

PART-A

1. Choose the most appropriate set of prepositions for the blanks given in the sentence below

Someone threw a stone _____ the speaker. It hit him _____ the head and knocked his glasses _____

- (A) on, at, off
- (B) at, on, of
- (C) at, on, off
- (D) at, in, off

2. Choose the pair which exhibits the same relationship as ROAD: FOOTPATH.

- (A) drawing room : kitchen
- (B) river : riverbank
- (C) box : lock
- (D) window : shutter

3. Choose the most appropriate word from the choice given below:

They were _____ over what to watch on TV.

- (A) crying
- (B) squabbling
- (C) working
- (D) asking

4. Doctor is related to patient in the same way as lawyer is related to _____

- (A) Customer
- (B) Accused
- (C) Magistrate
- (D) Client

5. Identify the word which is opposite in meaning to DEPICT.

- (A) Misrepresent
- (B) Portray
- (C) Misunderstand
- (D) Sketch

6. Identify the part of the body with which the disease Arthritis is associated.

- (A) Blood
- (B) Throat
- (C) Joints
- (D) Mouth

7. Select correct meaning of foreign expression status quo

- (A) existing state of affairs
- (B) social position

(C) acceptability

(D) relative importance

8. Identify the meaning of underlined word:

In our system of education, homework is imperative.

(A) unnecessary

(B) authorised

(C) a handicap

(D) compulsory

9. Identify the word which carries the meaning of following sentence

A place or scene of great disorder

(A) chaotic

(B) repulsive

(C) unpleasant

(D) regressive

10. Identify the word which carries the meaning of underlined idiom

The income tax department seized the property of all those who accumulated wealth through ill-gotten gains.

(A) theft

(B) money obtained through dishonesty

(C) gambling

(D) ancestry

11. Choose the correct alternative that will continue the same pattern 3, 9, 27, 81, _____.

(A) 324

(B) 243

(C) 210

(D) 162

12. If GIVE is coded as 5137 and BAT is coded as 924, how is GATE coded?

(A) 5427

(B) 5724

(C) 5247

(D) 2547

13. In a certain code, a number 15789 is written as AXBTC and 2346 is written as MDPU. How is 23549 written in that code?

(A) MPXDT

(B) MPADC

(C) MPXCD

(D) MPXDC

14. If sand is called air, air is called plateau, plateau is called well, well is called island and island is called sky, then from where will a woman draw water?

- (A) Well
- (B) Island
- (C) Sky
- (D) Air

15. $4.8\% \text{ of } 6000 + 3.3\% \text{ of } 5000 - 2.1\% \text{ of } ? = 432$

- (A) 1500
- (B) 1800
- (C) 1000
- (D) 2000

16. How much time will a train, 192m long to cross a tunnel of length 328m long if it is running at a speed of 72 km/h?

- (A) 26 seconds
- (B) 46 seconds
- (C) 36 seconds
- (D) 28 seconds

17. The least number by which 72 must be multiplied in order to produce a multiple of 112 is:

- (A) 6
- (B) 12
- (C) 14
- (D) 18

18. The L.C.M. of 148 and 185 is:

- (A) 680
- (B) 740
- (C) 2960
- (D) 3700

19. $(8 \div 88) \times 8888088 = ?$

- (A) 808008
- (B) 808080
- (C) 808088
- (D) 8008008

20. Who is the Ex-officio Chairman of Rajya Sabha?

- (A) President
- (B) Vice President
- (C) Prime Minister
- (D) Leader of Opposition

21. Recently a public charitable trust under the name of PM CARES Fund has been set up by the Government of India. In PM CARES Fund, the letter 'S' stands for

- (A) States
- (B) Stability
- (C) Sustainable
- (D) Situations

22. Our National Anthem Jana Gana Mana was first sung in

- (A) December 1909
- (B) December 1911
- (C) December 1935
- (D) December 1936

23. Who among the following is known as the Grand Old Man of India?

- (A) Madan Mohan Malaviya
- (B) Rabindranath Tagore
- (C) Dadabhai Naoroji
- (D) Sardar Vallabhbhai Patel

24. Who among the following was the first woman-film star nominated to the Rajya Sabha?

- (A) Madhubala
- (B) Shabana Azmi
- (C) Mala Sinha
- (D) Nargis Dutt

25. What is the second most abundant element in earth's crust?

- (A) Silicon
- (B) Carbon
- (C) Oxygen
- (D) Hydrogen

PART-B

26. If G is a group having an even number of elements, then the number of elements apart from the identity element that are their own inverses is

- (A) No element
- (B) Exactly one element
- (C) Exactly two elements
- (D) At least one element

27. Let G be a group and let $a \in G$ such that $a \neq e$ and $a \neq a^{-1}$. Then

- (A) $O(a) = 2$
- (B) $O(a) > 2$

(C) G is abelian
 (D) $O(a) > 2$ and a prime integer.

28. The order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 5 & 6 & 4 & 3 & 2 \end{pmatrix}$ is

(A) 2
 (B) 3
 (C) 4
 (D) 6

29. A group G has no proper subgroup if and only if it is

(A) a finite abelian group
 (B) a finite abelian group of prime order
 (C) a cyclic group of prime order
 (D) an infinite cyclic group

30. In the symmetric group S_n of $n!$ permutations, the number of distinct cycles of length $r \leq n$ is

(A) $\frac{n!}{(n-r)!}$
 (B) $\frac{n!}{(n-r)!r!}$
 (C) $\frac{n!}{r!}$
 (D) $\frac{1}{r} \frac{n!}{(n-r)!}$

31. The number of generators of a cyclic group of order 10 is

(A) 2
 (B) 4
 (C) 6
 (D) 8

32. Every finite group of prime order

(A) is abelian
 (B) is cyclic
 (C) has exactly two proper subgroups
 (D) has a cyclic proper subgroup

33. Let H and K be two distinct subgroups of a finite group G , each of order 2. Let N be the smallest subgroup of G containing H and K both. Then the order of N is always

(A) 2
 (B) 3
 (C) 4
 (D) 8

34. The number of generators of an infinite cyclic group is

- (A) at most 2
- (B) exactly 2
- (C) at least 2
- (D) infinite

35. If H is a subgroup of a group G and N is a normal subgroup of G , then

- (A) HK is a normal subgroup of G
- (B) $H \cap K$ is a normal subgroup of N
- (C) $H \cap K$ is a normal subgroup of H
- (D) $H \cap K = \{e\}$

36. If H and K are any two subgroups of a group G , whose orders are relatively prime, then

- (A) $O(H \cap K)$ is a prime integer
- (B) $HK = KH$
- (C) HK is a subgroup of G
- (D) $H \cap K = \{e\}$

37. Let H and K be any two subgroups of a group G , each having 12 elements. Which of the following numbers cannot be the cardinality of the set HK ?

- (A) 36
- (B) 48
- (C) 60
- (D) 72

38. Let R be a commutative ring with unity element 1. Then

- (A) Cancellation laws hold in R
- (B) R is an integral domain
- (C) R is an integral domain if and only if cancellation laws hold in R
- (D) Every ideal of R is a prime ideal of R

39. Mark the incorrect statement.

- (A) Every left, right or both sided ideal of a ring R is a sub ring of R
- (B) Every sub ring of the ring is an ideal of R
- (C) Every field is a simple ring
- (D) Every commutative division ring is a field

40. Every prime ideal of a ring R is a maximal ideal of R if R is

- (A) a finite commutative ring with unity element
- (B) a finite commutative ring with out zero divisor
- (C) a commutative ring with unity element
- (D) None of the above

41. Mark the false statement.

(A) Every vector space has at least two subspaces
 (B) The sum of any two subspaces of a vector space $V(F)$ is a subspace of $V(F)$
 (C) The union of any two subspaces of a vector space $V(F)$ is a vector space of $V(F)$.
 (D) The set $\{(x, x, x), x, R\}$ is a subspace of $R^3 (R)$

42. Mark the correct statement.

(A) If S and T are any two subsets of the vector space $V(F)$, then $S \neq T \Rightarrow L(S) \neq L(T)$
 (B) Every super set of a linearly independent set is linearly independent.
 (C) Every subset of a l.d. set is l.d.
 (D) Every superset of a l.d. set is a l.d. set

43. Which of the following subset of the space of all continuous functions on R is linearly independent?

(A) $S = \{\sin x, \cos x, \sin(1+x)\}$
 (B) $S = \{\sin^2 x, \cos 2x, 1\}$
 (C) $S = \{x, \sin x, \cos x\}$
 (D) $S = \{1, x - x^2, x + x^2, 3x\}$

44. Mark the correct statement.

(A) Every vector space has a unique basis
 (B) Any basis for $R^3(R)$ can be extended to a basis for $R^4(R)$
 (C) Every set of $n+1$ vectors in an n -dimensional vector space is linearly independent
 (D) Every set of n linearly independent vectors in an n -dimensional vector space is a basis.

45. Let $t: R^2 \rightarrow R^2$ be a linear transformation defined by $t(x, y) = (5x - 2y, 2x + y)$ and $B = \{(1, 0), (0, 1)\}$ be the basis for R^2 . Then the matrix of t w.r.t. B is

(A) $\begin{vmatrix} 5 & -1 \\ 2 & 1 \end{vmatrix}$
 (B) $\begin{vmatrix} 5 & 1 \\ 2 & -1 \end{vmatrix}$
 (C) $\begin{vmatrix} 5 & 2 \\ 1 & 1 \end{vmatrix}$
 (D) $\begin{vmatrix} 5 & 2 \\ -1 & 1 \end{vmatrix}$

46. Let $t: U \rightarrow V$ be a linear transformation of the vector space $U(F)$ into the vector space $V(F)$. Then

(A) $\text{Ker}(t) = \text{null space of } t$
 (B) $\text{rank}(t) = \dim[\text{ker.}(t)]$
 (C) $\text{rank}(t) = \dim[\text{range of } t]$
 (D) $\text{nullity of } t = \dim[\text{ker.}(t)]$

47. Let $t: R^3 \rightarrow R^2$ be a linear transformation where $t(1, 0, 0) = (2, -1)$, $t(0, 1, 0) = (3, 1)$ and $t(0, 0, 1) = (-1, 2)$. Then $t(-3, 4, 2)$ is

(A) (4, 11)
 (B) (4, -11)
 (C) (4, 13)

(D) (4, -13)

48. If A is a 7×5 matrix of rank 3 and B is a 5×7 matrix of rank 5, then the rank (AB) is

- (A) 3
- (B) 5
- (C) 8
- (D) 15

49. The rank of the skew-symmetric matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$ is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

50. The system of linear equations $x + y + z = 0$, $3x + 6y + z = 0$, $\alpha x + 2y + z = 0$ does not have a unique solution if α is equal to:

- (A) $-7/5$
- (B) $7/5$
- (C) $-5/7$
- (D) $5/7$

51. Let $AX = B$ be a system of n linear equations in n unknowns. If $|A| = 0$, then the system is consistent and will have infinitely many solutions if

- (A) $\text{adj } A = 0$
- (B) $\text{adj } A \neq 0$
- (C) $(\text{adj } A)B = 0$
- (D) $(\text{adj } A)B \neq 0$

52. The eigen values of a skew-symmetric matrix are always

- (A) all positive
- (B) 0
- (C) purely imaginary
- (D) purely imaginary or 0

53. The eigen values of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ are

- (A) 1, 1, 7
- (B) $\pm 1, 7$
- (C) $-1, -1, 7$
- (D) $1, -1, -7$

54. If the characteristic polynomial of the 3×3 matrix A is $f(\lambda) = \lambda^3 - \lambda^2 + 10\lambda - 8$, then trace (A) and $\det(A)$ of A are respectively

- (A) -1, -8
- (B) -1, 8
- (C) 1, -8
- (D) 1, 8

55. If the eigen values of a 3×3 matrix A are 1, 2 and -3, then A^{-1} is

- (A) $\frac{1}{6}[7I + A^2]$
- (B) $\frac{1}{6}[7I - A^2]$
- (C) $-\frac{1}{6}[7I - A^2]$
- (D) $-\frac{1}{6}[7I + A^2]$

56. The sequence $\langle e^n / n \rangle$ is

- (A) a monotonically increasing sequence
- (B) a monotonically decreasing sequence
- (C) bounded above but not bounded below
- (D) a bounded sequence

57. The sequence $\langle x_n \rangle$, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$

- (A) satisfies the Cauchy's criterion
- (B) does not satisfy the Cauchy's criterion
- (C) is a monotonically decreasing sequence
- (D) possesses the limit 0

58.

- (B) converges to 0
- (D) converges to $\frac{1}{4}$

59.

- (B) $p - q = 1$
- (D) $p - q > 1$

60. and $\sum_{n=1}^{\infty} b_n$ be two series, where $a_n = \frac{(-1)^n n}{2^n}$ and $b_n = \frac{(-1)^n}{\log(n+1)}$ for all $n \leq N$.

Then

- (A) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are absolutely convergent
- (B) $\sum_{n=1}^{\infty} a_n$ is absolutely convergent but $\sum_{n=1}^{\infty} b_n$ is conditionally convergent
- (C) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent but $\sum_{n=1}^{\infty} b_n$ is absolutely convergent
- (D) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are conditionally convergent

61. $\lim_{x \rightarrow 0} \frac{x - |x|}{x}$ equals

- (A) 0
- (B) 1
- (C) 2
- (D) limit does not exist

62. Let $f: R \rightarrow R$ be a continuous function satisfying $x + \int_0^x f(t) dt = e^{x-1}$, for all $x \in R$. Then the set $\{x \in R: 1 < f(x) \leq 2\}$ is the interval

- (A) $[\log 2, \log 3]$
- (B) $[2 \log 2, 3 \log 3]$
- (C) $[e-1, e-2]$
- (D) $[0, e]$

63. If $f(1) = 4$ and $f'(1) = 2$, then the value of the derivative of the function $\log(f(e^x))$ w.r.t x at $x = 0$ is

- (A) $\frac{1}{2}$
- (B) 1
- (C) 2
- (D) 4

64. Let $f(x) = e^x$ and $g(x) = e^{-x}$, for all $x \in [a, b]$. Then the value of the point $c \in (a, b)$ of the Cauchy's mean value theorem is

- (A) ab
- (B) $\frac{a+b}{2}$
- (C) $\frac{2ab}{a+b}$
- (D) \sqrt{ab}

65. The coefficient of $(x - 1)^2$ in the Taylor series expansion of the function $f(x) = xe^x, x \in R$ about the point $x = 1$ is

- (A) $\frac{e}{2}$
- (B) $\frac{3e}{2}$
- (C) $2e$
- (D) $3e$

66. The set $[a, b] \in [c, d]$ is

- (A) a closed set
- (B) an open set
- (C) neither closed nor an open set
- (D) neither an interval nor an open set

67. The Lagrange's form of remainder after n terms in Taylor's development of the function e^x in the interval $[a, a + h]$ is

- (A) $\frac{h^n}{n!} e^{a+\theta h}$
- (B) $\frac{h^n}{n!} e^{\theta h}$
- (C) $\frac{h^n(1-\theta)}{n!} e^{a+\theta h}$
- (D) $\frac{h^{n-1}}{n!} (1-\theta)^n e^{a+\theta h}$

68. If $u = (r)$, where $r^2 = x^2 + y^2 + z^2$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ equals

- (A) $r^2 f''(r) + \frac{1}{r} f'(r)$
- (B) $r^2 f''(r) + \frac{2}{r} f'(r)$
- (C) $f''(r) + \frac{1}{r} f'(r)$
- (D) $f''(r) + \frac{2}{r} f'(r)$

69. If $f(x, y) = x^5 y^2 \tan^{-1}(y/x)$, then $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ equals

- (A) $2f$
- (B) $3f$
- (C) $5f$
- (D) $7f$

70. The Lagrange's multiplier function $L(x, y)$ for the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$ is

- (A) $L(x, y) = x^2 + y^2 + \lambda(x^2 + 8xy + 7y^2)$
- (B) $L(x, y) = x^2 + 8xy + 7y^2 + \lambda(x^2 + y^2)$
- (C) $L(x, y) = x^2 + y^2 + \lambda(x^2 + 8xy + 7y^2 - 225)$
- (D) None of these

71. Define integral $\int_{-1}^1 (x - |x|) dx$ equals

- (A) -1
- (B) 0
- (C) 1
- (D) 2

72. Integral $\int_0^\pi 2 \frac{1}{1+\tan x} dx$ is equal to

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{2}$
- (C) π
- (D) 2π

73. The value of the triple integral $\int_{x=0}^1 \int_{y=0}^{x^2} \int_{z=0}^y (y + 2z) dx dy dz$ is

- (A) $\frac{1}{53}$
- (B) $\frac{2}{21}$
- (C) $\frac{4}{21}$
- (D) $\frac{1}{6}$

74. The value of $\int_{x=0}^1 \int_{y=0}^x \frac{1}{y} e^{-y} dx dy$ is equal to

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) $\frac{4}{3}$

75. The area of one loop of the limniscate $r^2 = a^2 \cos(2\theta)$ is

- (A) $\frac{1}{4}a^2$
- (B) $\frac{1}{2}a^2$
- (C) a^2
- (D) $2a^2$

76. The volume of the spindle shaped solid generated by revolving the asteroid $x = a \cos \theta$ about the x-axis is

- (A) $\frac{4\pi a^3}{105}$
- (B) $\frac{16\pi a^3}{105}$
- (C) $\frac{32\pi a^3}{105}$
- (D) $\frac{128\pi a^3}{105}$

77. The equation $\left[\frac{z-3}{z+3} \right] = 2$ represents a circle whose centre and radius are respectively

- (A) (3, 0) and 4
- (B) (-3, 0) and 4
- (C) (5, 0) and 4
- (D) (-5, 0) and 4

78. If $w = f(z) = u(x, y) + i v(x, y)$ is analytic in the complex region R, then $\frac{dw}{dz}$ is equal to

- (A) $-i \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}$
- (B) $\frac{\partial u}{\partial x} - i \frac{\partial v}{\partial x}$
- (C) $\frac{\partial u}{\partial y} - i \frac{\partial v}{\partial y}$
- (D) $i \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$

79. The Cauchy-Riemann conditions in polar form are

- (A) $\frac{\partial u}{\partial r} - r \frac{\partial v}{\partial \theta} \frac{\partial u}{\partial \theta} = -r \frac{\partial v}{\partial r}$
- (B) $\frac{\partial u}{\partial r} = r \frac{\partial v}{\partial \theta} \frac{\partial u}{\partial \theta} = -\frac{1}{r} \frac{\partial v}{\partial r}$
- (C) $\frac{1}{r} \frac{\partial u}{\partial r} = \frac{\partial v}{\partial \theta} \frac{\partial u}{\partial \theta} = \frac{1}{r} \frac{\partial v}{\partial r}$
- (D) $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta} \frac{\partial u}{\partial \theta} = -r \frac{\partial v}{\partial r}$

80. If $w = f(z) = u + iv$ is analytic in a given complex region, then its derivative $\frac{dw}{dz}$ is equal to

- (A) $e^{i\theta} \frac{\partial w}{\partial r}$
- (B) $\frac{1}{r} e^{i\theta} \frac{\partial w}{\partial \theta}$
- (C) $e^{-i\theta} \frac{\partial w}{\partial r}$
- (D) $-\frac{1}{r} e^{i\theta} \frac{\partial w}{\partial \theta}$

81. $u(x, y) = \frac{1}{2} \log(x^2 + y^2)$ is a harmonic function of x and y . The harmonic conjugate of

- (A) $\frac{1}{2} \tan^{-1} \left(\frac{xy}{x+y} \right) + C$
- (B) $\tan^{-1} \left(\frac{xy}{x^2-y^2} \right) - C$
- (C) $\tan^{-1} \left(\frac{x}{y} \right) + C$
- (D) $\tan^{-1} \left(\frac{y}{x} \right) - C$

82. If $C \mid z - 1 \mid = 1$, then in integral $\int_C (\bar{z})^2 dz$ equals

- (A) 0
- (B) πi
- (C) $2\pi i$
- (D) $4\pi i$

83. The integral $\int_{z=0}^{2i} \bar{z} dz$ equals

- (A) 1
- (B) i
- (C) 2
- (D) 2i

84. Let $u(x, y)$ and $v(x, y)$ be continuous and have continuous first partial derivatives at each point of a simply connected region R , bounded by the closed curve C . If $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$, then

- (A) $\int_C (v dx + u dy) = 0$
- (B) $\int_C (u dx + v dy) = 0$
- (C) $\int_C (u dx - v dy) = 0$

(D) $\iint_R \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dx dy = 0$

85. If C is a square whose vertices are $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$, then the complex integral $\int_C (\bar{z})^2 dz$ is equal to

- (A) $-1 + i$
- (B) $-1 - i$
- (C) $1 - i$
- (D) $1 + i$

86. The complex integral $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $C: |z| = 4$ is

- (A) $\frac{4\pi i}{3e^2}$
- (B) $\frac{4\pi i}{e^2}$
- (C) $\frac{8\pi i}{3e^2}$
- (D) $\frac{8\pi i}{e^2}$

87. The differential equation of the family of curves $y = c(x - c)^2$, where c is an arbitrary constant, is

- (A) $(y')^3 = 4y(xy' - y)$
- (B) $(y')^3 = y(4xy' - 2y)$
- (C) $(y')^3 = 4y(xy' - 2y)$
- (D) $(y')^3 = 2y(2xy' - y)$

88. The solution of the differential equation $x dy - (\log y - \log x + 1)y dx = 0$ is

- (A) $y = cx \log x$
- (B) $y = xe^{cx}$
- (C) $e^{y/x} = cx$
- (D) $xy = ce^x$

89. For what values of a , b and c , the differential equation $(ax^2 + b \times y + y^2)dx + (2x^2 + c \times y + y^2)dy = 0$ is exact?

- (A) $b = 2, c = 2a$
- (B) $b = 4, c = 2$
- (C) $b = 2, c = 4$
- (D) $b = 2a, c = 4$

90. If $\frac{1}{x}(C_1 + C_2 \log x)$ is the general solution of the differential equation $x^2 \frac{dy}{dx} + kx \frac{dy}{dx} + y = 0, x > 0$, then k equals

- (A) 3
- (B) -3
- (C) 2

(D) -1

91. The solution of the linear differential equation $(1 + y^2)dx = (\tan^{-1} y - z)dy$ is

(A) $y = Ce^{\tan^{-1} x}$
 (B) $y = \tan^I x I + Ce^{\tan^{-1} x}$
 (C) $y = C^{\tan^1 x} e^{\tan^1 x}$
 (D) $x = \tan^I y I + Ce^{\tan^{-1} y}$

92. The particular solution of the linear differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos 2x$ is

(A) $\frac{-1}{8} \sin 2x$
 (B) $\frac{1}{8} \sin 2x$
 (C) $\frac{1}{2} \cos 2x$
 (D) $\frac{1}{2} \sin 2x$

93. Let $y_1(X)$ and $y_2(X)$ be two linearly independent solutions of the differential equation $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$. If $W(X)$ is the Wronskian of the solutions of the differential equation, then $W(3) - W(2)$ is

(A) 1
 (B) 2
 (C) 3
 (D) 5

94. If $f(x, y, z) = x^2y + y^2z + z^2x$ for all $(x, y, z) \in R^3$, then the value of $\nabla \cdot (\nabla \times \nabla f) + \nabla \cdot (\nabla f)$ at $(1, 1, 1)$ is

(A) 0
 (B) 3
 (C) 6
 (D) 9

95. The directional derivative of the function $\phi(x, y, z) = x^2yz + 2xz^2$ at the point $(1, -2, -1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$ is

(A) $\frac{4}{3}$
 (B) $\frac{13}{3}$
 (C) $\frac{29}{3}$
 (D) $\frac{37}{3}$

96. Let C be the circle $x^2 + y^2 = 1$ taken in the anticlockwise direction. Then the value of the integral $\int_C [(2xy^3 + y)dx + (3x^2y^2 + 2x)dy]$ equals

- (A) 0
- (B) 1
- (C) $\frac{\pi}{2}$
- (D) π

97. Let V be the volume enclosed by the surface S and let $\iint_S \hat{r} \cdot \hat{n} dS = 1$. Then the volume V is

- (A) 1
- (B) $\frac{1}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{4}{3}$

98. A subset $S \subset \mathbb{C}^n$ is said to be a convex set if and only if for all $\lambda_1, \lambda_2 \in \mathbb{R}$.

- (A) $X_1, X_2 \in S \Rightarrow \lambda_1 X_1 + \lambda_2 X_2 \in S, \lambda_1 + \lambda_2 = 1$
- (B) $X_1, X_2 \in S \Rightarrow \lambda_1 X_1 + \lambda_2 X_2 \in S, \lambda_1 \geq 0, \lambda_2 \geq 0$
- (C) $X_1, X_2 \in S \Rightarrow \lambda_1 X_1 + \lambda_2 X_2 \in S, \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_1 + \lambda_2 = 1$
- (D) $X_1, X_2 \in S \Rightarrow \lambda_1 X_1 + \lambda_2 X_2 \in S, \lambda_1 > 0, \lambda_2 > 0, \lambda_1 + \lambda_2 = 1$

99. A basic feasible solution to the system $AX=b$ is said to be degenerate if

- (A) at most one of the basic variables vanishes
- (B) at any one of the basic variables vanishes
- (C) at least one of the basic variables vanishes
- (D) all the basic variables except one vanishes

100. If the optimality criterion is satisfied in the simplex algorithm, yet one or more artificial variables appear at positive level in the simplex tableau, then the problem is said to have

- (A) an optimal solution
- (B) no feasible solution
- (C) an unbounded solution
- (D) a degenerate optimal solution