

1. Choose the appropriate preposition to complete the sentence.

We have been living in this city _____ 2010.

- (A) for
- (B) since
- (C) from
- (D) about

2. Identify the meaning of the underlined idiom from the options given.

Sonal always gives a cold shoulder to Ritesh because of his anobbish behaviour.

- (A) insults him whenever she meets him
- (B) argues with him on any issue
- (c) shouts at him whenever she sees him
- (D) tries to be unfriendly to him

3. Choose the correct option to make a meaningful sentence

Sasha _____ that she would get a part in the inheritance left by her grandfather.

- (A) has hoped
- (B) is hoped
- (c) had hoped
- (D) have hoped

4. Identify the correct form of noun of the word given below.

Abject

- (A) abjectly
- (B) abjection
- (C) abjectively
- (D) abjects

5. Identify antonym for the underlined word in the sentence.

The NGO focused on those women who had succumbed to his sweet lies and personality.

- (A) identified
- (B) overcome
- (C) given up
- (D) surrendered

6. Find correctly spelt word from the options given below.

(ONLY one word is spelt correctly)

- (A) Harassment
- (B) Embarassment
- (c) Agreement
- (D) Alotment

7. Identify the correct passive voice for the sentence given below:

People consider that Sajesh is honest and trust worthy

- (A) It was considered that Sajesh was honest and trust worthy.
- (B) We were considered that Sajesh was honest and trust worthy.
- (C) It is considered that Sajesh is honest and trust worthy.
- (D) It will be considered that Sajesh is honest and trust worthy.

8. Identify the correct indirect speech for the sentence given below.

Manju said, "I printed the invitations and he faxed it".

- (A) Manju told that she printed the invitations and him faxed it.
- (B) Manju suggested that she printed the invitations he faxed it.
- (C) Manju asked that she printed the invitations they faxed it.
- (D) Manju said that she printed the invitations and he faxed it.

9. Given below are four sentences in jumbled order. Select the option that gives their correct order.

- (A) We ascribe to them supernatural powers.
- (B) But let's not be naive - cats do what they want.
- (C) We even make them embody our deepest wish, immortality.
- (D) We poor humans make cats depositories of our desires: we find them mysterious, we make them gods.

Choose the correct answer from the options given below :

- (A) (D), (A), (C), (B)
- (B) (D), (C), (B), (A)
- (C) (B), (D), (C), (A)
- (D) (A), (C), (B), (D)

10. From among the four options given, choose the grammatically correct sentence.

- (A) Millions of us take a multivitamin every day and we feel confident that we are getting results.
- (B) Millions of us taking a multivitamin every day and feeling confident that we are getting results.
- (C) Millions of us take a multivitamin every days and we felt confident that we is getting results.
- (D) Millions of you take a multivitamin every day and we feel confidence that we are get result.

11. Consider the following pairs:

- (A) Chapchar Kut - Mizoram
- (B) Khongjom Parba ballad - Manipur
- (C) Thang-Ta dance - Sikkim

Choose the correct answer from the options given below:

- (A) (A) and (B) only
- (B) (A) and (C) only
- (C) (B) and (C) only
- (D) (A), (B) and (C) only

12. Match List I with List II

List I	List II
--------	---------

(A) Fourth Schedule	(I) Administration of Tribal Areas of Assam, Meghalaya, Tripura and Mizoram
(B) Sixth Schedule	(II) Anti-Defection Laws
(C) Eighth Schedule	(III) Allocation of Seats for State and UT's in Rajya Sabha
(D) Tenth Schedule	(IV) Official Languages

Choose the correct answer from the options given below:

- (A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
 (B) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
 (C) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
 (D) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

13. Arrange the following in chronological order:

- (A) Tughlaqs
 (B) Lodhis
 (C) Sayyids
 (D) Ilbari Turks
 (E) Khaljis

Choose the correct answer from the options given below:

- (A) (B), (A), (C), (D), (E)
 (B) (E), (A), (D), (C), (B)
 (C) (D), (E), (A), (C), (B)
 (D) (D), (C), (B), (A), (E)

14. Who is won the Nobel Prize in Literature 2021

- (A) Haruki Murakami
 (B) Abdulrazak Gurnah
 (C) Paul Kegan
 (D) Joseph Stuart

15. Who has been appointed as the new vice-president of NITI Aayog?

- (A) Suman Berry
 (B) Rajiv Kumar
 (C) Surjit Bhalla
 (D) Jay Bhattacharya

16. If IMAGE is written as 0913010705, then how will BOUGHT be written as

- (A) 215021708112
 (B) 021521070820
 (C) 040603122612
 (D) 025121702180

17. In a family, there is a man, his wife, his father-in-law, two sons, their wives and two daughters. One son has two daughters and the other son has three sons. How many total members are in the family?

- (A) 12
- (B) 14
- (C) 15
- (D) 13

18. Six friends Geeta, Suresh, Mani, Pradeep, Deepti and Rekha are sitting in a circle facing the centre. Deepti is to the left of Pradeep. Mani is between Geeta and Suresh. Rekha is between Deepti and Geeta. Who is sitting immediate left of Suresh?

- (A) Deepti
- (B) Pradeep
- (C) Geeta
- (D) Rekha

19. Relationship between two words is given in a sentence. Use the same relationship to find out which choices complete the relationship with the third word?

Oar is to Rowboat as foot is to _____.

- (A) Running shoes
- (B) Exercising
- (C) Skateboard
- (D) Leg

20. Find the wrong number on the given sequence of numbers -

816, 844, 821, 849, 823, 854, 831

- (A) 854
- (B) 821
- (C) 849
- (D) 823

21. Arrange the following fractions (A-D) in their descending order.

- (A) $\frac{3}{5}$
- (B) $\frac{4}{7}$
- (C) $\frac{8}{9}$
- (D) $\frac{9}{11}$

Choose the correct answer from the options given below :

- (A) (C), (D), (B), (A)
- (B) (A), (D), (C), (B)
- (C) (D), (C), (B), (A)
- (D) (C), (D), (A), (B)

22. If $1^3 + 2^3 + 3^3 + \dots + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + \dots + (0.99)^3$ is close to:

- (A) 0.2695
- (B) 0.3695
- (C) 2.695
- (D) 3.695

23. The simplified value of

$\left(1 - \frac{1}{n}\right) + \left(1 - \frac{2}{n}\right) + \left(1 - \frac{3}{n}\right) + \dots$ upto n terms is:

- (A) $n/2$
- (B) $(n - 1)/2$
- (C) $n(n - 1)/2$
- (D) $1/n$

24. A, B, C and D share a property worth ₹77.500. If $A : B = 3 : 2$; $B : C = 5 : 4$ and $C : D = 3 : 7$, then what is the share of B ?

- (A) ₹ 20,000
- (B) ₹ 15,000
- (C) ₹ 25,000
- (D) ₹ 14,000

25. In a mixture, the ratio of the alcohol and water is 6 : 5. When 22 litre mixture is replaced by water, the ratio becomes 9 : 13. What is the quantity of water after replacement?

- (A) 62 litres
- (B) 50 litres
- (C) 40 litres
- (D) 52 litres

26. Let the eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ be written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is the value of $(a+b)$?

- (1) 0
- (2) 1
- (3) $\frac{1}{2}$
- (4) 2

27. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Then the determinant of $A^2 + A$ is

- (1) 196
- (2) 0
- (3) 144
- (4) 100

28. Which one of the following is correct?

- (1) Any abelian group of order 21 is cyclic
- (2) Any abelian group of order 27 is cyclic
- (3) Any abelian group of order 8 is cyclic
- (4) Any abelian group of order 9 is cyclic

29. Let $(\mathbb{Z}, +, \bullet)$ be the ring of integers. Let $\langle m \rangle$ for $m \in \mathbb{Z}$ be the ideal generated by m . Then which one is a field?

- (1) $\frac{\mathbb{Z}}{\langle 3 \rangle}$
- (2) $\frac{\mathbb{Z}}{\langle 4 \rangle}$
- (3) $\frac{\mathbb{Z}}{\langle 8 \rangle}$
- (4) $\frac{\mathbb{Z}}{\langle 6 \rangle}$

30. Let $\langle X \rangle$ be the ideal generated by the polynomial $p(x) = x$ in the ring $\mathbb{Z}[x]$. Then $\frac{\mathbb{Z}[x]}{\langle x \rangle}$ is

- (1) A field
- (2) Not a field but ring with zero divisor
- (3) Not an integral domain
- (4) An integral domain

31. The groups \mathbb{Z}_9 and $\mathbb{Z}_3 \times \mathbb{Z}_3$ are

- (1) Isomorphic
- (2) Both abelian
- (3) Both non-abelian
- (4) Both cyclic

32. Let $(\mathbb{F}, +, \bullet)$ be a finite field, then order of \mathbb{F} can not be

- (1) 6
- (2) 4
- (3) 9
- (4) 8

33. Given below are two statements:

Statement I:

There exists no integral domain of order 6.

Statement II:

The total number of distinct permutations on the set $S = \{1, 2, 3\}$ is 6.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both statement I and statement II are true
- (2) Both statement I and statement II are false
- (3) Statement I is true but statement II is false
- (4) Statement I is false but statement II is true

34. Given below are two statements:

Statement I:

For each positive integer n , the set $n\mathbb{Z} = \{0, \pm n, \pm 2n, \pm 3n, \dots\}$ is a subring of the ring of integers.

Statement II:

$(\mathbb{Z}_6, +_6, \times_6)$ is an integral domain.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both statement I and statement II are correct
- (2) Both statement I and statement II are incorrect
- (3) Statement I is correct but statement II is incorrect
- (4) Statement I is incorrect but statement II is correct

35. A. For a matrix A , $A^T A$ is always symmetric

B. $A + A^T$ is a symmetric matrix.

C. In a group G , if $a, b, c \in G$, are such that $ab = ac$, then $b = c$.

D. In the ring R , for $a, b \in R$, $(a + b)^2 = a^2 + b^2 + ab + ba$.

E. Every integral domain is a field.

Choose the correct answer from the options given below:

- (1) A, B, C, D only
- (2) A, B, C, D, E only
- (3) A, B only
- (4) B, C only

36. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

Let W_1 and W_2 be any two subspaces of a vector space V , then $W_1 \cap W_2$ is also a subspace.

Reason R:

Intersection of a subspace W of V with any other subset U of V that contains $\bar{0}$ is again a subspace of V .

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both A and R are true and R is the correct explanation of A
- (2) Both A and R are true but R is not the correct explanation of A
- (3) A is true but R is false
- (4) A is false but R is true

37. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

If a real matrix has one eigenvector, then it has an infinite number of eigenvectors.

Reason R:

Scalar multiple of an eigenvalue of a matrix is also an eigenvalue of the matrix.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both A and R are correct and R is the correct explanation of A

- (2) Both A and R are correct but R is not the correct explanation of A
 (3) A is correct but R is not correct
 (4) A is not correct but R is correct

38. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

Let G be an abelian group of order 15. Then G will have an element of order 3.

Reason R:

If a number n divides order of G , then G has an element of order n .

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both A and R are true and R is the correct explanation of A
 (2) Both A and R are true but R is not the correct explanation of A
 (3) A is false but R is true
 (4) A is true but R is false

39. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

The matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ have same eigenvalues.

Reason R:

The above matrices A and B are similar matrices.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both A and R are correct and R is the correct explanation of A
 (2) Both A and R are correct but R is not the correct explanation of A
 (3) A is correct but R is not correct
 (4) A is not correct but R is correct

40. Match List I with List II.

List I	List II
Vector Spaces	Dimensions
A. \mathbb{R} over \mathbb{R}	I. 4
B. \mathbb{C} over \mathbb{R}	II. 3
C. \mathbb{R}^3 over \mathbb{R}	III. 2
D. \mathbb{C}^2 over \mathbb{R}	IV. 1

Choose the correct answer from the options given below:

- (1) A-IV, B-III, C-II, D-I
 (2) A-I, B-IV, C-II, D-III
 (3) A-II, B-I, C-III, D-IV
 (4) A-I, B-II, C-III, D-IV

41. The value of $\lim_{n \rightarrow \infty} \sqrt[n]{n}$ is

- (1) 0
- (2) ∞
- (3) 1
- (4) 2

42. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if p is

- (1) 2
- (2) $\frac{1}{2}$
- (3) $\frac{1}{3}$
- (4) $\frac{1}{4}$

43. Let $a < b$ in \mathbb{R} . Suppose $f: [a, b] \rightarrow (a, b)$ is continuous.

- (1) f may be onto
- (2) f may be one-one
- (3) f may be one-one and onto
- (4) $f([a, b]) = \{c, d\}$ for some $c, d \in (a, b), c \neq d$

44. Rolle's theorem is applicable for

- (1) $f(x) = x^2$ in $[-1, 0]$.
- (2) $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$
- (3) $f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, if $[-\frac{1}{\pi}, \frac{1}{\pi}]$
- (4) $f(x) = |x|$ in $[-1, 1]$.

45. Let A and B be two non-empty subsets of \mathbb{R} . If \bar{A} , A° , and A' denote closure, interior and set of limit points of A respectively, then

- (1) $A^\circ \cap \bar{B}$ is open.
- (2) $A^\circ \cap B'$ is open.
- (3) $\bar{A} \cap B'$ is closed.
- (4) $\bar{A} \cap B$ is closed.

46. If $u(x, y) = \cos^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$, $0 < x < y < 1$.

Then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$

- (1) $-\frac{1}{2} \tan u$
- (2) $-\frac{1}{2} \cot u$
- (3) $\sin u$
- (4) $\cos u$

47. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(x, y) = \sqrt{|xy|}$, then

- (1) f_x, f_y do not exist at $(0, 0)$
- (2) directional derivative of f exists at $(0, 0)$ in every non-zero direction $u = (u_1, u_2)$ in \mathbb{R}^2
- (3) f is differentiable
- (4) f is not differentiable

48. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & \text{if } x^2 + y^2 \neq 0 \\ 0 & \text{if } x = 0 = y \end{cases}$$

Let $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y) = a$ and $\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y) = b$

Then

- (1) $a = b = 1$
- (2) $a = b = -1$
- (3) $a = 1$ and $b = -1$
- (4) $a = -1$ and $b = 1$

49. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

The sequence $\left\{(-1)^n \cdot \frac{n+1}{n}\right\}$ is not convergent

Reason R:

Every monotonic sequence is convergent

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both A and R are correct and R is the correct explanation of A
- (2) Both A and R are correct but R is not the correct explanation of A
- (3) A is not correct but R is correct
- (4) A is correct but R is not correct

50. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Let \mathbb{Q} be the set of rational numbers

Assertion A:

\mathbb{Q} is not complete.

Reason R:

The set $\{x \in \mathbb{Q} : x^2 < 2\}$ has no supremum in \mathbb{Q} .

In the light of the above statements, choose the correct answer from the options given below

- (1) Both A and R are true and R is the correct explanation of A
- (2) Both A and R are true but R is not the correct explanation of A
- (3) A is true but R is false
- (4) A is false but R is true

51. Match List I with List II

List I	List II
$f(x)$ in the interval	'c' in Rolle's Theorem, $f'(c) = 0$
A. $(x-1)(x-2)(x-3)$ in $[1,3]$	I. $\frac{2m-1}{2(m+n)-1}a$
B. $\sin x$ in $[0, \pi]$	II. $\frac{n}{m+n}a$
C. $x^{2m-1}(a-x)^{2n}$ in $[0, a]$	III. $2 - \frac{2}{\sqrt{3}}$
D. $x^{2n}(a-x)^{2m}$ in $[0, a]$	IV. $\frac{\pi}{2}$

Choose the correct answer from the options given below:

- (1) A-I, B-II, C-IV, D-III
 (2) A-II, B-III, C-I, D-IV
 (3) A-III, B-IV, C-I, D-II
 (4) A-III, B-IV, C-II, D-I

52. Given below are two statements:

Statement I:

Consider $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) := x^2$. Then f has a local extremum at $(0, 0)$.

Statement II:

Consider $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) := xy$. Then f has a saddle point at $(0, 0)$.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both statement I and statement II are true
 (2) Both statement I and statement II are false
 (3) Statement I is true but statement II is false
 (4) Statement I is false but Statement II is true

53. Given below are two statements:

Statement I:

$\frac{1}{x+y}$ is a homogeneous function of degree -1.

Statement II:

A function $f(x, y)$ is said to be a homogeneous function of degree n in x and y if it satisfies $f(x, y) = x^n g(x)$

In the light of the above statements, choose the correct answer from the options given below

- (1) Both statement I and statement II are correct
 (2) Both statement I and statement II are incorrect
 (3) Statement I is correct but statement II is incorrect
 (4) Statement I is incorrect but statement II is correct

54. A. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a Linear transformation such that $T(1,0) = (0,0) = T(0,1)$. Then $T(1,2) = (0,0)$.

B. The number of group homomorphisms from \mathbb{Z}_{10} to \mathbb{R}_7 is one.

C. The number of one dimensional subspaces of \mathbb{R}^2 over \mathbb{R} is two.

D. Let $D = \{z \in \mathbb{C} : |z| < 1\}$, $f : \mathbb{C} \rightarrow D$ be analytic, and $f\left(1 + \frac{i}{n}\right) = 1 - i$. Then $f\left(1 - \frac{i}{n}\right) = 1 + i$.

E. Every closed convex set is a feasible region of LPP.

Choose the correct answer from the options given below:

- (1) B, D, E only
- (2) A, E only
- (3) A, B only
- (4) A, B, C only

55. The value of $\int_C \frac{\cos(e^{2z})}{z} dz$, where $C : |z| = 2$ is

- (1) 0
- (2) $2\pi i$
- (3) $\pi i(e^i + e^{-i})$
- (4) $\pi i(e^i - e^{-i})$

56. Complex function $f(z) = \frac{z - \sin z}{z^2}$, has

- (1) Essential singularity at $z = 0$
- (2) Removable singularity at $z = 0$
- (3) Non-isolated essential singularity at $z = 0$
- (4) Simple pole at $z = 1$

57. Radius of convergence of the power series $\sum \left(\frac{n}{n+1}\right)^2 z^n$ is

- (1) zero
- (2) 2
- (3) 3
- (4) 1

58. The value of $\int_C \frac{z+4}{z^2+4z+13} dz$, where C is the circle $z + 2 = 1$ is

- (1) 0
- (2) $2\pi i$
- (3) $-2 + 3i$
- (4) $-2 - 3i$

59. Match List I with List II

List I	List I
Integral	Value
A. $\int_{ z =\frac{1}{2}} \frac{z+1}{z-\frac{3}{4}} dz$	I. $-\frac{86}{3} - 6i$
B. $\int_{ z+i =4} \frac{dz}{z-3}$	II. $2\pi i$
C. $\int_{1+i}^{2+4i} z^2 dz$ along parabola $x = t, y = t^2$	III. $-2\pi i$
D. $\int_{ z-1 =3} \frac{\cos z}{z-\pi} dz$	IV. 0

Choose the correct answer from the options given below:

- (1) A-I, B-IV, C-II, D-III
- (2) A-II, B-I, C-IV, D-III
- (3) A-IV, B-II, C-I, D-III
- (4) A-III, B-II, C-IV, D-I

60 Match List I with List II.

List I	List II
Complex Functions	Singular Points
A. $f(z) = \frac{\sin z}{z^2 - 5z + 6}$	I. Pole of order 3 at $z=2$
B. $f(z) = \frac{e^z}{(z-1)^4}$	II. Simple poles at $z=2, z=3$
C. $f(z) = \frac{z^2}{(z-3)^3}$	III. Pole of order 4 at $z = 1$
D. $f(z) = \frac{z^2}{(z-2)^3}$	IV. Pole of order 3 at $z=3$

Choose the correct answer from the options given below:

- (1) A-I, B-II, C-IV, D-III
- (2) A-II, B-III, C-IV, D-I
- (3) A-II, B-IV, C-III, D-I
- (4) A-II, B-I, C-IV, D-III

61. Given below are two statements:

Statement I:

The singularity of $f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3(3z+2)^2}$ at $z = \infty$ is a pole of order 3.

Statement II:

The singularity of $f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3(3z+2)^2}$ at $z = 1$ is a pole of order 3.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both statement I and statement II are true
- (2) Both statement I and statement II are false
- (3) Statement I is true but statement II is false
- (4) Statement I is false but statement II is true

62. A. $f(z) = z$ is not analytic $\forall z \in \mathbb{C}$.

B. $f(z) = \bar{z}$ is not analytic $\forall z \in \mathbb{C}$.

C. $f(z) = z\bar{z}$ is differentiable at $z = 0$ but not analytic in any region.

D. $f(z) = \sin z$ is unbounded function.

E. $f(z) = \cos z$ is bounded function.

Choose the most appropriate answer from the options given below:

- (1) A, B, E only
- (2) A, C, E only
- (3) D, E only
- (4) A, B, C, D only

63. Value of the integral $\int_0^{\pi/2} \log \tan x \, dx$ is

- (1) zero
- (2) $\frac{\pi}{2}$
- (3) $\frac{\pi}{4}$
- (4) ∞

64. What is the value of

$$\lim_{n \rightarrow \infty} \left[\frac{1}{n^2+1^2} + \frac{2}{n^2+2^2} + \frac{3}{n^2+3^2} + \dots + \frac{1}{2n} \right] ?$$

- (1) $\log 2$
- (2) $\frac{1}{2} \log 2$
- (3) $\log 3$
- (4) $\frac{1}{2} \log 3$

65. Value of the double integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$, is

- (1) $\frac{\pi}{3}$
- (2) $\frac{1}{6}$
- (3) $\frac{\pi}{4}$
- (4) $\frac{\pi}{8}$

66. The formula of volume in spherical coordinates is

- (1) $\iiint r^2 \sin \theta \, dr \, d\theta \, d\phi$
- (2) $\iiint \sin \theta \, dr \, d\theta \, d\phi$
- (3) $\iiint dr \, d\theta \, d\phi$
- (4) $\iiint r^3 \, dr \, d\theta \, d\phi$

67. The formula for the volume of the solid generated by the revolution of the area bounded by the curve $r = f(\theta)$ and radii vectors $\theta = \alpha; \theta = \beta$, about the initial line ($\theta = 0$) is equal to

- (1) $\int_\alpha^\beta \frac{2\pi}{3} r^3 \sin \theta \, d\theta$
- (2) $\int_\alpha^\beta \frac{2\pi}{3} r^3 \cos \theta \, d\theta$
- (3) $\int_\alpha^\beta \frac{2\pi}{3} r^3 \sin^3 \theta \, d\theta$
- (4) $\int_\alpha^\beta \frac{2\pi}{3} r^3 \cos^3 \theta \, d\theta$

68. The length of the arc of the curve $x = f(t); y = \phi(t)$ between the points from $t = a$ to $t = b$, is equal to

- (1) $\int_a^b \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{1/2} dt$

(b) $\int_a^b \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{1/2} dt$

(c) $\int_a^b \left[y^2 \left(\frac{dx}{dt} \right)^2 + x^2 \left(\frac{dy}{dt} \right)^2 \right]^{1/2} dt$

(d) $\int_a^b \left[1 + \left(\frac{dx}{dt} \right)^2 \right]^{1/2} dt$

69. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

Area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is πab .

Reason R:

Area is defined as $\iint_R dx dy$ for the region R.

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both A and R are correct and R is the correct explanation of A
- (2) Both A and R are correct but R is not the correct explanation of A
- (3) A is correct but R is not correct
- (4) A is not correct but R is correct

70. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

The derivative of $F(x) = \int_{\frac{1}{x}}^{\sqrt{x}} \cos t^2 dt$ ($x > 0$) at $x = 1$ is $\frac{3}{2} \cos 1$

Reason R:

Since $\frac{d}{dx} \int_{\Psi(x)}^{\Phi(x)} f(t) dt = f(\Phi(x)) - f(\Psi(x))$

In the light of the above statements, choose the correct answer from the options given below

- (1) Both A and R are true and R is the correct explanation of A
- (2) Both A and R are true but R is not the correct explanation of A
- (3) A is true but R is false
- (4) A is false but R is true

71. If $z = f(x, y)$ is a homogeneous function of x, y of degree n , then

A. $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$

B. $z = x^n \phi\left(\frac{y}{x}\right)$

C. $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz$

D. $\frac{\partial z}{\partial x} + y \frac{\partial^2 z}{\partial x \partial y} = n \frac{\partial z}{\partial y}$

E. $\frac{\partial z}{\partial x} + x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x \partial y} = n \frac{\partial z}{\partial x}$

Choose the correct answer from the options given below:

- (1) A, B, D only
- (2) A, C, D only
- (3) B, C, E only
- (4) A, B, E only

72. The bending moment M of a beam is given by $\frac{dM}{dx} = -w(l - x)$, where w and l are constants. For $x = 0, M = \frac{1}{2}wl^2$, then determine M in terms of x .

- (1) $\frac{1}{2}w(l - x)^2$
- (2) $-\frac{1}{2}w(l - x)^2$
- (3) $\frac{1}{2}w(l - x)$
- (4) $-\frac{1}{2}w(l - x)$

73. The solution of the differential equation $\frac{d^2y}{dx^2} - 4y = x \sinh x$ is

- (1) $y = c_1 e^{2x} + c_2 e^{-2x} - \frac{x}{3} \sinh x - \frac{2}{9} \cosh x$
- (2) $y = c_1 e^{-2x} + c_2 e^x - \frac{x}{3} \sinh x - \frac{2}{9} \cosh x$
- (3) $y = c_1 e^x + c_2 e^{-x} - \frac{x}{3} \sinh x - \frac{2}{9} \cosh x$
- (4) $y = c_1 e^{2x} + c_2 e^{-2x} - \frac{x}{3} \sinh x + \frac{2}{9} \cosh x$

74. The integrating factor (IF) of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) \frac{dx}{dy} = 1$ is :

- (1) $e^{2\sqrt{x}}$
- (2) $e^{-2\sqrt{x}}$
- (3) $e^{\sqrt{x}}$
- (4) $e^{-\sqrt{x}}$

75. The integrating factor of the differential equation $x dy - (y + xy(1 + \log x)) dx = 0$ is

- (1) x
- (2) x^2
- (3) $\log x$
- (4) e^x

76. The Wronskian of the functions $1, \sin x, \cos x$ is

- (1) 0
- (2) 1
- (3) $\frac{1}{2}$
- (4) -1

77. The orthogonal trajectories of the family of curves $r^n = a \sin n\theta$ is

- (1) $r^n = b \sin n\theta$

(2) $r^n = b \cos n\theta$

(3) $r = b \sin n\theta$

(4) $r = b \cos n\theta$

78. The complete solution (CS.) of the differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ is :

(1) $y = c_1 e^x + c_2 \cos x + c_3 \sin x$

(2) $y = c_1 e^{-x} + c_2 \cos x + c_3 \sin x$

(3) $y = c_1 e^{-x} + c_2 \cos 2x + c_3 \sin 2x$

(4) $y = c_1 e^x + c_2 \cos 2x + c_3 \sin 2x$

79. The particular integral (PI) of the differential equation $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 5e^{(3/2)x}$ is given as

(1) $Xe^{-(3/2)x}$

(2) $e^{(3/2)x}$

(3) $xe^{(3/2)x}$

(4) $e^{-(3/2)x}$

80. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

The solution of differential equation $\frac{d^2y}{dx^2} + 4y = 0$ is $c_1 e^{2x} + c_2 e^{-2x}$

Reason R:

If $y_1(x)$ and $y_2(x)$ are any two solutions of the differential equation $\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$, then their linear combination $c_1 y_1 + c_2 y_2$ where c_1 and c_2 are constants, is also a solution of the differential equation.

In the light of the above statements, choose the correct answer from the options given below

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is not the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

81. Given below are two statements:

Statement I:

In the equation $M(x, y) dx + N(x, y) dy = 0$, if $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ is a function $f(y)$ of y only, then $\int_e f(y) dy$ is an integrating factor.

Statement II:

In the equation $M(x, y) dx + N(x, y) dy = 0$, if $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M}$ is a function $f(y)$ of y only, then $\int_e f(y) dy$ is an integrating factor.

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both statement I and statement II are correct
 (2) Both statement I and statement II are incorrect
 (3) Statement I is correct but statement II is incorrect
 (4) Statement I is incorrect but statement II is correct

82. Match List I with List II:

List I	List II
Differential Equation	Integrating factor (IF)
A. $\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$	I. $\frac{1}{x^2y^2}$
B. $y^{-3} \frac{dy}{dx} + \frac{y^{-2}}{x} = x^2$	II. x^{-2}
C. $x^{-2} \frac{dx}{dy} - yx^{-1} = y^3$	III. $e^{y^2/2}$
D. $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$	IV. $\frac{1}{x+1}$

Choose the correct answer from the options given below:

- (1) A-II, B-I, C-IV, D-III
 (2) A-IV, B-II, C-I, D-III
 (3) A-I, B-III, C-IV, D-II
 (4) A-IV, B-II, C-III, D-I

83. Value of $\nabla^2(r^n \vec{r})$ is

- (1) $(n+3)r^{n-2}\vec{r}$
 (2) $(n+3)r^{n-1}\vec{r}$
 (3) $n(n+3)r^{n-2}\vec{r}$
 (4) $nr^{n-2}\vec{r}$

84. Value of $\int_{(1,2)}^{(3,4)} (6xy^2 - y^3)dx + (6x^2y - 3xy^2)dy$ is

- (1) 206
 (2) 136
 (3) 216
 (4) 236

85. If $\phi = x^2yz^2$ and $\vec{A} = xz\hat{i} - y^2\hat{j} + 2x^2y\hat{k}$, the value of $\text{curl}(\phi\vec{A})$ is

- (1) $(4x^4yz^3 + x^2y^3z^2)\hat{i} + (4x^3yz^3 - 8x^3y^2z^3)\hat{j} - (2xy^3z^3 + x^3z^4)\hat{k}$
 (2) $4x^4yz^3\hat{i} + 4x^3yz^3\hat{j} - 2xy^3z^3\hat{k}$
 (3) $(4x^4yz^3 - 9x^2y^3z)\hat{i} + (4x^3yz^3 - 8x^2yz^3)\hat{j} - (2xy^3z^3 + x^3z^2)\hat{k}$
 (4) $3x^2y^3z^2\hat{i} + 4x^3yz^2\hat{j} + x^3z^4\hat{k}$

86. The value of $\nabla^2\left(\frac{x}{r^3}\right)$ is

- (1) 3
 (2) -3
 (3) 0

(4) 1

87. What is the value of $\iint (x^2 + y^2) dS$, where S is the surface of the paraboloid $z = 2(x^2 + y^2)$ above the xy plane?

- (1) $\frac{149\pi}{6}$
 (2) $\frac{139\pi}{30}$
 (3) $\frac{149\pi}{30}$
 (4) $\frac{139\pi}{6}$

88. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

The vector field $\vec{v} = -(x + y + 2)\hat{i} - 2\hat{j} + (x + y)\hat{k}$ is solenoidal.

Reason R:

If $\text{div}(\vec{v}) = 0$, the vector field \vec{v} is said to be solenoidal.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both A and R are true and R is the correct explanation of A
 (2) Both A and R are true but R is not the correct explanation of A
 (3) A is true but R is false
 (4) A is false but R is true

89. Which of the following are correct?

- A. If \vec{F} is a vector field, $\nabla \cdot \vec{F}$ is a vector field.
 B. If \vec{F} is a vector field, $\nabla \times \vec{F}$ a vector field.
 C. If \vec{F} is conservative, $\nabla \times \vec{F} = 0$.
 D. $\text{Curl}(\text{div } \vec{F})$ is not a meaningful expression.
 E. If \vec{F} is conservative on a simply connected region D , there is some function on D such that $\nabla f = \vec{F}$.
 Choose the correct answer from the options given below:

- (1) A, B only
 (2) A, B, C only
 (3) B, C, E only
 (4) B, C, D, E only

90. Match List I with List II:

List I	List II
Operator on functions	Values
A. $\text{grad}(xy^2)$	I. $2xy^2 + ye^{yz}$
B. $\text{curl}(zx^2\hat{i} + y^2\hat{j} + z\hat{k})$	II. $\vec{0}$
C. $\text{curl}(\text{grad}(xy))$	III. $\hat{i}y^2 + \hat{j}(2xy)$
D. $\text{div}(x^2y^2\hat{i} + 2xz\hat{j} + e^{yz}\hat{k})$	IV. $x^2\hat{j}$

Choose the correct answer from the options given below:

- (1) A-I, B-III, C-IV, D-II
- (2) A-II, B-III, C-I, D-IV
- (3) A-III, B-IV, C-II, D-I
- (4) A-I, B-IV, C-III, D-II

91. A feasible solution to a linear programming problem

- (1) must satisfy all of the problem's constraints simultaneously.
- (2) need not satisfy all the constraints, only some of them.
- (3) must be a corner point of the feasible region.
- (4) must optimize the value of the objective function.

92. Which set of the following is a convex set?

- (1) $\{(x_1, x_2) \in R^2 | 2x_1 + 4x_2 \leq 4, 8x_1 + x_2 \leq 8\}$
- (2) $\{(x_1, x_2) \in R^2 | x_1^2 + x_2^2 \leq 1, x_1^2 \geq x_2\}$
- (3) $\{(x_1, x_2) \in R^2 | -x_1x_2 + 1 \geq 0; x_1, x_2 \geq 0\}$
- (4) $\{(x_1, x_2) \in R^2 | x_1x_2 \leq 4; x_1x_2 \leq 1; x_1, x_2 \geq 0\}$

93. For the linear programming problem $\text{Max } z = x + 3y$, the coordinates of the corner points of the bounded feasible region are A(3, 3), B(20, 3), C(20, 10), D(18, 12) & E(12, 12). The max value of z will be at the point

- (1) (3, 3)
- (2) (20, 3)
- (3) (12, 12)
- (4) (18, 12)

94. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A:

Some linear programming problems have more than one optimal solutions.

Reason R:

If the feasible region of a linear programming problem is unbounded, it must have more than one optimal solution.

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both A and R are correct and R is the correct explanation of A
- (2) Both A and R are correct but R is not the correct explanation of A
- (3) A is not correct but R is correct
- (4) A is correct but R is not correct

95. Given below are two statements:

Statement 1:

Every LPP admits a unique optimal solution.

Statement II:

The set of all feasible solutions of a LPP may or may not be a convex set.

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both Statement I and Statement II are correct
- (2) Both Statement I and Statement II are incorrect
- (3) Statement I is correct but Statement II is incorrect
- (4) Statement I is incorrect but Statement II is correct

96. Which one of the following is a linear programming problem?

- A. $\max 2x_1 + 4x_2$; subject to, $x_1 + x_2 \leq 4$; $x_1, x_2 \geq 0$
- B. $\min 2x_1 + 4x_2$; subject to, $x_1 - x_2 \leq 4$; $x_1, x_2 \geq 0$
- C. $\max x_1$; subject to, $|x_1| + |x_2| \leq 4$; $x_1, x_2 \geq 0$
- D. $\max x_1^2$; subject to, $x_1^2 + 8x_2 \leq 4$; $x_1 - x_2 \geq 0$
- E. $\max x_1 x_2$; subject to $x_1 + x_2 \leq 4$; $x_1, x_2 \geq 0$

Choose the correct answer from the options given below:

- (1) A, C only
- (2) A, D only
- (3) C, E only
- (4) A, B, C only

97. Match List I with List II:

List I	List II
Convex Set	No. of Extreme Points
A. $S_1 = \{(x_1, x_2): X_1^2 + X_2^2 \leq 1\}$	I. 3
B. $S_2 = \{(x_1, x_2): 2x_1 + x_2 \leq 4; x_1, x_2 \geq 0\}$	II. infinite
C. $S_3 = \{0 \leq x_1 \leq 4; 0 \leq x_2 \leq 4\}$	III. 5
D. $S_4 = \{(x_1, x_2): x_1 + x_2 \leq 6; 2x_1 - x_2 \leq 6; x_2 \leq 4; x_1, x_2 \geq 0\}$	IV. 4

Choose the correct answer from the options given below:

- (1) A-II, B-I, C-III, D-IV
- (2) A-I, B-II, C-III, D-IV
- (3) A-II, B-I, C-IV, D-III
- (4) A-IV, B-I, C-II, D-III

98. In the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$

- A. The value of Wronskian of the solution is ex
- B. e^{3x} and xe^{3x} are two linearly independent solutions of the differential equation
- C. $PI = e^{3x} (\log x + 1)$
- D. $W = e^{-6x}$, W = Wronskian of solution of the differential equation
- E. $PI = -e^{3x} (\log x + 1)$

Choose the most appropriate answer from the options given below:

- (1) B, C only
- (2) A, B, E only
- (3) B, C, D only
- (4) A, C, D only

99. Given below are two statements:

Statement I:

In a linear programming problem, more than one degenerate basic feasible solutions may correspond to the same corner point.

Statement II:

The system of equations $x_1 + x_2 + x_3 = 3, x_1 - x_2 - x_4 = 0, x_1, x_2, x_3 \geq 0$ has 3 degenerate basic feasible solutions.

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both statement I and statement II are correct
- (2) Both statement I and statement II are incorrect
- (3) Statement I is correct but statement II is incorrect
- (4) Statement I is incorrect but statement II is correct

100. Match List I with List II:

List I	List II
Function	
A. $f(x, y) = 2x + 3y$	I. convex function
B. $f(x, y) = xy - 1$	II. concave function
C. $f(x, y) = x^2 + y^2 - 1$	III. both convex and concave function
D. $f(x, y) = -x^2 - y$	IV. neither convex nor concave

Choose the correct answer from the options given below:

- (1) A-I, B-III, C-IV, D-II
- (2) A-III, B-II, C-IV, D-I
- (3) A-I, B-IV, C-III, D-II
- (4) A-III, B-IV, C-I, D-II