USEFUL NOTATION

$\mathbb{N}$	$\{1, 2, 3, \cdots\}$, the set of all natural numbers.
$\mathbb{Z}$	$\{0, \pm1, \pm2, \cdots\}$, the set of all integers.
$\mathbb{Q}$	Set of all rational numbers.
$\mathbb{R}$	Set of all real numbers.
$\mathbb{C}$	Set of all complex numbers.
$\mathbb{R}^n$	n-dimensional Euclidean space $\{(x_1, x_2, \ldots, x_n) : x_k \in \mathbb{R}, 1 \le k \le n\}$.
$GL_2(\mathbb{R})$	general linear group of 2×2 invertible matrices over $\mathbb{R}$.
$\mathbb{Z}(G)$	center of a group G.
$C[0, 1]$	set of all continuous real/complex-valued functions defined on the closed interval $[0, 1]$.
S_n	group of all permutations on n distinct symbols under composition of mappings.
A_n	alternating group, a subgroup of S_n.
$\mathbb{Z}_n$	group of congruence classes of integers modulo n.
$\mathbb{Z}[i]$	$\{a + ib : a, b \in \mathbb{Z}\}$, the Gaussian integers.
$\mathbb{Z}[\sqrt{-5}]$	$\{a + b\sqrt{-5} : a, b \in \mathbb{Z}\}$.
$\mathbb{Z}[\sqrt{p}]$	$\{a + b\sqrt{p} : a, b \in \mathbb{Z}$, p is a prime number$\}$.
$\mathbb{Z}_n[x]$	set of all polynomials with coefficients in $\mathbb{Z}_n$.
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of positive x, y and z axes in three dimensional rectangular coordinate system.

1. The value of $\int_0^{\pi/2} \sin^2(x)\cos^4(x)\,dx =$ ________.

(A) $\frac{\pi}{24}$

(B) $\frac{\pi}{32}$

(C) $\frac{2\pi}{15}$

(D) $\frac{3\pi}{32}$

2. Which one of the following statements is ***incorrect*** ?

(A) A point that separates the convex part of a continuous curve from concave part is called as point of inflection.

(B) The equation of asymptote to the curve $y^2x^2 = x^2 - 1$ is $y = \pm 1$.

(C) The curve $x^3 - y^3 = 3xy$ is symmetrical about the line $y = -x$.

(D) The equation of the tangent to the curve $x^2y^2 = x^2 - 1$ at the point (1, 0) is $y = x + 1$.

3. What is the surface area of the solid obtained by revolving the curve : $y = \sqrt{9 - x^2}$, $-2 \le x \le 2$ about the x-axis ?

(A) 18π

(B) 22π

(C) 24π

(D) 30π

4. Which one of the following equations is obtained from the equation: $xy = -8$ by rotating the axes counterclockwise through an angle 45° ? (X, Y) : new coordinates.)

(A) $X^2 - Y^2 + 8 = 0$

(B) $X^2 - Y^2 + 16 = 0$

(C) $X^2 - Y^2 + 24 = 0$

(D) $X^2 - Y^2 + 32 = 0$

5. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors satisfying : $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 1$ and $\vec{a} \times (\vec{a} \times \vec{b}) + \vec{c} = 0$, then what is the angle between the vectors $\vec{a}$ and $\vec{b}$?

(A) 30°

(B) 45° $\sqrt{2}$

(C) 60°

(D) 75°

6. Consider the following statements :

(I) The set $\left\{\sin\left(\frac{1}{x}\right) : x \in (0, 1)\right\}$ is uncountable.

(II) The set $\{(x, y) \in \mathbb{R}^2 : x.y \in \mathbb{Z}\}$ is uncountable.

(III) The set of all 2×2 real matrices with rational eigenvalues is uncountable.

Choose the ***correct*** answer from the options given below :

(A) Only (I) is true

(B) Only (I) and (II) are true

(C) Only (II) and (III) are true

(D) All (I), (II) and (III) are true

7. For the propositions : p and q, consider the following statements :

(I) $(p \vee q) \wedge (\neg p \vee \neg q)$ is a tautology.

(II) $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction.

Choose the ***correct*** answer from the options given below :

(A) Only (I) is true
(B) Only (II) is true
(C) Both (I) and (II) are true
(D) Both (I) and (II) are false

8. Which one of the following options is the general solution(s) of the linear congruences : $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$ and $x \equiv 3 \pmod 7$?

(A) $x \equiv 17 \pmod{105}$
(B) $x \equiv 23 \pmod{105}$
(C) $x \equiv 31 \pmod{105}$.
(D) $x \equiv 52 \pmod{105}$

9. In how many ways can 12 distinct people be arranged around a circular table, if exactly two of them must always have at least one person between them ?

(A) $9 \times 9!$
(B) $9 \times 10!$
(C) $10 \times 10!$
(D) $8 \times 9!$

10. If a, b are integers and p is a prime number, then which of the following statements is/are ***false*** ?

(I) If $(a, p) = (b, p) = 1$ and $a^k \equiv b^k \pmod p$ $(k \in \mathbb{N})$, then $a \equiv b \pmod p$.

(II) If $(a, p) = 1$ and $ax \equiv 1 \pmod p$, then $a \equiv a^{p-2} \pmod p$.

(A) Only (I)
(B) Only (II)
(C) Both (I) and (II)
(D) Neither (I) nor (II)

11. What is the value of a_{32} from the recurrence relation : $a_1 = 4$, $a_n = 5n + a_{n-1}$ $(n \geq 2)$?

(A) 2369
(B) 2469
(C) 2569
(D) 2639

12. Which one of the following statements is ***true*** for every square matrix with only real eigenvalues ?

(A) If the trace of the matrix is positive and the determinant of the matrix is negative, then at least one of its eigenvalues is negative.

(B) If the trace of the matrix is positive, then all its eigenvalues are positive.

(C) If the determinant of the matrix is positive, then all its eigenvalues are positive.

(D) If the product of the trace and determinant of the matrix is positive, then all its eigenvalues are positive.

13. For a system : $AX = b$ of linear equations to have infinitely many solutions, which one of the following options must be ***true*** ?

(A) Determinant of the matrix A is non-zero.

(B) The rank of the matrix A is less than the rank of the augmented matrix $(A|b)$.

(C) The rank of the matrix A is equal to the rank of the augmented matrix $(A|b)$, but less than the number of variables.

(D) The system must be homogeneous.

14. In a simple graph, which of the following statements is/are ***true*** ?

(I) Adjacency matrix is symmetric.

(II) Trace of the adjacency matrix is 1.

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

15. Which one of the following options is always ***true*** ?

(A) Every Eulerian graph is also Hamiltonian.

(B) Every Hamiltonian graph is also Eulerian.

(C) If the sum of degrees of vertices is odd, the graph is Hamiltonian.

(D) If all vertices have even degree, the graph has an Eulerian circuit.

16. Which one of the following options is ***incorrect*** for the set $S = \{x \in \mathbb{Q} : x^2 < 5\}$?

(A) S is an open set in $\mathbb{Q}$.

(B) The closure of S in $\mathbb{Q}$ is itself.

(C) The limit points of S are $-\sqrt{5}$ and $\sqrt{5}$.

(D) S has no least upper bound in $\mathbb{Q}$.

17. Which one of the following statements is ***incorrect*** ?

(A) The set $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is neither an open set nor a closet in $\mathbb{R}$.

(B) The set $\left\{x \in \mathbb{R} : \sin\left(\frac{1}{x}\right) = 0\right\}$ is is a closet in $\mathbb{R}$.

(C) The set $\{(x, y) \in \mathbb{R}^2 : xy = 0\}$ has no interior points.

(D) The limit points of the set $\left\{\frac{1}{n} + \frac{1}{m} : n, m \in \mathbb{N}\right\}$ is $\{0\} \cup \left\{\frac{1}{k} : k \in \mathbb{N}\right\}$.

18. Consider the sequences: $\{a_n\}_{n\in\mathbb{N}}$, $\{b_n\}_{n\in\mathbb{N}}$ and $\{c_n\}_{n\in\mathbb{N}}$ defined by :

(I) $a_n = n^2 \sin\left(\frac{1}{n}\right)$

(II) $b_n = 1 + \frac{(-1)^n}{n}$

(III) $c_n = n\cos\left(\frac{1}{n}\right)$ $(n \in \mathbb{N})$

Pick out the ***correct*** option.

(A) Only (II) is a Cauchy sequence

(B) Only (I) and (II) are Cauchy sequences

(C) Only (II) and (III) are Cauchy sequences

(D) Only (I) and (III) are Cauchy sequences

19. Let the limits of the sequences : $\{x_n\}_{n\geq 1}$, $\{y_n\}_{n\geq 1}$, respectively, be λ and λ^3. If the sequence : $x_1, y_1, x_2, y_2, x_3, y_3, \ldots$ has a limit, then its value is ________.

(A) λ^3

(B) $\lambda + \lambda^3$

(C) 0 or 1

(D) -1, 0 or 1

20. Which of the following assertions is ***correct*** ?

(I) $\lim\limits_{x\to\infty}\left(1+\frac{1}{n}\right)^{n^2} e^{-n} = 1$

(II) For $0 < a < 1 < b$, $\lim\limits_{x\to\infty}\left(a+\frac{b}{n}\right)^n = e^{b/a}$

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

21. Consider the following statements :

(I) If $a_n \geq 0$ for each $n \in \mathbb{N}$ and the series $\sum\limits_{n=1}^{\infty}$ converges, then the series $\sum\limits_{n=1}^{\infty} \frac{\sqrt{a_n}}{n^p}$ converges for $p > \frac{1}{2}$.

(II) The series $\sum\limits_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$ is conditionally convergent.

Which one of the options given below is ***correct*** ?

(A) Only (I) is true (B) Only (II) is true

(C) Both (I) and (II) are true (D) Neither (I) nor (II) is true

22. For a function $f : \mathbb{R} \to \mathbb{R}$, consider the following statements :

(I) If $\lim\limits_{h \to 0} \{f(x+h) - f(x-h)\} = 0$, then f is continuous on $\mathbb{R}$.

(II) If $\lim\limits_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} = 0$, then f is differentiable on $\mathbb{R}$.

Choose the ***correct*** answer from the options given below :

(A) Both the statements (I) and (II) are false

(B) Both the statements (I) and (II) are true

(C) Only statement (I) is true

(D) Only statement (II) is true

23. Which one of the following options is ***true*** for the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \min\{|x|, x^2 - 1\}, x \in \mathbb{R}$?

(A) f is a discontinuous function in $\mathbb{R}$.

(B) f is continuous and differentiable everywhere in $\mathbb{R}$.

(C) f is continuous and differentiable everywhere except at one point of $\mathbb{R}$.

(D) f is continuous and differentiable everywhere except at two points of $\mathbb{R}$.

24. If a function $f : [0, 1] \to \mathbb{R}$ is differentiable, $f'(0) = -1$ and $f'(1) = 5$, then which one of the following statements is ***true*** ?

(A) There exists a point $c \in (0, 1)$ such that $f'(c) = -1$.

(B) f' must be continuous on $[0, 1]$.

(C) There exists a point $c \in (0, 1)$ such that $f'(c) = 4$.

(D) f' must be differentiable on $(0, 1)$.

25. Consider the improper integrals given below and the statements that follow :

(a) $I_\lambda = \int_0^1 \frac{dx}{(1-x)^\lambda}$

(b) $J_\lambda = \int_1^\infty \frac{dx}{x^\lambda}$ $(\lambda \in \mathbb{R})$

(I) For $\lambda = 1$, I_λ converges, but J_λ diverges.

(II) For $\lambda = 2$, I_λ diverges, but J_λ converges.

Choose the correct option :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

26. The value of the integral : $\int_0^2 \left(\left[x^2 \right] - [x]^2 \right) dx$ is ________.

(A) 0

(B) 1

(C) $4 - \sqrt{3} - \sqrt{2}$

(D) $2 - \sqrt{3}$

27. Consider the following statements for a function $f : [a, b] \to \mathbb{R}$:

(I) f is Riemann integrable, if and only if it is bounded on [a, b].

(II) f is Riemann integrable, if and only if it is monotonic on [a, b].

(III) f is Riemann integrable, if and only if it is continuous on [a, b].

Which one of the options given below is ***correct*** ?

(A) Only (III) is true.

(B) Only (I) and (II) are true.

(C) Only (II) and (III) are true.

(D) All (I), (II) and (III) are false.

28. The coefficient of x^5 in the Taylor series expansion of the function $f(x) = \tan(x)$ about the point $x = 0$ is ________.

(A) $\frac{2}{15}$

(B) $\frac{1}{5!}$

(C) $\frac{1}{5}$

(D) 1

29. Consider the following statements :

(I) The sequence $\{f_n\}_{n \geq 1}$ of functions defined by $f_n(x) = x^n$ is both point-wise and uniformly convergent on [0, 1].

(II) The sequence $\{f_n\}_{n \geq 1}$ of functions defined by $f_n(x) = \frac{\ln(1 + nx)}{n}$ is both point-wise and uniformly convergent on [0, 1].

Choose the ***correct*** answer from the options given below.

(A) Only (I) is true.

(B) Only (II) is true.

(C) Both (I) and (II) are true.

(D) Neither (I) nor (II) is true.

30. Which of the following series does ***not*** converge uniformly on $(0, 1]$?

(I) $\sum_{n=1}^{\infty} \frac{x^n}{n+x}$ (II) $\sum_{n=1}^{\infty} \frac{x}{n^2+x}$

(III) $\sum_{n=1}^{\infty} \frac{x^n}{1+x^n}$

(A) Only (I) (B) Only (II)

(C) Only (I) and (III) (D) Only (II) and (III)

31. Which one of the following is ***not*** a valid metric on R ?

(A) $d(x, y) = |x-y|$ for all $x, y \in \mathbb{R}$

(B) $d(x, y) = \sqrt{|x-y|}$ for all $x, y \in \mathbb{R}$

(C) $d(x, y) = |x-y| + |x^2-y^2|$ for all $x, y \in \mathbb{R}$

(D) $d(x, y) = \max\{|x-y|, |x^2-y^2|\}$ for all $x, y \in \mathbb{R}$

32. Let (X, d) be metric space. If S and T are subsets X, then which one of the options given below is **not** necessarily true ? (For any $Y \subseteq X$, Int(Y) : interior of Y and Clos(Y) : Closure of Y.)

(A) $X \setminus Int(S) = Clos(X \setminus S)$ (B) $X \setminus Clos(S) = Int(X \setminus S)$

(C) $Int(S \cup T) = Int(S) \cup Int(T)$ (D) $Clos(S \cup T) = Clos(S) \cup Clos(T)$

33. Consider the metric spaces: $(C[0, 1], d_1)$, $(C[0, 1], d_\infty)$, under the metrices $d_1(f, g) = \int_0^1 |f(x)-g(x)|dx$ and $d_\infty(f, g) = \sup_{x \in [0,1]} \{|f(x)-g(x)|\}$ $(f, g \in C[0, 1])$.

Then which one of the options given below is ***correct*** ?

(A) The metrics d_1 and d_∞ are equivalent.

(B) Every open ball in $(C[0, 1], d_1)$ is open in $(C[0, 1], d_\infty)$.

(C) $(C[0, 1], d_1)$ is a complete metric space.

(D) $(C[0, 1], d_\infty)$ is a complete metric space.

34. Consider the following statements :

(I) $(0, 1)$ and R are homeomorphic.

(II) $(0, 1)$ and $(0, 1]$ are homeomorphic.

(III) $(0, 1)$ and $[0, 1]$ are homeomorphic.

Choose the ***correct*** option.

(A) Only (I) and (II) are false. (B) Only (II) and (III) are false.

(C) Only (I) and (III) are false. (D) All (I), (II) and (III) are false.

35. Which one of the following options is necessarily ***true*** ?

(A) Every contraction map on a metric space is uniformly continuous.

(B) The image of an open set under a continuous function is an open set.

(C) Every complete metric space is compact.

(D) Every metric space is second countable.

36. Which one of the following statements is ***correct*** for the function :

$$f(x, y) = \begin{cases} (x^2 + y^2)\sin\dfrac{1}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases} ?$$

(A) f_x and f_y exist at (0, 0), but are unbounded in any neighborhood of (0, 0).

(B) f is not continuous at (0, 0).

(C) f is continuous, but not differentiable at (0, 0).

(D) f is continuous and differentiable at (0, 0).

37. For the function $f(x, y) = x^3 + y^3 - 3xy$, consider the following statements :

(I) f has 3 critical points.

(II) f has two saddle points and one point for local minimum.

(III) f has only one saddle point, a point for local minimum and a point for local maximum.

Choose the ***correct*** answer from the options given below.

(A) Only (I) is true

(B) Only (II) is true

(C) Only (I) and (II) are true

(D) Only (I) and (III) are true.

38. A unit vector that maximizes the directional derivative of the function :
$f(x, y) = g(2x + y)$, $g'(3) = 3$ at the point (1, 1) is ________.

(A) $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$

(B) $\frac{1}{\sqrt{5}}(-2\hat{i} + \hat{j})$

(C) $\frac{1}{\sqrt{5}}(2\hat{i} - \hat{j})$

(D) $-\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$

39. What is the value of the integral : $\iint_D dA$, where D is the region bounded by the curve $y = x^2$ and the line $y = 4$?

(A) $\frac{8}{3}$

(B) $\frac{11}{3}$

(C) $\frac{16}{3}$

(D) $\frac{32}{3}$

40. The iterated integral : $\int_0^2 \int_0^{4-y^2} \int_0^{2-x} dz\,dy\,dx$ represents the volume of which one of the following solid regions ?

(A) The portion of the ellipsoid $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{9}$ above the plane $z=4$.

(B) The region bounded by the parabolic cylinder $x=4-y^2$ and the plane $z=2-x$.

(C) A paraboloid bounded by $x^2+y^2=z$ and the plane $z=4$.

(D) The top half of a sphere $x^2+y^2+z^2=25$ cut by the plane $z=4$.

41. Which of the following statements is/are is ***true*** ?

(I) The vector field $\vec{F}(x, y, z)=(yz, zx, xy)$ is solenoidal.

(II) The vector field $\vec{F}(x, y, z)=(x^2-y^2, 2xy, z^3)$ is conservative.

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

42. Consider the following assertions :

(I) $\iint_S (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{n}\,dS=2V$, where $\hat{n}$ is the unit outward surface normal and S is a closed surface enclosing a volume V.

(II) $\int_C (yz\hat{i}+xz\hat{j}+xy\hat{k})\cdot\vec{dr}=3$, where C is the boundary of a surface S.

Choose the ***correct*** option.

(A) (I) is correct, but (II) is false

(B) (I) is false, but (II) is correct

(C) Both (I) and (II) are false

(D) Both (I) and (II) are correct

43. Consider the following statements :

(I) The function $f(z)=\dfrac{x^2-y^2+2ixy}{x^2+y^2}$ $(z=x+iy)$ has a limit as $z\to 0$.

(II) The function $f(z)=\begin{cases}\dfrac{\text{Re}(z)}{|z|}, & z\neq 0\\ 1, & z=0\end{cases}$ is continuous at $z=0$.

Choose the ***correct*** answer from the options given below :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

44. Which of the statements given below is/are ***false*** ?

(I) $f(z) = x^2 + y^2 + 2ixy$ $(z = x + iy)$ is differentiable only at the points that lie on the x-axis.

(II) $f(z) = |z - 1|^2$ is differentiable at $z = 1$, but not analytic at $z = 1$.

(III) There exists an analytic function in $\mathbb{C}$ whose imaginary part is x^2.

(A) Only (III)
(B) Only (I) and (II)
(C) Only (II) and (III)
(D) All (I), (II) and (III)

45. Which one of the following options is ***true*** ?

(A) If Log(z) denotes the principal value of the logarithm, then $\mathrm{Log}(zw) = \mathrm{Log}(z) + \mathrm{Log}(w)$ for all $z, w \in \mathbb{C} \setminus \{0\}$.

(B) The solutions set of the equation: $e^{iz} = -1$ is $\{(2k+1)\pi : k \in \mathbb{Z}\}$

(C) $\cos(z)$ is bounded in the whole complex plane.

(D) A Mobius transformation with three fixed points is a constant.

46. The radius of convergence of the power series : $\sum_{n=1}^{\infty}\left(\frac{2^n}{n} + \frac{3^n}{n^2}\right) z^n$ is ________.

(A) e
(B) 1
(C) $\frac{1}{2}$
(D) $\frac{1}{3}$

47. If C is a simple closed curve not passing through the points -1, 0 and 1 in the complex plane, then the set of all possible value(s) of the integral : $\int_C \frac{dz}{z(1-z^2)}$ is ________.

(A) $\{0, \pm i\pi, \pm 2i\pi\}$
(B) $\{0, \pm i\pi, 2i\pi\}$
(C) $\{0, \pm i\pi\}$
(D) $\{0\}$

48. Which one of the options given below is ***false*** ?

(A) If f is analytic in a simply connected domain D, then its line integral along any two different paths in D connecting the same endpoints P and Q are equal.

(B) There exists an entire function f such that $f\left(\frac{1}{n}\right) = \frac{2n}{1+3n}$ for $n \in \mathbb{N}$.

(C) If f is an entire function satisfying $|f(z)| \le z^2$ for all $|z| > 1$, then f is a polynomial of degree at most 2.

(D) If T is a triangle with 0, 1 and i as its vertices, then $\int_T z\, dz = 0$.

49. For an entire function f, which one of the following statements is **false** ?

(A) f is constant, if the range of f is contained in a straight line.

(B) f is constant, if f has uncountably many zeros.

(C) f is constant, if f is bounded on $\{z \in \mathbb{C} : \operatorname{Re}(z) \leq 0\}$.

(D) f is constant, if the real part of f is bounded.

50. If $\sum_{n=-\infty}^{\infty} a_n z^n$ is the Laurent series expansion of the function : $f(z) = \dfrac{1}{2z^2 - 13z + 15}$

in the annulus $\left\{z \in \mathbb{C} : \frac{3}{2} < |z| < 5\right\}$, then $\dfrac{a_1}{a_2} =$ ________.

(A) -5

(B) $-\frac{1}{5}$

(C) $\frac{1}{5}$

(D) 5

51. Which of the following statements is/are **true** ?

(I) The function $f(z) = \sin\left(\dfrac{1}{\cos(1/z)}\right)$ has an isolated singularity at $z = 0$.

(II) The value of the integral: $\int_{|z|=1} \dfrac{dz}{z\sin(z)}$ is zero.

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

52. Consider the following statements :

(I) Residue of $f(z) = e^z$ at $z = \infty$ is -1

(II) If a meromorphic function f has 5 simple zeros and 2 simple poles inside $|z| = 1$, then $\int_{|z|=1} \dfrac{f'(z)}{f(z)} dz = 6i\pi$.

Pick out the ***correct*** option.

(A) (I) is true, but (II) is false

(B) (I) is false, but (II) is true.

(C) Both (I) and (II) are true.

(D) Both (I) and (II) are false.

53. In the Dihedral group $D_5 = \{r, s : r^5 = e, s^2 = e, sr = r^{-1}s\}$ under composition as the binary operation, which of the following options is ***not*** true ?

(A) D_5 is a non-commutative group

(B) Order of D_5 is 10

(C) $Z(D_5) = \{e\}$

(D) Number of normal subgroups of D_5 is 2

54. In a group G, let $x^5 = e$ (e : identity element of G) and $xyx^{-1} = y^2$ for $x, y \in G$. If $y \neq e$, then the order of y is ________.

(A) 25
(B) 30
(C) 31
(D) 32

55. Let G be a finite Abelian group. If the subgroups H and K are of index 3 each in G, then what is the index of the subgroup $H \cap K$ in G ?

(A) 9
(B) 6
(C) 5
(D) 3

$B \cap K =$
$A \vee B' = (A \vee B)'$

56. Which of the following statements is/are *true* ?

(I) The maximum possible order of an element in the group (S_5, o) is 6.

(II) If $f : (S_3, o) \to (Z_6, +_6)$ is a group homomorphism, then the order of $f(S_3)$ is 1, 2 or 3.

(A) Both (I) and (II)
(B) Only (I)
(C) Neither (I) nor (II)
(D) Only (II)

57. Which one of the following statements is **incorrect** ?

(I) The group $\mathbb{Z}_{15}$ is isomorphic to $\mathbb{Z}_3 \times \mathbb{Z}_5$

(II) $Inn(A_3)$ is isomorphic to A_3

(A) Only (I)
(B) Only (II)
(C) Both (I) and (II)
(D) Neither (I) nor (II)

58. Which of the following statements is/are *correct* ?

(I) In $GL_2(R)$, matrices with the same determinant always belong to the same conjugacy class.

(II) The class equation of A_5 (the alternating group on 5 elements) is $60 = 1 + 6 + 10 + 15 + 28$.

(A) Only (I)
(B) Only (II)
(C) Both (I) and (II)
(D) Neither (I) nor (II)

59. Which of the following statements is *false* ?

5, 7

(I) $\mathbb{Z}_5 \times \mathbb{Z}_7$ is a cyclic group.

(II) The order of the element $(2, 5) \in \mathbb{Z}_5 \times \mathbb{Z}_7$ is 10.

(III) $Aut(\mathbb{Z}_5 \times \mathbb{Z}_7)$ is isomorphic to $\mathbb{Z}_{35}$.

(A) Only (I)
(B) Only (II)
(C) Only (II) and (III)
(D) Only (I) and (III)

60. Which one of the following options is ***correct***? (p is a prime number.)

(A) There are 3 non-isomorphic Abelian groups of order 45.

(B) Every p-group is Abelian.

(C) The group $\mathbb{Z}_2 \times \mathbb{Z}_3$ is a p-group.

(D) A group of order 42 must have elements of orders 6 and 7 only.

61. If G is a group of order 30, then the number of Sylow 5-subgroups in G must be ________.

(A) 1 or 2 (B) 2 or 3

(C) 3 or 5 (D) 1 or 6

6

5+6

62. On $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$ with the usual addition and multiplication of matrices, which of the following statements is ***true***?

(A) R is a ring without zero-divisors.

(B) R is a ring with zero-divisors.

(C) R is a commutative ring.

(D) Every non-zero element in R has a multiplicative inverse.

63. Which of the following statements is/are ***false***?

(I) $I = \left\{ \begin{pmatrix} a & 0 \\ c & d \end{pmatrix} : a, c, d \in \mathbb{R} \right\}$ is both a subring and an ideal of $M_2(\mathbb{R})$.

(II) $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ is an integral domain, but not a field.

(A) Only (I) (B) Only (II)

(C) Both (I) and (III). (D) Neither (I) nor (II)

64. Which one of the following options is ***correct*** for the ideal $I = \langle x^2 + 5 \rangle$ in the ring $\mathbb{Q}[x]$?

(A) I is a prime ideal, but not a maximal ideal.

(B) I is a maximal ideal, but not a prime ideal.

(C) I is both a prime ideal and a maximal ideal.

(D) I is neither a prime ideal nor a maximal ideal.

65. Consider the following statements :

(I) The kernel of the ring homomorphism $f : \mathbb{Z}[x] \to \mathbb{Z}$ given by $f(p(x)) = p(1)$ is $\{(x-1)q(x) : q(x) \in \mathbb{Z}[x]\}$.

(II) The ring $\mathbb{Z}_4$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Choose the ***correct*** answer from the options given below.

(A) Only (I) is true (B) Only (II) is true

(C) Both (I) and (II) are true. (D) Both (I) and (II) are false

66. Which one of the following options is **incorrect** ?

(A) In an integral domain, every prime element is irreducible.

(B) In a unique factorization domain, every irreducible element is prime.

(C) The polynomial $p(x) = x^3 - 6x + 9$ is irreducible over $\mathbb{Q}$.

(D) The polynomial $p(x) = x^3 + 4x + 7$ is irreducible over $\mathbb{Z}_5$.

67. Which of the statements given below is/are ***true*** ?

(I) $\mathbb{Z}[i]$ is a Principal ideal domain and Euclidean domain.

(II) $\mathbb{Z}\left[\sqrt{-5}\right]$ is neither a Principal ideal domain nor a Unique factorization domain.

(A) Only (I)

(B) Only (II)

(C) Neither (I) nor (II)

(D) Both (I) and (II)

68. Let *V* be the real vector space consisting of all polynomials in one real variable with real coefficients of degree at most 6, including the zero polynomial. Then which one of the following options is ***true*** ?

(A) $W_1 = \left\{p \in V : p\left(\frac{1}{2}\right) \notin \mathbb{Q}\right\}$ is a subspace of *V*.

(B) $W_2 = \left\{p \in V : p\left(\frac{1}{2}\right) = 1\right\}$ is a subspace of *V*.

(C) $W_3 = \left\{p \in V : p\left(\frac{1}{2}\right) = p(\text{[illegible]})\right\}$ is a subspace of *V*.

(D) $W_4 = \left\{p \in V : p'\left(\frac{1}{2}\right) = 1\right\}$ is a subspace of *V*.

69. For a real 4×3 matrix M and the standard basis $\{e_1, e_2, e_3\}$ of $\mathbb{R}^3$, which of the following statements is/are **true** ?

(I) If rank(M) = 1, then $\{Me_1, Me_2\}$ is a linearly independent set in R^4.

(II) If rank(M) = 2, then $\{Me_1, Me_2\}$ is a linearly independent set in R^4.

(III) If rank(M) = 3, then $\{Me_1, Me_2\}$ is a linearly independent set in R^4.

(A) Only (III)

(B) Both (I) and (II)

(C) Both (I) and (III)

(D) Both (II) and (III)

70. Let V be a 7–dimensional vector space. If U, W are subspaces of V with dimensions 4 and 5, respectively, then which of the following is **not** a possible value of dimension of $U \cap W$?

(A) 4

(B) 3

(C) 2

(D) 1

71. For a linear transformation $L : V \to W$ with $\dim(V) = 10$ and $\dim(W) = 8$, which one of the following options is **correct** ?

(A) L is always injective

(B) L is always surjective

(C) Ker(L) is at least 2-dimensional

(D) L is always invertible

72. Let L be a linear transformation defined on the vector space $\mathbb{R}^6$ over the field $\mathbb{R}$. If Rank(L) = 4 and Nullity(L^2) = 3, then Rank (L^2) is ________ (L^2 denotes the composition of L with itself).

(A) 4

(B) 3

(C) 2

(D) 1

73. Consider the following statements for the linear transformation L on $\mathbb{R}^3$ defined by $L(x, y, z) = (x, y, 0)$ for all $(x, y, z) \in \mathbb{R}^3$:

(I) Rank of L is 2.

(II) The only eigenvalues of L are 0 and 1.

(III) The number of linearly independent eigenvectors of L is 2.

Choose the correct answer from the options given below.

(A) Only (I) and (II) are true

(B) Only (II) and (III) are true

(C) Only (I) and (III) are true

(D) All (I), (II) and (III) are true

74. On the vector space $\mathbb{R}^3$ over the field R with the standard basis $\{e_1, e_2, e_3\}$, what is the annihilator (S^o) of the subspace $S = \text{span}\{e_1, e_2\}$?

(A) $S^o = \{0\}$, the zero functional on R^3

(B) $S^o = \text{span}\{f_1, f_2\}$, where f_1, f_2 are the dual basis elements corresponding to the basis elements e_1, e_2

(C) $S^o = \text{span}\{f_3\}$, where f_3 is the dual basis element corresponding to the basis element e_3

(D) $S^o = (\mathbb{R}^3)^*$, the entire dual space of $\mathbb{R}^3$

75. If $\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues of the matrix : $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$, then the value of $\lambda_1^2 + \lambda_2^2 + \lambda_3^2$ is ________.

(A) 40

(B) 43

(C) 45

(D) 48

76. If the eigenvalue λ of a matrix M has algebraic multiplicity 3 and geometric multiplicity 2, then which one of the following options is **true** ?

(A) M is diagonalizable

(B) The characteristic polynomial of M has degree 3

(C) The eigenspace associated with λ has dimension 3

(D) M is not diagonalizable

77. Which one of the following matrices has $p(x) = x^3 - 8x^2 + 5x + 7$ as the minimal polynomial ?

(A) $\begin{pmatrix} 0 & 0 & 7 \\ 1 & 0 & 5 \\ 0 & 1 & 8 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 & 8 \\ 0 & 0 & 5 \\ 0 & 1 & 7 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 0 & -7 \\ 1 & 0 & -5 \\ 0 & 1 & 8 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 0 & 7 \\ 1 & 0 & 5 \\ 0 & 1 & -8 \end{pmatrix}$

78. Let $(V, <\, ,\, >)$ be an inner product space and let $< x, y > = ||x||\, ||y||$ for all $x, y \in V$. Then which one of the following options is **true** ?

(A) $\{x, y\}$ is a linearly independent set. (B) $\{x, y\}$ is a linearly dependent set.

(C) $\{x, y\}$ is an orthogonal set. (D) $\{x, y\}$ is an orthonormal set.

79. The projection of the vector $u = (2, -1, 3) \in \mathbb{R}^3$ onto the vector $v = (1, 2, -1)$ of the vector space $\mathbb{R}^3$ is ________.

(A) $\left(-\frac{1}{2}, -1, \frac{1}{2}\right)$ (B) $\left(\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}\right)$

(C) $\left(\frac{2}{3}, \frac{4}{3}, -\frac{2}{3}\right)$ (D) $\left(\frac{1}{5}, \frac{2}{5}, -\frac{1}{5}\right)$

80. Which one of the following statements is ***false*** ?

(I) The linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects a vector across the line $y = x$ is both self-adjoint and normal.

(II) The linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates a vector by an angle $\frac{\pi}{4}$ in the anti-clockwise direction is both self-adjoint and normal.

(A) Only (I) (B) Only (II)

(C) Both (I) and (II). (D) Neither (I) nor (II)

81. What does the differential equation: $(2x + y + 1)\,dx + (x + 2y + 1)\,dy = 0$ represents ?

(A) A family of circles
(B) A family of parabolas
(C) A family of hyperbolas
(D) A family of ellipses

$\frac{dy}{dx} = f$

82. If the non-homogeneous term in an ordinary differential equation (ODE) is: xe^{3x} and 3 is a root of the characteristic equation with multiplicity 2, then what is form of the particular solution (i.e., y_P) ? (c is a constant.)

(A) $y_P = -ce^{3x}$
(B) $y_P = cxe^{3x}$
(C) $y_P = cx^2e^{3x}$
(D) $y_P = cx^2e^{3x}$

83. The Bernoulli equation: $\frac{dy}{dx} + P(x)y = Q(x)y^3$ is best solved by making which of the following substitution ?

(A) $v = y^{-2}$
(B) $v = y^{-1}$
(C) $v = y^2$
(D) $v = y^3$

84. The general solution of the ODE : $(D^2 + 6D + 9)y = \frac{e^{-3x}}{x^3}$, $D = \frac{d}{dx}$ (c_1 and c_2 are arbitrary constants) is ________.

(A) $y(x) = (c_1 + c_2x)e^{3x} + \frac{e^{3x}}{2x}$
(B) $y(x) = (c_1 + c_2x)e^{-3x} + \frac{e^{3x}}{2x}$
(C) $y(x) = (c_1 + c_2x)e^{-3x} + \frac{e^{-3x}}{2x}$
(D) $y(x) = (c_1 + c_2x)e^{3x} + \frac{e^{-3x}}{2x}$

$\frac{d^2w}{dx^2} + 6$

$\frac{dy}{dx} +$

85. Which one of the following options is ***true*** for the initial value problem :

$\frac{dy}{dx} = 2y^{\frac{1}{3}}, y(0) = 0$?

(A) It has no solution.
(B) It has exactly one solution.
(C) It has more than one, but finite number of solutions.
(D) It has infinitely many solutions.

86. If $u = x^3$ and $v = y^2$ transforms the ODE:

$3x^5dx - y(y^2 - x^3)dy = 0$ into $\frac{dy}{du} = \frac{\lambda u}{(u - v)}$, then what is the value of λ ?

(A) 4
(B) 2
(C) –2
(D) –4

87. If the roots of the characteristic equation of the Euler's ODE has a repeated root m, then what is the **correct** form of the general solution ? (c_1 and c_2 are arbitrary constants.)

(A) $y(x) = c_1 x^m + c_2 x^{-m}$

(B) $y(x) = c_1 x^m + c_2 x^{-m} \ln x$

(C) $y(x) = c_1 e^{mx} + c_2 e^{-mx}$

(D) $y(x) = c_1 \cos mx + c_2 \sin mx$

88. The general solution of the linear partial differential equation (PDE) :

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = z$ is ________ (ϕ is an arbitrary function).

(A) $\phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

(B) $\phi(x^2 - z^2, x^3 - y^3) = 0$

(C) $\phi\left(\frac{y}{z}, x^2 + y^2 + z^2\right) = 0$

(D) $\phi(x + y + z, xyz) = 0$

89. In solving the heat equation using separation of variables, the eigenvalues correspond to ________.

(A) The Fourier coefficients

(B) The characteristic equation of the wave equation

(C) The eigenvalues of the Laplacian operator

(D) The energy of the system

90. Which of the following statements is/are ***true*** for the PDE :

$(1 + x^2)u_{xx} + (1 + y^2)u_{yy} + xu_x + yu_y = 0$?

(I) It is classified as parabolic PDE.

(II) The canonical equation is $u_{\xi\xi} + u_{\eta\eta} = 0$.

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

91. Solution of the PDE: $u_x + yu_y = 0$ with the initial condition $u(0, y) = y^3$ is ________.

(A) $u(x, y) = y^3 e^{-5x}$

(B) $u(x, y) = y^3 e^{-4x}$

(C) $u(x, y) = y^3 e^{-3x}$

(D) $u(x, y) = y^3 e^{-2x}$

$\frac{du}{dx} + y\frac{du}{dy} = 0$

$\int \frac{du}{dx} = \int -y\frac{du}{dy}$

$-\frac{y^2}{}$

$u = xy\frac{du}{dx}$

92. A string of length 1 meter is fixed at both ends and obeys the wave equation : $u_{tt} = 4u_{xx}$ with initial conditions: $u(x, 0) = \sin(\pi x)$, $u_t(x, 0) = 0$. Then it's solution $u(x, t)$ is ________.

(A) $\sin(\pi t)\cos(\pi x)$

(B) $\sin(2\pi t)\cos(\pi x)$

(C) $\sin(2\pi x)\cos(\pi t)$

(D) $\sin(\pi x)\cos(2\pi t)$

93. The solution of the system of ODEs : $\frac{dy}{dt} = x + 2y$, $\frac{dy}{dt} = 3x + 2y$ with initial conditions : $x(0) = 6$ and $y(0) = 4$ is ________.

(A) $x(t) = 4e^{4t} + 2e^{t}, y(t) = 6e^{4t} - 2e^{t}$

(B) $x(t) = 4e^{4t} + 2e^{-t}, y(t) = 6e^{4t} - 2e^{-t}$

(C) $x(t) = 6e^{4t} - 2e^{-t}, y(t) = 2e^{4t} + 2e^{-t}$

(D) $x(t) = 2e^{4t} + 4e^{-t}, y(t) = 2e^{4t} + 2e^{-t}$

94. Regula Falsi method is used to find a root of the equation: $f(x) = x^3 - x - 1$ by choosing the initial guesses: $x_0 = 1$ and $x_1 = 2$. What would be the value of the next iteration (i.e., x_3) up to two decimal places ?

(A) 1.76

(B) 1.56

(C) 1.36

(D) 1.25

95. Using Newton's divided difference method, the second divided difference for the function values f(1) = 2, f(2) = 3, f(4) = 7 is approximately equal to ________.

(A) 1.5

(B) 1

(C) 0.5

(D) 0.33

96. Which of the following statements is/are ***true*** ?

(I) Gauss-Seidel method for solving a linear systems converges faster than Gauss-Jacobi method.

(II) In the Gauss-Jordan method for solving a linear systems, the coefficient matrix be transformed into upper triangular matrix.

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

97. Using Lagrange interpolation and with the following function values :

x	f(x)
0	1
1	2
3	4

what would be the approximate value of f (2) ?

(A) 2.13

(B) 3.00

(C) 3.13

(D) 3.33

98. What is the approximate value of the integral: $\int_0^1 (x + x^2)dx$, when Simpson's $\frac{1}{3}$–rd rule is applied with 2 sub-intervals ?

(A) 0.83

(B) 0.75

(C) 0.67

(D) 0.58

99. Which one of the following statements about the central difference and averaging operators is ***correct*** ?

(A) The central difference operator is used for approximating the second derivative, while the averaging operator is used for approximating the first derivative.

(B) Both the operators approximate derivative with the same order of accuracy.

(C) The central difference operator is more accurate than the averaging operator for approximating the derivative.

(D) The averaging operator is used for approximating the second derivative, while the central difference operator approximates the first derivative.

100. For what value(s) of the constants: a, b and c, the quadrature formula :

$\int_{-1}^{1} f(x)\, dx \approx af(-1) + bf(0) + cf(1)$ is exact for polynomials of degree up to 3 ?

(A) $a = \frac{1}{3}, b = \frac{4}{3}$ and $c = \frac{1}{3}$

(B) $a = \frac{4}{3}, b = \frac{1}{3}$ and $c = \frac{1}{3}$

(C) $a = \frac{1}{3}, b = \frac{1}{3}$ and $c = \frac{4}{3}$

(D) $a = \frac{1}{3}, b = \frac{4}{3}$ and $c = -\frac{1}{3}$