups, internal direct products, group homomorphisms, definition and examples, properties of homomorphisms, the first isomorphism theorem.

#### **Unit III**

Characteristic of a ring, ideals, factor rings, prime ideals and maximal ideals, ring homomorphisms, definition and examples, the field of quotients, polynomial rings, notations and terminology, division algorithm and consequences.

## Unit IV

Factorization of polynomials, reducibility test, irreducibility test, unique factorization in  $\mathbb{Z}[x]$ , divisibility in integral domains, irreducible, primes, unique factorization domain, Euclidean domain.

### Books Recommended:

- ✓ *Joseph A. Gallian, Contemporary Abstract Algebra (9th Edition), Narosa Publishing House, New Delhi, 2010.*
- ✓ *I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.*

### Books for Reference:

- ✓ *John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.*
- ✓ *D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley-India edition, 2013.*
- ✓ *Joseph I. Rotman, An Introduction to the Theory of Groups, 4th Ed., Springer Verlag, 1995.*
- ✓ *M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source <http://ndl.iitkgp.ac.in> ; <http://ocw.mit.edu> ; <http://mathforum.org>*