PART-A

1. The culprit denied having________ the crime.
A) commit
B) committing
C) committed
D) had committed

2. A horse is kept in a________
A) kennel
B) shed
C) yard
D) stable

3. I have been living here_______the last two years.
A) since
B) about
C) for
D) over

4. Can I stay________ the week end?
A) until
B) by
C) to
D) along

5. I____ hardly hear what you are saying.
A) can't
B) don't
C) can
D) do

6. ______ do you think you are, any way?
A) How
B) Whom
C) Why
D) Who

7. I think he did________down and hurt himself.
A) fell
B) fall
C) felt
D) fallen

8. Much__________since he left the town.
A) had happen
B) was happened
C) had happened
D) was happen

9. I am sure he is not telling the truth, he has____ to his friends.
A) lyed
B) lied
C) lieyed
D) lying

10. ________money is better than none.
A) Little
B) A little
C) The little
D) Most

11. Choose the appropriate answer for the following:
Roentgen : X-Rays : : Becquerel : ?
A) Uranium
B) Radioactivity
C) Fission
D) Superconductivity

12. Which number will come in the blank space?
1, 2, 3, 5, 8,___
A) 9
B) 11
C) 13
D) 15

13. Which of the following is not a member of SAARC?
A) Bhutan
B) Burma
C) Bangladesh
D) Maldives

14. In a group of 15 people, 7 read French, 8 read English while 3 of them read none of these two. How many of them read French and English both?
A) 0
B) 3
C) 4

D) 5

15. How many rectangles are there in the following figure?

A) 6
B) 7
C) 8
D) 9

16. Select the most suitable synonym for TACT.
A) cunningness
B) diplomacy
C) intelligence
D) discrimination

17. Select the most suitable antonym for DEPICT.
A) misrepresent
B) portray
C) misunderstand
D) sketch

18. Identify the meaning of idiom "Be in two minds".
A) be burdened
B) be indifferent
C) be mischievous
D) be undecided

19. Who is the author of the book titled "The Z Factor: My Journey as the Wrong Man at the Right Time"?
A) Mahendra Verma
B) Vijay Joshi
C) Narayan Pandit
D) Subhash Chandra

20. Choose the correct option

$$\frac{1260}{15/7} = ?$$

A) 12
B) 58
C) 122

D 588

21. The average of 7 consecutive numbers is 20. The largest of these numbers is
A) 20
B) 22
C) 23
D) 24

22. What percent of Rs.2,650 is Rs. 1,987.50?
A) 60%
B) 75%
C) 80%
D) 90%

23. A sells an article which costs him Rs.400 to B at a profit of 20%. B then sells it to C, making a profit of 10% on the price he paid to A. How much does C pay to B?
A) Rs.472
B) Rs.476
C) Rs.528
D) Rs.532

24. If $0.75 : x :: 5 : 8$, then x is equal to
A) 1.12
B) 1.20
C) 1.25
D) 1.30

25. A and B can do a piece of work in 72 days; B and C can do it in 120 days; A and C can do it in 90 days. In what time can A alone do it?
A) 80 days
B) 100 days
C) 120 days
D) 150 days

PART-B

26. Consider the set $S = \left\{x \in R: \frac{2x+1}{x+2} < 1\right\}$, where R is the set of reals. Determine which one of the following statements about S is correct.
A) S is bounded below but not above and infS = -2
B) S is bounded above but not below and supS = 1
C) S is bounded both below and above with infS= -2, supS = 1
D) S is neither bounded below nor above

27. Consider the set $= \{\frac{mn}{1+m+n} : m, n\ are\ natural\ numbers\}$. Then determine which one of the following statements is correct.
A) S is a bounded set
B) S is bounded below with infS = 1/3 but not bounded above
C) S is bounded above with supS = 1 but not below
D) S is neither bounded below nor above

28. Let $p.q$ be two reals such that $p > q > 0$. Define the sequence (x_n), where $x_1 = p + q$ and $x_n = x_1 - \frac{pq}{x_{n-1}}$ for $n \geq 2$. Then for all n, x_n is equal to one of the following and determine it.
A) $X_n = \frac{p^{n+1}-q^{n+1}}{p^n-q^n}$
B) $X_n = \frac{p^{n+1}+q^{n+1}}{p^n+q^n}$
C) $X_n = \frac{(pq)^n}{p^n+q^n}$
D) $X_n = \frac{(pq)^n}{p^n-q^n}$

29. Which one of the following statements is wrong?
A) Every convergent sequence of reals is necessarily bounded
B) Every sequence of reals has a monotone subsequence
C) Every monotone increasing sequence which is bounded above is convergent
D) Every sequence which is bounded above has a convergent subsequence

30. The sequence
A) Is bounded but not convergent
B) Is convergent and converges to 0
C) Is convergent and converges to 1
D) Is monotone increasing

31. The minimum value of the sum $\sum_{n=1}^{a} a_k^2$ of reals satisfying $\sum_{k=1}^{a} a_k = 1$ is
A) $\frac{1}{\sqrt{n}}$
B) $\frac{1}{n}$
C) $\frac{1}{\sqrt{n^2}}$
D) $\frac{1}{n^3}$

32. Consider the sequences (a_n) and (b_n), where $a_a = \left(1 + \frac{1}{n}\right)^a$ and $b_n = \left(1 + \frac{1}{n}\right)^{a+1}$ for all $n \in N$.
Then,
A) both sequences are monotone increasing
B) both sequences are monotone decreasing
C) one of these two sequences is monotone increasing and the other one is monotone decreasing
D) both the sequences are unbounded

33. The series $\sum_{n=1}^{\infty} \frac{n}{3.5.7...(2n+1)}$ converges to

A) 1/2

B) 1/3

C) 1/4

D) 1/5

34. The series $\sum_{k=2}^{\infty} \frac{1}{k(\log K)^{\alpha}}$ where α is a real no. and $log\ k\ = \log_e k$

A) Converges for all α

B) Converges only for $\alpha \leq 0$

C) Converges only for all a satisfying $0\ < \alpha \leq 1$

D) Converges only for all $\alpha >\ 1$

35. The $\lim_{x\to 0} \frac{\log(\cos x)}{\sin^2 x}$

A) does not exist

B) exists and its value is -1/2

C) exists and its value is 0

D) exists and its value is 1/2

36. Consider the function $f(x) = \frac{1}{1+e^{1/\pi}}$ for $x \neq 0$. Then

A) Left hand limit $\lim_{x\to 0-} f(x)$ at $x\ =\ 0$ exists but the right hand $\lim_{x\to 0+} f(x)$ at $x = 0$ does not exist

B) $\lim_{x\to 0+} f(x)$ at $x\ =\ 0$ exists but $\lim_{x\to 0-} f(x)$ does not exist

C) Both $\lim_{x\to 0-} f(x)$ and $\lim_{x\to 0+} f(x)$ at $x\ =\ 0$ exist and they are equal

D) Both $\lim_{x\to 0-} f(x)$ and $\lim_{x\to 0+} f(x)$ at $x\ =\ 0$ exist and they are not equal

37. The value of $\lim_{x\to\infty} x\left(log\left(1+\frac{x}{2}\right) - log\left(\frac{x}{2}\right)\right)$ is

A) 2

B) 1

C) 0

D) -1

38. Define the function $f: R\ \to\ R$ by $f(x) = \begin{cases} \sin\ |x| & if\ x\ is\ rational \\ 0, & otherwise \end{cases}$

Then f is continuous

A) at all rational points

B) at all irrational points

C) at all $x = k\pi$, where k is any integer

D) at all $x \neq k\pi$, where k is any integer

39. Let f and $g[a, b] \rightarrow R$ be two continuous functions such that $f(a) < g(a)$ and $f(b) > g(b)$. Then

A) there is a $c \in (a, b)$ such that $f(c) + g(c) = 0$

B) there is a $c \in (a, b)$ such that $f(c) - g(c) = 0$

C) for all $x \in (a, b), f(x) = g(x)$

D) for all $x \in (a, b), f(x) \neq g(x)$

40. Which of the following functions is uniformly continuous on $[0, \infty]$?

A) $f(x) = x\ sinx$

B) $g(x) = sinx^2$

C) $h(x) = e^x$

D) $k(x) = sin\ (sinx)$

41. Let $f: (1, \infty) \rightarrow R$ be a function defined by $f(x) = \log_x 2$. Then the derivative of f is

A) $\frac{1}{x \log x} f(x)$

B) $-\frac{1}{x \log x} f(x)$

C) $\frac{1}{\log x} f(x)$

D) $-\frac{1}{\log x} f(x)$

42. Let $f: R \rightarrow R$ be a function defined by $f(x) = \{\begin{matrix} ax + b, if x \leq 1 \\ ax^2 + c, if\ 1 < x \leq 2 \\ \frac{dx^2+1}{x}, if\ x > 2 \end{matrix}$ where a, b, c, d are constants. The values of a, b, c, d so that f is differentiable on R, are

A) $a = 0, b = c = 1, d = 1/4$

B) $a = 0, b = c = -1, d = ½$

C) $a = 1, b = c = -1, d = 1/4$

D) $a = -1, b = c = 0, d = ½$

43. $Let\ g\ (0, \infty) \rightarrow R$ be a differentiable function such that $g(x) = \frac{1}{x}$ for all x. Define on $(0, \infty)$ by $f(x) = \left(g(x^2)\right)^3$. Then f is differentiable and $f'(x)$ is equal to

A) $6x\left(g(x^2)\right)^2$

B) $6x^2\left(g(x^2)\right)^2$

C) $6\left(g(x^2)\right)^2/x$

D) $6\left(g(x^2)\right)^2/x^2$

44. Define the function f on R by $f(x) = \sum_{i=1}^{0}(a_1 - x)^2$ where $a_1, a_2, \dots, a_3$ are real constants. Then f' has a relative extremum at the point

A) $x = \sum_{i=1}^{0} a_1$

B) $x = \frac{1}{n}\sum_{i=1}^{0} a_1$

C) $x = \sum_{i=1}^{n} a_1^2$
D) $x = \frac{1}{n}\sum_{i=1}^{0} a_1^2$

45. Let $f: (-1,1) \to R$ be a function defined by $f(x) = \begin{cases} 2x^4 + x^4 \sin 1/x & for\ x \neq 0 \\ 0 & for\ x = 0 \end{cases}$

Then
A) $f(x)$ has no point of extremum in (-1, 1)
B) $f(x)$ is a decreasing function
C) $f(x)$ is an increasing function
D) $f(x)$ is neither a decreasing nor an increasing function in (-1, 1)

46. The value of the integral $\int_0^{\pi/2} \frac{\sin^0 x}{\sin^0 x + \cos^0 x} dx$, where n is a natural number is
A) π
B) $\pi/2$
C) $\pi/3$
D) $\pi/4$

47. Let f be a positive monotone decreasing function on $[1, \infty]$. Then the sequence (a_n), where $a_n = \sum_{n=1}^{a} f(k) = \int_1^a f(x)\, dx$ for $n \in \mathbb{N}$
A) is not bounded
B) is not a monotone sequence
C) is convergent
D) is oscillatory

48. Let f be a continuously differentiable function defined on an interval [a, b] such that $f(a) = f(b) = 0$ and $\int_a^b f^2(x)\, dx = 1$. Then the value of $\int_a^b x\, f(x) f'(x)\, dx$ is
A) -½
B) 0
C) ½
D) 1

49. The improper integral $\int_1^\infty \frac{dx}{x^n}$
A) converges if $n < 1$
B) converges if $n = 1$
C) converges if $n > 1$
D) converges for all values of n

50. The improper integral $\int_0^\infty e^{-\pi j} sin\ xdx$, where $y > O$ satisfies which one of the following?
A) does not converge for some $y > 0$
B) converges for all $y > 0$ but not uniformly on $[a, \infty)$ for any $a > 0$
C) converges uniformly on $(0, \infty)$

D) converges uniformly on $[a, \infty)$, where $a > 0$

51. Consider the function $f : R^2 \to R$ of two variables defined by

$$f(x,y) = \begin{cases} 0, & if\ (x,y) = (0,0) \\ \dfrac{xy^2}{x^2+y^2}, & if\ (x,y) \neq (0,0) \end{cases}$$

Determine which one of the following facts about f is not true.

A) f is differentiable at (0,0)

B) f is continuous at (0, 0)

C) f has directional derivative at (0, 0) in the direction of any vector $u = (a,b) \neq (0,0)$

D) partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ at (0,0,) exist

52. Let $f: R^2 \to R$ and $g: R^2 \to R$ be two functions given by $f(x,y) = (x^2 + y^2, x^2 - y^2)$ and $g(x,y) = 2xy$, Define $h: R^2 \to R$ by $h = gof$. Then h is differentiable at each $(x,y) \in R^2$ and it is a 1×2 matrix given by

A) $h'(x,y) = (2x^3\ 2y^3)$

B) $h'(x,y) = (8x^3 - 8y^3)$

C) $h'(x,y) = (-8x^3\ 8y^3)$

D) $h'(x,y) = (4x^3\ 4y^3)$

53. The function $f: R^2 \to R$ given by $f(x,y) = xy$

A) has a critical point at (0, 0) which is a relative minimum

B) has a critical point at (0, 0) which is a relative maximum

C) has a critical point at (0, 0) which is a saddle point

D) has no critical point

54. The area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

A) $ab/2$

B) ab

C) $\pi ab/2$

D) πab

55. The value of the integral $\int_C \frac{dx}{x+y}$ where C is the curve whose parametric representation is $x = at^2, y = 2at, 0 \leq t \leq 2$, is

A) log 2

B) $\frac{1}{2}$ log 2

C) $\frac{1}{3}$ log 2

D) 2 log 2

56. The value of the integral $\iint_x x^2y^2dxdy$, where S is the region $x \geq 0, y \geq 0$ and $x^2 + y^2 \leq 1$, is

A) $\pi/96$
B) $\pi/48$
C) $\pi/24$
D) $\pi/12$

57. The value of the integral $\iiint_D (a^2b^2c^2 - b^2c^2x^2 - c^2a^2y^2 - a^2b^2z^2)^{1/2} dxdydz$ where D is the region $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$, is
A) $a^2b^2c^2\pi^2$
B) $\frac{1}{4}a^2b^2c^2\pi^2$
C) $\frac{1}{3}a^2b^2c^2\pi^2$
D) $\frac{1}{2}a^2b^2c^2\pi^2$

58. The gradient vector ∇f of $f(x, y, z) = e^{xy} - xcos(yz^2)$ at $(1, 0, 0)$ is
A) $\vec{i} + \vec{j}$
B) $\vec{i} - \vec{j}$
C) $-\vec{i} + \vec{j}$
D) $(-\vec{i} + \vec{j} + \vec{k})$

59. A unit normal to the surface $sinxy = e^x$ at $\left(1, \frac{\pi}{2}, 0\right)$ is
A) $(\vec{i} + \vec{j} + \vec{k})/\sqrt{3}$
B) $\vec{i}$
C) $\vec{j}$
D) $\vec{k}$

60. The equation of the tangent plane to the surface $3xy + z^2 = 4$ at $(1, 1, 1)$ is
A) $3x + 3y + 2z = 8$
B) $3x - 3y + 2z = 8$
C) $3x + 3y - 2z = 8$
D) $-3x + 3y + 2z = 8$

61. Suppose y is a differentiable function of x satisfying $e^{x-y} + x^2 - y = 1$. Then the value of $\frac{dy}{dx}$ at (0, 0) is
A) 0
B) 1/2
C) 1/3
D) 1/4

62. The divergence of the vector field given by $\vec{F} = x^2y\vec{i} + z\vec{j} + xyz\vec{k}$ is
A) xy

B) $2xy$
C) $3xy$
D) $4xy$

63. The curl of the vector field $\vec{F}(x,y,z) = xy\vec{\imath} - sinz\,\vec{\jmath} + \vec{k}$ is
A) $cosz\vec{\imath} + x\vec{\jmath}$
B) $cosz\vec{\imath} + x\vec{k}$
C) $cosz\vec{\jmath} + x\vec{k}$
D) $cosz\vec{\imath} - x\vec{k}$

64. Which one of the following vector fields is not a gradient vector field?
A) $\vec{F}(x,y,z) = (y + z)\,\vec{\imath} + (z + x)\vec{\jmath} + (x + y)\vec{k}$
B) $\vec{F}(x,y,z) = y\vec{\imath} - x\vec{\jmath}$
C) $\vec{F}(x,y,z) = 2xy^2\vec{\imath} + 2(x^2 + z^2)y\,\vec{\jmath} + 2y^2z\vec{k}$
D) $\vec{F}(x,y,z) = \frac{y}{x^2+y^2}\vec{\imath} - \frac{x}{x^2+y^2}\vec{\jmath}, where\ (x,y) \neq (0,0)$
65. Let $f(x,y,z)$ and $g(x,y,z)$ be two function defined on R^2 having second order partial derivatives with respect to x, y, z. Determine which one of the following fact about the Laplacian operator Vis true.
A) $\nabla^1(fg) = (\nabla^2 f)\,(\nabla^2 g)$
B) $\nabla^2(fg) = g\nabla^2 f + f\nabla^2 g)$
C) $\nabla^2(fg) = g\nabla^2 f + f\nabla^2 g + \nabla f.\nabla g$
D) $\nabla^2(fg) = g\nabla^2 f + f\nabla^2 g + 2(\nabla f.\nabla g)$

66. The initial value problem $\frac{dy}{dx} = \sqrt{|y|}, y(0) = 0$ has
A) no non-trivial solution
B) only trivial solution
C) two solutions
D) more than two solutions

67. The primitive of the differential equation
$\left(2x\ sin\ h\frac{y}{x} + 3y\ cos\ h\frac{y}{x}\right) dx - 3x\ cos\ h\frac{y}{x}\ dy = 0$ is given by
A) $x^2 = K \sinh^3\frac{y}{x}$
B) $x^2 = K \sinh^2\frac{y}{x}$
C) $x^2 = K\ cosh^3\frac{y}{x}$
D) $x^2 = K \cosh^2\frac{y}{x}$
Where K is an arbitrary constant.

68. An integrating factor of the differential equation $y(2xy + 1)\,dx + x\,(1 + 2xy - x^3y^3)dy = 0$ is

A) $\frac{1}{x^5y^5}$
B) $\frac{1}{x^4y^4}$
C) $\frac{1}{x^3y^3}$
D) $\frac{1}{x^2y^2}$

69. The primitive of the differential equation $6y^2\left(\frac{dy}{dx}\right)^2 + 3x\frac{dy}{dx} - y = 0$ is given by
A) $y^3 = Kx^2 + \frac{1}{3}K^2$
B) $y^3 = Kx^2 + \frac{2}{3}K^2$
C) $y^3 = Kx^3 + \frac{1}{3} K^2$
D) $y^3 = Kx + \frac{2}{3}K^2$
Where K is an arbitrary constant.

70. Given below four sets (f_1, f_2, f_3) of functions defined on R. Determine which set is linearly dependent.

A) $(f_1(x) = x^2, f_2(x) = x^4, f_3(x) = x^{-2})$
B) $(f_1(x) = x, f_2(x) = x + 1, f_3(x) = x + 2)$
C) $(f_1(x) = cosx, f_2(x) = sinx, f_3(x) = 1\}$
D) $(f_1(x) = e^x, f_2(x) = e^{-x}, f_3(x) = 1)$

71. The general solution of the differential equation $x^2y" + xy' - y = (x^3 + 3x^2)\, e^x$ is
A) $y = C_1x + C_2x^2 + xe^x$
B) $y = \frac{C_1}{x} + C_2x + e^x$
C) $y = \frac{C_1}{x} + C_2x + xe^x$
D) $y = \frac{C_1}{x} + \frac{C_2}{x^2} + xe^x$
Where C_1, C_2 are arbitrary constants.

72. Let V be the vector space of all functions from the interval [-1, 1] into R. Determine which one of the following subsets of V is not a subspace of V.
A) $V_1 = \{f \in V: f(x^2) = f(x)^2$ for all $x \in [-1,1]\}$
B) $V_2 = \{f \in V: f(x) + f(-x) = 0$ for all $x \in [-1,1]\}$
C) $V_3 = \{f \in V: f(0) = f(1)\}$
D) $V_4 = f \in V: f\ is\ a\ continuous\ function\}$

73. Determine which one of the following sets of vectors from R^3 does not form a basis for R^3.
A) $((1,0,-1), (2,5,1), (0,-4,3))$
B) $((1,2,-1), (1,0,2), (2,1,1))$
C) $((-1,3,1), (2,-4,-3), (-3,8,2))$

D) $(2, -4, 1), (0, 3, -1), (6, 0, -1))$

74. The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 0 & 2 & -3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$ is

A) 1
B) 2
C) 3
D) 4

75. Let $P = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{pmatrix}$ be a 3×3 matrix over R. Then for a given vector $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in R^{3\times 1}$ the vector space of all 3×1 matrices over R, the system $PX = Y$ has a solution if

A) $y_1 - y_2 + y_3 = 0$
B) $2y_1 - y_2 + y_3 = 0$
C) $y_1 + y_2 - y_3 = 0$
D) $2y_1 + y_2 - y_3 = 0$

76. Let V be a finite dimensional vector space over a field F and T: V→ V be a linear operator. Which one of the following statement is true?

A) If T has an eigen-vector then it has infinitely many distinct eigen-vectors
B) Sum of two eigen-values of T is an eigen-value of T
C) Sum of two eigen-vectors of T is an eigen-vector of T
D) Eigen-values of T are necessarily non-zero scalars

77. Which one of the following statements about similar matrices is wrong?

A) Two similar matrices have the same determinant
B) Every square matrix is similar to its transpose
C) Two similar matrices have the same minimal polynomial
D) If two nxn matrices have the same characteristics polynomial then they are similar

78. The minimal polynomial of the matrix $\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$ is

A) $x^2(x^2 - 4)$
B) $x^2(x - 2)$
C) $x(x^2 - 4)$
D) $x^2(x + 2)$

79. Which one of the following statements is not correct?

A) A linear operator T on a finite dimensional vector space V is diagonalizable if and only if the multiplicity of each eigen-value λ of T is equal to the dimension of the eigen-space $W = \{x \in V: Tx = \lambda x\}$ corresponding to λ

B) Two distinct eigen-vectors corresponding to the same eigen-value of a linear operator are always linearly dependent

C) If $\lambda_1 \lambda_2$ are two distinct eigen-values of a linear operator T on a finite dimensional vector space V then $Tx - \lambda_1 x = Tx - \lambda_2 x = 8$ where $x \in V$ implies $x = 0$, the zero vector of V

D) If a vector space V is the direct sum of subspaces $M_1, M_2, \dots, M_k$ of V then, for $1 \leq i, j \leq k$ and $i \neq j, M_1 \cap M_1 = \{0\}$.

80. Let $X = \{x \in R : x \neq 1, 2, \dots, 100\}$ be a subset of R. Determine which one of the following statements is true.

A) Integers 1, 2,...,100 are the only limit points of X

B) No limit point of X lies between 1 and 100

C) X is a closed subset of R

D) X is an open subset of R

81. Let $X = \left\{\frac{1}{n} : n \in Z, n \neq 0\right\}$ be a subset of R (Z is the set of all integers). Determine which one of the following properties of X is true.

A) X is a bounded set

B) X is an open subset of R

C) X is a closed subset of R

D) X has no limit point in R

82. The set Z of all integers

A) is an open subset of R

B) is a closed subset of R

C) is a compact subset of R

D) has infinitely many limit points in R

83. Determine which one of the following subsets of R is connected.

A) $(-\infty, 0) \cup (0, \infty)$

B) The set Q of all rational numbers

C) $\cup_{n=1}^{\infty}(-n, n)$

D) R Z

84. Consider the set $X = [-1, 1]$ with the subspace topology relative to R. Which one of the following subsets of X is open in X and in R ?

A) $\left\{x \in X: \frac{1}{2} \leq |x| < 1\right\}$

B) $\left\{x \in X: \frac{1}{2} < |x| \leq 1\right\}$

C) $\left\{x \in X: |x| > \frac{1}{2}\right\}$

D) $\left\{x \in X: \frac{1}{2} < |x| < 1\right\}$

85. Which one of the following statements is true
A) Every closed interval in R is homeomorphic to R
B) Every open interval of the type (a, b) is homeomorphic to R
C) Every interval of the type [a, b) is homeomorphic to R
D) Every interval of the type (a, b) is homeomorphic to R

86. The radius of convergence of the power series $\sum_{n=1}^{\infty} n^{2n}\, x^n$ is
A) 0
B) 1/2
C) 1
D) ∞

87. Consider the power series $Z_{n=1}^{\infty} a_n x^n$, where $a_n = 1\; if\; n = k^2$ for somek $k \in N, a_n = 0$, otherwise. Then the region of convergence for this series is
A) (0)
B) (-1, 1)
C) (-2, 2)
D) (-∞, ∞)

88. Let $\sum_{n=1}^{\infty} a_n x^a$ be a power series. Determine which one of the following formulas does not define the radius of convergence ρ.
A) $\rho = \frac{1}{\lim \sup |a_n| 1/n}$
B) $\rho = \frac{1}{\lim \sup |n a_n| 1/n}$
C) $\rho = \frac{1}{\lim \sup \left|\frac{a_n+1}{a_n}\right|}$
D) $\rho = \frac{1}{\lim \inf |a_n| 1/n}$

89. If power series $\sum_{n=1}^{\infty} a_n x^n$ and $\sum_{n=1}^{\infty} b_n x^n$ have radius of convergence ρ_1, ρ_2 respectively, then the radii of convergence ρ of the power series $\sum_{n=1}^{\infty}(a_a + b_a)x^a$ is given by
A) $\rho = \rho_1 + \rho_2$
B) $\rho = \max\{\rho_1, \rho_2\}$
C) $\rho = \min\{\rho_1, \rho_2\}$
D) $\rho = |\rho_1 - \rho_2|$

90. Let $\sum_{n=1}^{\infty} a_n x^n$ be a power series with ρ its radius of convergence where $0 < \rho < \infty$. Determine which one of the following facts is correct.
A) The series converges absolutely and uniformly on any closed interval in $(-\rho, \rho)$
B) The series converges absolutely and uniformly on any subinterval of $(-\rho, \rho)$
C) The series converges uniformly on $(-\rho, \rho)$

D) The series converges absolutely on $[-\rho, \rho]$

91. Let G be the set of all rationals $\frac{P}{q}$, where q is an odd integer. With respect to the usual multiplication of reals, G is not a group because
A) The closure property does not hold
B) The associative property does not hold
C) No element of G can be an identity element
D) Not every element of G can have an inverse

92. Let G be a group and $a, b \in G$. Then which one of the following statements is not true?
A) If a, b and ab have same order then ab = ba
B) If $a^3 = e$, the identity element of G, and $aba^{-1} = b^2$ then the order of b is 16
C) ab and ba have same order
D) b and aba^{-1} have same order

93. Let $G = \{(a, b) \in R^2 : a \neq 0\}$. On G define a binary operation by $(a, b)o(c, d) = (ac, bc + d)$. With respect to this operation on G, which one of the following is true?
A) G is a group with identity (1, 1) and inverse of (a, b) is (a^{-1}, ba^{-1})
B) G is a group with identity (1, 1) and inverse of (a, b) is $(a^{-1}, -ba^{-1})$
C) G is a group with identity (1, 0) and inverse of (a, b) is $(a^{-1}, -ba^{-1})$
D) G is not a group

94. Which one of the following subsets of G, given in the problem 93, is not a subgroup of G?
A) $H_1 = \{(1,0)\}$
B) $H_2 = \{(a, b) \in G : a = 1\}$
C) $H_3 = \{(a, b) \in G : a \text{ is rational}\}$
D) $H_4 = \{(a, b) \in G : a \text{ is irrational}\}$

95. Let G be a cyclic group of order 9. Then
A) G has nine generators
B) G has six generators
C) G has five generators
D) G has three generators

96. Let P_4 be the permutation group of 4 elements. Then the order of the element $(1\ 3)(2\ 4) \in P_4$ is
A) 6
B) 4
C) 3
D) 2

97. Let G be the group of non-zero real numbers under multiplication. Determine which one of the following functions $f: G \to G$ is not a homomorphism

A) $f(x) = x^2, x \in G$
B) $f(x) = |x|, x \in G$
C) $f(x) = \sqrt{x}|, x \in G$
D) $f(x) = 2^x, x \in G$

98. Let $a_1, a_2, \dots, a_n$ be the roots of a polynomial $x^n + x^{n-1} + \dots + x + 1$, where $a_1 \neq 1$ for $i = 1, 2, \dots\dots n$. Then the value of $\sum_{i=1}^{n} \frac{1}{1-a_1}$ is
A) $\frac{n}{2}$
B) $\frac{n}{3}$
C) $\frac{n}{4}$
D) n

99. Let P_3 be the permutation group of 3 elements. Then the number of elements in P_3 which are conjugate to $(2, 3) \in P_3$ is
A) 1
B) 2
C) 3
D) 6

100. Let G be a group of order 12. Then the maximal number of subgroups of order 4 in G can be
A) 6
B) 5
C) 4
D) 3