

1. Select the correct word that can best complete the given sentence:

By training children at a young age we can bring out the _____ talents in them.

- (1) artificial
- (2) budding
- (3) superficial
- (4) inherent

2. From the choices given below, select the pair which exhibits the same relationship as the one in capitalized pair of words:

SOUP: APPETIZER

- (1) coffee : bean
- (2) pudding : dessert
- (3) breakfast : cereal
- (4) tea : drink

3. Select the most suitable synonym:

MUDDE

- (1) whisper
- (2) horde
- (3) disorder
- (4) speculate

4. Identify the part of the body with which the disease is associated:

Leukemia

- (1) heart
- (2) skin
- (3) lungs
- (4) lymphatic system

5. Identify the meaning of the expression below from the options given:

Per se

- (1) by word of mouth
- (2) gossip
- (3) by itself
- (4) spontaneous

6. Identify the meaning of the underlined word:

Home appliances manufacturer 'B' electrical Limited is looking for acquisitions in the domestic market.

- (1) obtainment
- (2) apprentice
- (3) attrition
- (4) attention

7. Identify the meaning of the phrase below from the options given:

Leave to a person by a will

- (1) beseech
- (2) billow
- (3) blanch
- (4) bequeath

8. Select the most suitable antonym:

SKIMPY

- (1) glaring
- (2) modest
- (3) affluent
- (4) generous

9. Identify the meaning of the idiom from the options given:

A man of the world

- (1) headstrong and arrogant
- (2) highly trustworthy
- (3) very popular because of success
- (4) highly experienced in many fields

10. Select the correct word from the answer:

Now a days, it is difficult to ___ good books in English.

- (1) track
- (2) verify
- (3) find
- (4) know

11. If Neena says "Anita's father Raman is the only son of my father-in-law Mahipal", then how is Bindu, who is the sister of Anita, related to Mahipal?

- (1) Neice
- (2) Daughter
- (3) Wife
- (4) None of these

12. Choose the missing term out of the given alternatives.

Y, W, U, S, Q, __, __

- (1) N, J
- (2) M, L
- (3) J, R
- (4) O, M

13. Arrange the given words in alphabetical order and tick the one that comes last.

- (1) Abandon
- (2) Actuate
- (3) Accumulate
- (4) Acquit

14. Raman ranks sixteenth from the top and forty nine from the bottom in a class. How many students are there in the class?

- (1) 64
- (2) 65
- (3) 66
- (4) None of these

15. If \times means \div , $-$ means \times , \div means $+$ and $+$ means $-$, then

$$(3 - 15 \div 19) \times 8 + 6 = ?$$

- (1) 8
- (2) 4
- (3) 2
- (4) -1

16. Choose the correct alternative based on relationship:

Botany : Plants :: Entomology : ?

- (1) Snakes
- (2) Insects
- (3) Birds
- (4) Germs

17. Choose the correct answer:

$$9572 - 4018 - 2164 = ?$$

- (1) 3300
- (2) 3390
- (3) 3570
- (4) 7718

18. The H.C.F. (Highest Common Factor) of 2923 and 3239 is:

- (1) 37
- (2) 47
- (3) 73
- (4) 79

19. One fifth of a number exceeds one seventh of the same by 10. The number is:

- (1) 125
- (2) 150
- (3) 175
- (4) 200

20. If a number, divided by 4, is reduced by 21, the number is:

- (1) 18
- (2) 20
- (3) 28
- (4) 38

21. The newly launched 100 rupees notes have the motif of _____ on the reverse side.

- (1) Mangalyaan
- (2) Sanchi stupa
- (3) Hampi with chariot
- (4) Rani ki vav

22. The 'Mission Purvodaya' initiative is related to the development of which of the following?

- (1) Agriculture sector
- (2) Space sector
- (3) Dairy sector
- (4) Steel sector

23. Generally wooden doors are difficult to open or close during rainy season because of

- (1) Diffusion
- (2) Imbibition
- (3) Osmosis
- (4) Photosynthesis

24. Mahabhasya, an outstanding work in the fields of Sanskrit grammar, is attributed to

- (1) Ghosha
- (2) Ashwins
- (3) Patanjali
- (4) Kalidas

25. Who among the following passed the Bengal Sati Regulation Act, 1829, which declared the practice of 'Sati' a punishable offence?

- (1) Lord William Bentinck
- (2) Lord Dalhousie
- (3) Lord Wellesley
- (4) Warren Hastings

PART B-MATHEMATICS

26. Let G be a group of order 121, then

- (1) G is non-abelian
- (2) G is cyclic
- (3) Center of G has order 121

(4) None of these

27. The number of subgroups of \mathbb{Z}_{48} is

- (1) 10
- (2) 48
- (3) 2
- (4) 100

28. How many normal subgroups does a non-abelian group G of order 21 have other than the identity subgroup $\{e\}$ and G ?

- (1) 0
- (2) 7
- (3) 3
- (4) 1

29. Let G be a finite abelian group of order n . Which one of the following is correct?

- (1) If d divides n , there exists an element of order d in G
- (2) If d divides n , there exists a subgroup of G of order d
- (3) If every proper subgroup of G is cyclic then G is cyclic
- (4) None of the above

30. Total number of group homomorphisms from \mathbb{Z}_5 to \mathbb{Z}_7 is

- (1) 35
- (2) 7
- (3) 5
- (4) 1

31. The Ring M of 3×3 matrices with elements from the set of real numbers. The two operations in the ring M are usual addition and multiplication of matrices. Then M is a

- (1) Commutative ring with zero divisors, without unity
- (2) Non-commutative ring with zero divisors, with unity
- (3) Commutative ring with unity
- (4) Field

32. The cardinality of a finite integral domain cannot be

- (1) 21
- (2) 7
- (3) 5
- (4) 3

33. Which one of the following is not prime ideal of the ring \mathbb{Z} of integers?

- (1) $2\mathbb{Z}$
- (2) $4\mathbb{Z}$

(3) $3\mathbb{Z}$
 (4) $5\mathbb{Z}$

34. Which one of the following sets of vectors $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ in \mathbb{R}^n is a subspace of \mathbb{R}^n ($n \geq 3$)?

(1) all α such that $\alpha_1 \geq 0$
 (2) all α such that $\alpha_1 + 3\alpha_2 = \alpha_3$
 (3) all α such that $\alpha_2 = \alpha_1^2$
 (4) all α such that $\alpha_1\alpha_2 = 0$

35. The dimension of the vector space of all symmetric matrices of order $n \times n$ with real entries and trace equal to zero is

(1) $\frac{n(n+1)}{2} - 1$
 (2) $\frac{n(n-1)}{2} + 1$
 (3) $\frac{n(n+1)}{2} + 1$
 (4) $\frac{n(n-1)}{2} - 1$

36. Suppose V is a finite dimensional non-zero vector space over the complex field \mathbb{C} and $T: V \rightarrow V$ is a linear transformation such that range $(T) =$ null space (T) . Then which one of the following is correct?

(1) The dimension of V may be 7
 (2) The dimension of V may be 9
 (3) The dimension of V may be 11
 (4) The dimension of V may be 6

37. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear operator defined by $T(x, y) = (x, 0)$. Then the matrix of T relative to the ordered basis $B = \{(0,1), (1,0)\}$ is

(1) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$
 (2) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
 (3) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
 (4) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

38. Let A and B be 2×2 matrices over the real field. Let $\det A$ and $\det B$ denote the determinants of the matrices A and B respectively. Then which one of the following is true?

(1) $\det(A + B) + \det(A - B) = 2\det A + 2\det B$
 (2) $\det(A + B) - \det(A - B) = 2\det A - 2\det B$
 (3) $\det(A + B) + \det(A - B) = 2\det A - 2\det B$
 (4) $\det(A + B) - \det(A - B) = 2\det A + 2\det B$

39. Let $A = \begin{pmatrix} 2 & 0 & 5 \\ 1 & 2 & 3 \\ -1 & 5 & 1 \end{pmatrix}$. Let $X, Y \in \mathbb{R}^3$. The system of linear equations $AX = Y$ has a solution

(1) only for $Y = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}, x \in \mathbb{R}$

(2) for all $Y \in \mathbb{R}^3$

(3) only for $Y = \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}, y, z \in \mathbb{R}$

(4) only for $Y = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}, y \in \mathbb{R}$

40. Which one of the following is correct?

(1) $S = \{(1, 0, 0), (0, -1, 0), (1, 1, 0)\}$ is a linearly independent set of vectors in \mathbb{R}^3

(2) $S = \{(1, 0, 0), (0, 2, 0), (1, 1, 0)\}$ is a linearly independent set of vectors in \mathbb{R}^3

(3) A subset of a linearly dependent set of vectors is linearly independent

(4) A subset of a linearly independent set of vectors is linearly independent

41. Let W be a subspace of \mathbb{R}^4 given by $W = \{(x, y, z, w): y + z + w = 0\}$. Then the dimension of W is

(1) 3

(2) 2

(3) 1

(4) 4

42. The eigenvalues of a skew-symmetric matrix are

(1) of absolute value 1

(2) real

(3) purely imaginary or zero

(4) negative

43. For the matrix $A = \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, A^{-1} is given by

(1) $A^2 + 2A + 3I$

(2) $A - 3I$

(3) $A^2 - 2A$

(4) $A^2 - 2A - I$

44. Let M^T denotes the transpose of M . Given that $M = \begin{pmatrix} \frac{3}{5} & \frac{3}{5} \\ x & \frac{3}{5} \end{pmatrix}$, where $M^T = M^{-1}$, then the value of x is

(1) $-\frac{4}{5}$

(2) 1

(3) 0

(4) $-\frac{3}{5}$

45. Let $\{a_n\}_{n=1}^{\infty}$ be a bounded sequence of real numbers. Then

- (1) There is a subsequence of $\{a_n\}_{n=1}^{\infty}$ which is convergent
- (2) Every subsequence of $\{a_n\}_{n=1}^{\infty}$ is convergent
- (3) There is exactly one subsequence of $\{a_n\}_{n=1}^{\infty}$ which is convergent
- (4) None of these

46. Which one of the following is incorrect?

- (1) Every subsequence of a convergent sequence of real numbers is convergent
- (2) Every convergent sequence of real numbers is bounded
- (3) Every bounded infinite set of real numbers has at least one limit point
- (4) Every bounded sequence of real numbers is convergent

47. Which one of the following series is convergent?

- (1) $\sum_{n=1}^{\infty} n^2$
- (2) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
- (3) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$
- (4) $\sum_{n=1}^{\infty} n^3$

48. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function. If $\int_0^x f(2t)dt = \frac{x}{\pi} \sin(\pi x)$ for all $x \in \mathbb{R}$ then $f(2)$ is equal to

- (1) 2
- (2) 1
- (3) 0
- (4) -1

49. Which one of the following is incorrect?

- (1) A function which is uniformly continuous on an interval is continuous on that interval
- (2) If a function is continuous in a closed and bounded interval, then it is bounded therein
- (3) The function defined by $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is not continuous at $x = 0$
- (4) A function which is derivable at a point is necessarily continuous at that point

50. Let \mathbb{Q} denotes the complement of the set of rational numbers \mathbb{Q} . The set of all boundary points of \mathbb{Q} in the set of real numbers \mathbb{R} is

- (1) \mathbb{Q}^c

(2) \mathbb{R}
 (3) ϕ
 (4) \mathbb{Q}

51. The set $U = \{x \in \mathbb{R} \mid \sin x = \frac{1}{2}\}$ is

(1) closed
 (2) open
 (3) neither open nor closed
 (4) both open and closed

52. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x - 2)^4$ is

(1) $\frac{1}{256}$
 (2) $\frac{2^4}{256}$
 (3) $\frac{e^4}{256}$
 (4) $\frac{e^2}{256}$

53. Let $x \in \mathbb{R}$. The set of all x at which the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ converges is

(1) $[-1, 1]$
 (2) $(-1, 1)$
 (3) \mathbb{R}
 (4) $[0, 1]$

54. Let $(x, y) \in \mathbb{R}^2$. Which one of the following is true?

(1) $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy^2}{x^2+y^4}$ exists
 (2) $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy^2}{x^2+y^4}$ does not exist
 (3) $f(x, y) = \frac{2xy^2}{x^2+y^4}$ is continuous at $(0,0)$
 (4) None of these

55. Let $(x, y) \in \mathbb{R}^2$. Consider the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ such that $f(x, y) = x^2 + 2xy + y^2$. Then

(1) The function f is a non-homogeneous function
 (2) The function f is a homogeneous function of degree 1
 (3) The function f is a homogeneous function of degree 2
 (4) The function f is a homogeneous function of degree 0

56. If $f: \mathbb{C} \rightarrow \mathbb{C}$ such that $f(z) = u + iv$ with $z = x + iy$. Then which one of the following is true for the function f to be analytic

(1) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$
 (2) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

(3) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$

(4) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$

57. Which one of the following is an analytic function?

- (1) z
- (2) \bar{z}
- (3) Real part of z
- (4) Imaginary part of z

58. The radius of convergence of the power series $\sum_{n=1}^{\infty} n^n z^n$ is

- (1) 1
- (2) ∞
- (3) 0
- (4) None of these

59. The maximum modulus of e^{z^2} on the set $S = \{z \in \mathbb{C} : 0 \leq \operatorname{Re}(z) \leq 1, 0 \leq \operatorname{Im}(z) \leq 1\}$ is

- (1) e
- (2) 1
- (3) $\frac{1}{e}$
- (4) ∞

60. Which one of the following is incorrect?

- (1) Every bounded entire function must be constant
- (2) Every non-constant single-variable polynomial with complex coefficients has at least one complex root
- (3) If $f(z)$ is analytic everywhere within a simply connected region D , then $\oint_C f(z) dz = 0$ for every simple closed path C lying in the region D
- (4) None of these

61. The value of integral $\oint_{|z-t|=2} \frac{1}{z^2+4} dz$ is

- (1) 0
- (2) π
- (3) $\frac{\pi}{2}$
- (4) ∞

62. Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be an entire function. If real part of f is bounded then

- (1) $f = 0$
- (2) f is nonzero constant
- (3) Imaginary part of f is constant
- (4) f is constant

63. Which one of the following is correct?

- (1) $S_1 = \{z \in \mathbb{C} : 1 < |z| < 2\}$ is connected
- (2) $S_2 = \{z \in \mathbb{C} : |z| < 1 \text{ and } |z - 2| < 1\}$ is connected
- (3) $S_3 = \{z \in \mathbb{C} : \operatorname{Im}(z) > 1\}$ is not connected
- (4) $S_4 = \{z \in \mathbb{C} : \operatorname{Im}(z) = 1\}$ is not connected

64. About the definite integrals, which one of the following is incorrect?

- (1) $\int_a^b f(x)dx = - \int_a^b f(t)dt$
- (2) $\int_a^b f(x)dt = - \int_b^a f(t)dx$
- (3) If $a < c < b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
- (4) $\int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx & \text{, when } f(x) \text{ is even} \\ 0 & \text{, when } f(x) \text{ is odd} \end{cases}$

65. The value of integral $\int_{-\pi/2}^{\pi/2} x \cos x dx$ is

- (1) $\pi - 2$
- (2) $\pi + 2$
- (3) 0
- (4) $\frac{\pi}{2}$

66. Which one of the following is incorrect?

- (1) If a real valued function f is monotonic on $[a, b]$, then it is integrable on $[a, b]$
- (2) A real valued bounded function f , having a finite number of points of discontinuity on $[a, b]$, is not integrable on $[a, b]$
- (3) If a real valued function f is integrable on $[a, b]$, then f^2 is also integrable on $[a, b]$
- (4) Every real valued continuous function f on $[a, b]$ is integrable on $[a, b]$

67. The value of integral $\int_0^{\pi} x \sin x dx$ is

- (1) 2π
- (2) 0
- (3) $\frac{\pi}{2}$
- (4) π

68. Which one of the following is the statement of fundamental theorem of calculus?

- (1) Every continuous function is integrable
- (2) Functions possessing primitives are necessarily continuous
- (3) If a real valued function is bounded and integrable on $[a, b]$, and there exists a function F such that $\frac{d}{dx} F(x) = f$ on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$
- (4) Every monotonically increasing function is integrable

70. After the change of order of integral, the double integral $\int_0^8 \int_{x^3}^2 dy dx$ becomes

- (1) $\int_{x^3}^2 \int_0^8 dx dy$
- (2) $\int_0^2 \int_0^{y^3} dx dy$
- (3) $\int_8^0 \int_2^{x^{\frac{1}{3}}} dx dy$
- (4) $\int_0^2 \int_{y^3}^0 dx dy$

70. The area of the region bounded by the curves $x = y^2$ and $y = x^2$ is

- (1) 1
- (2) $\frac{2}{3}$
- (3) $\frac{1}{3}$
- (4) $\frac{4}{3}$

71. The value of the triple integral $\int_{x=0}^1 \int_{y=0}^1 \int_{z=0}^1 (x^2 + y^2 + z^2) dz dy dx$ is

- (1) 0
- (2) $-\frac{1}{3}$
- (3) $-\frac{1}{4}$
- (4) 1

72. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

- (1) 1
- (2) 2
- (3) 3
- (4) Not defined

73. The differential equation $x \frac{dx}{dy} + y = 0$, represents a family of

- (1) hyperbolas
- (2) exponential curves
- (3) parabolas
- (4) circles

74. The family of orthogonal trajectories of the family of parabolas $y = cx^2$ is

- (1) $x^2 + 2y^2 = k^2$, where k is an arbitrary constant
- (2) $2x^2 + y^2 = k^2$, where k is an arbitrary constant
- (3) $x^2 + y^2 = k^2$, where k is an arbitrary constant
- (4) $x^2 = ky$ where k is an arbitrary constant

75. For $a, b, c \in \mathbb{R}$, if the differential equation $(ax^2 + bxy + y^2)dx + (2x^2 + cxy + y^2)dy = 0$ is exact, then

- (1) $b = 2, c = 4$
- (2) $a = b, c = 20$
- (3) $b = 4, c = 2$
- (4) $b = 2, a = 2c$

76. Which one of the following is a linear differential equation?

- (1) $\frac{dy}{dx} = \sin y$
- (2) $y \frac{dy}{dx} = x$
- (3) $\left(\frac{dy}{dx}\right)^2 = 1$
- (4) $\frac{dy}{dx} = e^x$

77. Which one of the following is an integrating factor of the differential equation $ydx + 2xdy = 0$?

- (1) $\mu(x, y) = x$
- (2) $\mu(x, y) = y$
- (3) $\mu(x, y) = 1$
- (4) $\mu(x, y) = 2$

78. The general solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ is

- (1) $c_1 e^{-2x} + c_2 e^x$
- (2) ce^{-2x}
- (3) $c_1 e^x + c_2 x + c_3 x^3$
- (4) $c_1 e^{2x} + c_2 e^x$

79. The differential equation $\frac{d^2y}{dx^2} + \sin(x + y) = \sin x$ is

- (1) Linear and non-homogeneous
- (2) Linear and homogeneous
- (3) Nonlinear and homogeneous
- (4) Nonlinear and non-homogeneous

80. The solution of the differential equation $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 4y = 0$ satisfying the conditions $y(0) = 4$

and $\left.\frac{dy}{dx}\right|_{x=0} = 8$ is

- (1) e^{2x}
- (2) $4e^{-2x}$
- (3) $(4 + 16x)e^{-2x}$
- (4) $4e^{2x}$

81. Let $y(x)$ be the solution of the initial value problem $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 0, x > 0, y(2) = 0,$

$\left. \frac{dy}{dx} \right|_{x=2} = 4$. Then value of $y(4)$ is

- (1) 32
- (2) 0
- (3) 1
- (4) $\frac{1}{32}$

82. $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants, is a solution of

- (1) $B \frac{d^2y}{dx^2} + Ay = 0$
- (2) $A \frac{d^2y}{dx^2} + 4y = 0$
- (3) $\frac{d^2y}{dx^2} + 2y = 4$
- (4) $\frac{d^2y}{dx^2} - 4y = 0$

83. Let P and Q be real constants. Then $y = e^{mx}$ is a solution of differential equation $\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0$, if

- (1) $m^2 + Pm - Q = 0$
- (2) $m^2 + Pm + Q = 0$
- (3) $m^2 - Pm + Q = 0$
- (4) $m^2 - Pm - Q = 0$

84. Let $y_1(x)$ and $y_2(x)$, defined on $[0,1]$, be twice continuously differentiable functions satisfying

$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$. Let $W(x)$ be Wronskian of y_1 and y_2 which satisfies $W\left(\frac{1}{2}\right) = 0$. Then

- (1) $W(x) \neq 0$ for all $x \in \left[0, \frac{1}{2}\right] \cup \left(\frac{1}{2}, 1\right]$
- (2) $W(x) > 0$ for all $x \in \left[\frac{1}{2}, 1\right]$
- (3) $W(x) = 0$ for all $x \in [0,1]$
- (4) $W(x) < 0$ for all $x \in \left[0, \frac{1}{2}\right]$

85. The two linearly independent solutions of the differential equation $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = 0$ are

- (1) e^x and $2e^x$
- (2) x and $5x^2$
- (3) $\sin x$ and $9\sin x$
- (4) e^x and xe^x

86. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$. If $\phi(x, y, z)$ is a solution of the Laplace's equation then the vector field $(\nabla\phi + \vec{r})$ is

- (1) solenoidal but not irrotational
- (2) neither solenoidal nor irrotational

(3) irrotational but not solenoidal
 (4) both solenoidal and irrotational

87. Let ϕ and \vec{f} be differentiable scalar and vector functions, respectively and both have continuous second partial derivatives. Then which one of the following is true?

(1) The curl of the gradient of ϕ is never zero
 (2) The divergence of the curl of \vec{f} is zero
 (3) \vec{f} is irrotational if its divergence is zero
 (4) \vec{f} is called solenoidal if curl of \vec{f} is zero

88. The magnitude of the gradient of function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ such that $f(x, y, z) = xyz^2$ at the point $(1, 0, 2)$ is

(1) 4
 (2) 3
 (3) 1
 (4) 0

89. Suppose $\vec{r} = x\hat{i} + y\hat{j}$ be a position vector of the point $P(x, y)$. Let $\vec{f} = -3x^2\hat{i} + 5xy\hat{j}$ and let C be the curve $y = 2x^2$ in the xy -plane. Then the value of the line integral $\int_C \vec{f} \cdot d\vec{r}$ from the point $P_1(0, 0)$ to $P_2(1, 2)$ is

(1) 0
 (2) $\frac{1}{7}$
 (3) 1
 (4) 7

90. If \vec{F} is a conservative vector field, then

(1) \vec{F} is solenoidal
 (2) curl of \vec{F} is nonzero
 (3) \vec{F} is irrotational
 (4) divergence of \vec{F} is zero

91. Let $\vec{F} = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$. The value of $\iiint_V \vec{F} dV$, where V is the region bounded by the surfaces $x = 0, y = 0, y = 6, z = x^2, z = 4$ is

(1) 1
 (2) $8\hat{i} + 4\hat{j} + 4\hat{k}$
 (3) $128\hat{i} - 24\hat{j} + 385\hat{k}$
 (4) $128\hat{i} - 24\hat{j} + 384\hat{k}$

92. Which one of the following is true?

(1) Green's theorem in the plane is a special case of Stokes' theorem
 (2) Stokes' theorem is a special case of Green's theorem
 (3) Fundamental theorem of integral calculus is the generalization of Green's theorem in plane

(4) Green's theorem in plane is the generalization of Gauss' divergence theorem

93. Which one of the following is incorrect?

- (1) The intersection of two convex sets is a convex set
- (2) The intersection of any finite number of convex sets is a convex set
- (3) The set $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ is a convex set
- (4) The union of two convex sets is also a convex set

94. Let $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ and $T = \{(x, y) \in \mathbb{R}^2 : y \geq x^2\}$. Then which one of the following is correct

- (1) S is a convex set but T is not a convex set
- (2) T is a convex set but S is not a convex set
- (3) Both S and T are not convex sets
- (4) Both S and T are convex sets

95. Which one of the following is in the convex hull of the points $(0, 1)$, $(1, 0)$ and $(1, 1)$?

- (1) $(0, 0)$
- (2) $\left(0, \frac{3}{2}\right)$
- (3) $\left(\frac{1}{2}, \frac{1}{2}\right)$
- (4) $(-10, 10)$

96. Let $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ and $T = \{(0,1), \left(\frac{1}{2}, \frac{1}{2}\right), (1,0)\}$, then the convex hull of $S \cup T$ is

- (1) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$
- (2) $S \cup T$
- (3) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$
- (4) $S \cap T$

97. Which one of the following is true?

- (1) A boundary point of a convex set is the extreme point of the convex set
- (2) An extreme point of a convex set is the boundary point of the convex set
- (3) Convex hull of a set $S \subset \mathbb{R}^n$ is the largest convex set containing S
- (4) A hyperplane in \mathbb{R}^n is not a convex set

98. One of the vertex of the convex set $\{(x, y) \in \mathbb{R}^2 : x + 2y \geq 2, 2x + 3y \leq 6, x \geq 0, y \geq 0\}$ is

- (1) $(0, 0)$
- (2) $(1, 1)$
- (3) $(2, 0.5)$
- (4) $(3, 0)$

99. The value of the objective function at an optimal solution of the linear programming problem $\min z = x_1 + x_2$ subject to the conditions $x_1 - x_2 = -5, x_1 \geq 0, x_2 \geq 0$ will be

- (1) 0
- (2) 10
- (3) 5
- (4) -5

100. The linear programming problem $\max z = -x_1 + 2x_2$ subject to conditions $-x_1 + x_2 \leq 1, -x_1 + 2x_2 \leq 4, 0 \leq x_1 \leq 5, x_2 \geq 0$ has

- (1) multiple optimal solutions
- (2) unique optimal solution
- (3) no solution
- (4) unbounded solution

