## NOTATION

| | |
|---|---|
| $\mathbb{N}$ | set of all natural numbers {1, 2, 3, .....}. |
| $\mathbb{Z}$ | set of all integers {0, ±1, ±2, ±3 .....}. |
| $\mathbb{Q}$ | set of all rational numbers. |
| $\mathbb{R}$ | set of all real numbers. |
| $\mathbb{C}$ | set of all complex numbers. |
| $\mathbb{R}^n$ | n-dimensional Euclidean space $\{(x_1, x_2, ..... x_n) : x_k \in \mathbb{R}, 1 \le k \le n\}$. |
| $\mathbb{S}_n$ | group of permutations on n distinct symbols under composition of mappings. |
| $\mathbb{Z}_n$ | additive group of congruence classes of integers modulo n. |
| $\mathbb{M}_n(\mathbb{R})$ | set of all n × n matrices with entries in $\mathbb{R}$. |
| O(G) | order of a set G(i.e., number of distinct elements in G). |
| $\hat{i}, \hat{j}, \hat{k}$ | unit vectors having the directions of positive X, Y and Z axes in three dimensional rectangular coordinate system. |

1. Which one of the following is true for the sequence $\{a_n\}_{n\ge1}$ given by $a_n = 1 + (-1)^n + \frac{1}{2^n}$ ?

   (A) $\lim_{n\to\infty} \sup a_n = 1$

   (B) $\lim_{n\to\infty} \inf a_n = 1$

   (C) $\{a_n\}_{n\ge1}$ is a congervent sequence

   (D) $\lim_{n\to\infty} \sup a_n \ne \lim_{n\to\infty} \inf a_n$

2. Which one of the following is true for the set $\Omega = \{x \in \mathbb{Q} : x^2 < 4\} \subset \mathbb{R}$ ?

   (A) $\Omega$ is bounded above, but not bounded below

   (B) $\Omega$ is bounded above and $\sup(\Omega) = 2$

   (C) $\Omega$ is bounded above, but does not have a supremum

   (D) $\Omega$ is not bounded above

3. For a sequence $\{a_n\}_{n\geq 1}$ of real numbers, consider the following statements :

P : If $|a_{n+1} - a_n| \leq \frac{1}{n}$ for $n \geq 1$, then $\{a_n\}_{n\geq 1}$ is a Cauchy sequence.

Q : If $|a_{n+1} - a_n| \leq \frac{1}{2^n}$ for $n \geq 1$, then $\{a_n\}_{n\geq 1}$ is a Cauchy sequence.

Which of the above statements is / are true ?

(A) Only P

(B) Both P and Q

(C) Only Q

(D) Neither P nor Q

4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \min\{x - [x], [x] - x + 1\}$, where $[x]$ is the greatest integer less than or equal to x. If :

$P = \{x \in [0, 3] : f \text{ is discontinuous}\}$ and $Q = \{x \in (0, 3) : f \text{ is not differentiable}\}$, then the sum of the number of elements in P and the number of elements in Q is ________.

(A) 6

(B) 5

(C) 4

(D) 3

5. The sequence $\{a_n\}_{n\geq 1}$ defined by $a_n = \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$ converges to ________.

(A) 1

(B) 2

(C) 3

(D) 5

6. For the series : $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ $(p \in \mathbb{R})$, which one of the following is true ?

(A) The series converges for all values of p.

(B) The series converges for $p > 0$ and diverges for $p \leq 0$.

(C) The series does not converge for any value of p.

(D) The series converges for $p > 1$ and diverges for $p \leq 1$.

7. Which one of the following integrals is convergent ?

(A) $\int_1^\infty \frac{dx}{x^2}$

(B) $\int_1^\infty \frac{dx}{\sqrt{x}}$

(C) $\int_0^1 \frac{dx}{x^2}$

(D) $\int_0^\infty \frac{dx}{\sqrt{x}}$

8. Consider the following statements :

P : The sequence $\{f_n\}_{n\geq 1}$ where $f_n(x) = \frac{nx}{1+(nx)^2}$ $(x \in \mathbb{R})$ is uniformly convergent on any closed interval containing 0.

Q : The series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ is uniformly convergent for all $x \in \mathbb{R}$.

Choose the correct option.

(A) P is true, but Q is false

(B) P is false, but Q is true

(C) Both P and Q are false

(D) Both P and Q are true

9. If f is a real-valued continuous function on the closed interval [0, 1], differentiable on the open interval (0, 1) such that $f(0) = -1$ and $f(1) = \frac{1}{2}$, then which one of the following is true ?

(A) There exists a $c \in (0, 1)$ such that $f'(c) = \frac{3}{2}$

(B) There exists a $c \in (0, 1)$ such that $f'(c) = -2$

(C) There exists a $c \in (0, 1)$ such that $f'(c) = -\frac{1}{2}$

(D) There exists a $c \in (0, 1)$ such that $f'(c) = 1$

10. $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$ can be expressed as :

(A) $\frac{1}{2}\int_0^1 \frac{dx}{\log(1+x)}$

(B) $\int_0^1 \frac{dx}{\log(1+x)}$

(C) $\int_0^2 \frac{dx}{\log(1+x)}$

(D) $2\int_0^1 \frac{dx}{\log(1+x)}$

11. If $f : \mathbb{R} \to \mathbb{R}$ is continuous function satisfying $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, then which one of the following is true ?

(A) f is increasing, if $f(1) \geq 0$ and decreasing, if $f(1) \leq 0$

(B) f is increasing, if $f(1) \geq 0$ and decreasing, if $f(1) \geq 0$

(C) f is a not an increasing function

(D) f is neither an increasing nor a decreasing function

12. Let $f : \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be a continuous function. If $\int_0^{\sin x} f(t)dt = \frac{\sqrt{3}}{2}x$ for $0 \leq x \leq \frac{\pi}{2}$, then what is the value of $f\left(\frac{1}{2}\right)$ ?

(A) $\frac{1}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$

(D) 1

13. For the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x < 0 \\ 0 & x \geq 0 \end{cases}$, which one of the following is false ?

(A) f has derivative of all orders at every point $x \in \mathbb{R}$.

(B) $f^{(n)}(0) = 0$ for each $n \in \mathbb{N}$.

(C) Taylor series expansion of f about $x = 0$ converges to f for all $x \in \mathbb{R}$.

(D) Taylor series expansion of f about $x = 0$ converges to f for all $x \geq 0$.

14. Which one of the following statements is true for the function $f(x) = \frac{x^2 - 3x}{x - 5}$ ?

(A) f has a horizontal asymptote, but no vertical asymptote.

(B) f has a vertical asymptote and a horizontal asymptote.

(C) f has a vertical asymptote, but no horizontal and slant asymptotes.

(D) f has a vertical asymptote and a slant asymptote, but no horizontal asymptote.

15. How many point(s) of inflexion does the function $f(x) = \sin^2(x)$ have in the interval $(0, \pi)$ ?

(A) 3

(B) 2

(C) 1

(D) 0

16. For the following assertions :

P : If $I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,dx\ (n \in \mathbb{N})$, then $I_n = \left(\frac{n-1}{n}\right) I_{n-2},\ n \geq 2,$

Q : If $J_n = \int_0^{\frac{\pi}{2}} \cos^n(x)\,dx\ (n \in \mathbb{N})$, then $J_n = \left(\frac{n-1}{n}\right) I_{n-2},\ n \geq 2,$

the correct option is :

(A) Only P is true

(B) Only Q is true

(C) Neither P nor Q is true

(D) Both P and Q are true

17. Let R be the region bounded above by the graph of $f(x) = \frac{1}{x}$ and below by the X-axis over the interval [1, 3]. What is the volume of the solid of revolution formed by revolving R around the Y-axis ?

(A) $4\pi$ units$^3$

(B) $3\pi$ units$^3$

(C) $2\pi$ units$^3$

(D) $\pi$ units$^3$

18. Choose the correct option to fill the blank in the following truth table :

| p | q | $q \rightarrow p$ |
|---|---|---|
| T | T | T |
| T | F | – |
| F | T | – |
| F | F | – |

(A) F, T, F

(B) T, F, T

(C) F, F, T

(D) T, T, F

(Turn over)



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19. What is the value of the expresion : $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60}$ (mod 7) ?

(A) 5

(B) 3

(C) 1

(D) 0

20. A graph G has 21 edges, 3 vertices are of degree 4 and other vertices are of degree 3. Then, the total number of vertices in G is ________.

(A) 10

(B) 11

(C) 13

(D) 14

21. In a playground, 3 sisters and 8 other girls are playing a game together. In a particular game, how many ways can all the girls be seated around a circle so that the three sisters are not seated together ?

(A) 8 × 11 !

(B) 128 × 9 !

(C) 84 × 8 !

(D) 210 × 7 !

22. Consider the recurrence relation $a_1 = 4$, $a_n = 5n + a_{n-1}$, $n \geq 2$. The value of $a_{64}$ is ________.

(A) 10399

(B) 23760

(C) 75100

(D) 53700

23. Let $P \in M_2(\mathbb{R})$, the set of all 2 × 2 matrices with entries in $\mathbb{R}$ such that the trace of P = – 3 and the determinant of P = 2. Which one of the following is the characteristic polynomial (i. e., c(λ)) of the matrix $P^{-1}$ ?

(A) $c(\lambda) = \lambda^2 + 3\lambda + 2$

(B) $c(\lambda) = \lambda^2 + \frac{3}{2}\lambda + \frac{1}{2}$

(C) $c(\lambda) = \frac{1}{2}\lambda^2 + \frac{3}{2}\lambda + 1$

(D) $c(\lambda) = \lambda^2 + \frac{3}{2}\lambda + 1$

24. Which of the following statements is / are false ?

P : A system of linear equations with more equations than unknowns is always inconsistent.

Q : A system of linear equations with more unknowns than equations has infinitely many solutions.

R : Every system of linear equations has either no solution, unique solution or infintely many solutions.

(A) Only P

(B) Only Q

(C) Only Q and R

(D) Only P and Q

( Continued )

25. If sin(x) is the integrating factor of the differential equation : $\frac{dy}{dx} + P(x)y = Q(x)$, then

$P(x) =$ ________.

(A) cos(x)

(B) tan(x)

(C) cot(x)

(D) cosec(x)

26. If the differential equation : $(ax^2 + bxy + y^2)dx + (2x^2 + cxy + y^2)dy = 0$ $(a, b, c \in \mathbb{R})$ is exact, then which one of the following options is correct ?

(A) $b = 2, c = 2a$

(B) $b = 4, c = 2$

(C) $b = 2, c = 4$

(D) $b = 2, a = 2c$

27. The initial value problem : $x\frac{dy}{dx} - 2y = 0$, $y(0) = 0$, $x > 0$ has ________.

(A) no solution

(B) a unique solution

(C) extra two solutions

(D) infinitely many solutions

28. Consider the following statements with regard to the solutions $y_1(x) = \sin(x)$ and $y_2(x) = \cos(x)$ of the differential equation : $\frac{d^2y}{dx^2} + y = 0$.

P : $y_1(x)$ and $y_2(x)$ are linearly dependent solutions of the given differential equation.

Q : Wronskian of $y_1(x)$ and $y_2(x)$ is equal to – 1.

R : $y_1(x)$ and $y_2(x)$ are linearly independent solutions of the given differential equation.

Which of the above statements is/are true ?

(A) Only Q and R

(B) Only P and Q

(C) Only Q

(D) Only R

29. The particular solution of the differential equation $y'' + 4y = \sin(2x)$ is ________.

(A) $-\frac{1}{4}\cos(2x)$

(B) $\frac{1}{4}\cos(2x)$

(C) $-\frac{1}{4}x\cos(2x)$

(D) $\frac{1}{4}x\cos(2x)$

30. Which one of the following is the general solution of the following partial differential equation :

$$y\frac{\partial u}{\partial y} - x\frac{\partial u}{\partial x} = 1 \,?$$

(A) $u = \log(y) + f(x^2y^2)$, where f is an arbitrary differentiable function.

(B) $u = \log(y) + f(xy)$, where f is an arbitrary differentiable function.

(C) $u = 2\log(y) + f(xy)$, where f is an arbitrary differentiable function.

(D) $u = \log(y) + f\left(\sqrt{xy}\right)$, where f is an arbitrary differentiable function.

31. Which one of the following is true for the partial differential equation (PDE) :

$$y^3\frac{\partial^2 u}{\partial x^2} - (x^2 - 1)\frac{\partial^2 u}{\partial y^2} = 0 \,?$$

(A) The PDE is hyperbolic in $\{(x, y) : y > 0, x > 1\}$

(B) The PDE is parabolic in $\{(x, y) : x < 0\}$.

(C) The PDE is elliptic in $\mathbb{R}^2$.

(D) The PDE is parabolic in $\{(x, y) : x > 0, y > 0\}$.

32. The solution of the partial differential equation : $\frac{\partial^2 u}{\partial t^2} = 36\frac{\partial^2 u}{\partial x^2}$ subject to the initial conditions $u(x, 0) = 5x$ and $\frac{\partial u}{\partial t}(x, 0) = 1$ at the point $(x, t) = (2, 1)$ is ________.

(A) 7

(B) 11

(C) 21

(D) 31

33. Consider the following statements in connection with the numerical evaluation of the definite integral : $I = \int_a^b x^2 dx$, where a and b are given :

P : The value of I obtained by using the Trapezoidal rule is always greater than or equal to the exact value of the definite integral.

Q : The value of I obtained by using the Simpson's rule is always equal to the exact value of the definite integral.

Which of the above statements is / are true ?

(A) Only P

(B) Only Q

(C) Both P and Q

(D) Neither P nor Q

34. The Newton-Raphson iteration method is used to determine the approximate value of the reciprocal of the number $x = 4$. If the initial guess is $x_0 = 0.2$, then what is the numerical value of the reciprocal after the first iteration (i. e., the value of $x_1$) ?

(A) 0.21
(B) 0.23
(C) 0.24
(D) 0.26

35. What is the order of convergence of the Regula-Falsi method for finding the approximate numerical value of a real root of an equation $f(x) = 0$ ?

(A) 1.0
(B) 1.618
(C) 2.0
(D) 2.231

36. Which one of the following is the fastest iterative method for solving simultaneous linear algebraic equations ?

(A) Gauss-Elimation method
(B) Gauss-Jordan method
(C) Gauss-Seidel method
(D) Gauss-Jacobi method

37. Assuming that the interval of differencing is 1, the value of $\Delta^2\left(\frac{1}{x}\right)$ is ________.

(A) $\frac{2}{x(x+1)(x+2)}$
(B) $\frac{1}{x(x+1)}$
(C) $\frac{1}{(x+1)(x+2)}$
(D) $\frac{2}{x(x+2)}$

38. Which one of the following is the linear Lagrange polynomial that passes through the points (a, f, (a)) and (b, f(b)) ?

(A) $p(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{a-b} f(b)$
(B) $p(x) = \frac{x}{b-a} f(a) + \frac{x}{b-a} f(b)$
(C) $p(x) = f(a) + \frac{f(b)-f(a)}{b-a}$
(D) $p(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

39. On the set $\mathbb{R}$ of real numbers, which one of the following is not a metric ?

(A) $d(x, y) = |x^2 - y^2| \ (x, y \in \mathbb{R})$

(B) $d(x, y) = |x - y| \ (x, y \in \mathbb{R})$

(C) $d(x, y) = \dfrac{|x - y|}{1 + |x - y|} \ (x, y \in \mathbb{R})$

(D) $d(x, y) = \min\{1, |x - y|\} \ (x, y \in \mathbb{R})$

40. If S and T are subsets of a metric space (X, d), then which one of the following options is false ? (For $S \subseteq X$, $S^o$ denotes the set of all interior points of S).

(A) If $S \subseteq T$, then $S^o \subseteq T^o$

(B) If S is an open set in X and $S \subseteq T$, $S \subseteq T^o$

(C) $(S \cup T)^o = S^o \cup T^o$

(D) $(S \cap T)^o = S^o \cap T^o$

41. Consider the following statements :

P : $\mathbb{Z}$, the set of all integers with the metric d defined by $d(m, n) = |m - n|$ is a complete metric space.

Q : $\mathbb{R}$, the, set of all real numbers with the metric d defined by $d(x, y) = |\tan^{-1}(x) - \tan^{-1}(y)|$ is a complete metric space.

Which of the above statements is / are false ?

(A) Both P and Q

(B) Neither P nor Q

(C) Only P

(D) Only Q

42. Let (X, d) and (Y, ρ) be metric space. If $f : X \to Y$ is continuous, then which one of the following is necessarily true ?

(A) f(U) is open in Y, whenever U is open in X

(B) If (X, d) is complete, then so is (Y, ρ)

(C) f(K) is compact in Y for all compact set $K \subset X$

(D) $f^{-1}(K)$ is compact in X for all compact set $K \subset Y$

43. Consider the metric space $(\mathbb{R}^2, d)$ with the discrete metric given by $d(x, y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}$.

Then which one of the following is true ?

(A) Every subset of $\mathbb{R}^2$ is dense in $(\mathbb{R}^2, d)$

(B) $(\mathbb{R}^2, d)$, is separable

(C) $(\mathbb{R}^2, d)$ is compact, but not connected

(D) Every subspace of $(\mathbb{R}^2, d)$ is complete

44. Which one of the following rings is not an integral domain ?

(A) $\mathbb{Q}$, the set of all rational numbers

(B) $Z_{15}$ with addition modulo 15 and multiplication modulo 15

(C) $\mathbb{R}[x]$, the set of all polynomials with real coefficients

(D) $\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z}\}$, the set of all Gaussian integers

45. Which one of the following statements is false ?

(A) The characteristic of the ring $\mathbb{Z}_2$ is 0

(B) $\mathbb{Z}$ is a subring of the ring $\mathbb{Q}$

(C) $\mathbb{Z}$ is not an ideal of the ring $\mathbb{Q}$

(D) There exists a field with 8 elements

46. The ideal $I = \langle x \rangle$ (the ideal generated by the element $x \in \mathbb{Z}$) of the polynomial ring $\mathbb{Z}[x]$ is ________.

(A) A maximal ideal, but not a prime ideal

(B) A prime ideal, but not a maximal ideal

(C) Both a prime and a maximal ideal

(D) Neither a prime nor a maximal ideal

47. The set of all ring homomorphisms $f : \mathbb{Z} \to \mathbb{Z}$ :

(A) Is an empty set

(B) Is a singleton set

(C) Has exactly two elements

(D) Is an infinite set

48. Consider the following classes of commutative rings with unity.

ED is the class of Euclidean domain, PID is the class of principal ideal domain, UFD is the class of unique factorization domain and ID is the class of integral domain.

Then, which one of the following inclusion relations is correct ?

(A) $PID \subset ED \subset UFD \subset ID$

(B) $ED \subset UFD \subset PID \subset ID$

(C) $ED \subset PID \subset UFD \subset ID$

(D) $UFD \subset PID \subset ED \subset ID$

49. For the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$, which one of the following is true ?

(A) f is continuous on $\mathbb{R}^2$

(B) f is continuous at all points of $\mathbb{R}^2$ except (0, 0)

(C) $f_x(0, 0) = f_y(0, 0)$

(D) f is bounded

50. In which direction, the directional derivative of the scalar point function $\phi(x, y, z) = x^2 y^2 z$ from the point (1, 1, 1) will be maximum ?

(A) $2\hat{i} + \hat{j} + 2\hat{k}$

(B) $2\hat{i} + 2\hat{j} + \hat{k}$

(C) $\hat{i} + \hat{j} + \hat{k}$

(D) $-2\hat{i} + \hat{j} - 2\hat{k}$

51. Consider the following statements for a vector field $\vec{F}$.

P : If $\text{curl}\left(\vec{F}\right) = 0$, then the field $\vec{F}$ is solenoidal.

Q : If $\text{div}\left(\vec{F}\right) = 0$, then the field $\vec{F}$ is irrotational.

Choose the correct option :

(A) P is true, but Q is false

(B) P is false, but Q is true

(C) Both P and Q are true

(D) Both P and Q are false

52. Pick out the correct option for the function $f(x, y) = x^3 + 3x^2 + 4xy + y^2$.

(A) f has a minimum at (0, 0)

(B) f has a maximum at (0, 0)

(C) f has a minimum at $\left(\frac{2}{3}, -\frac{4}{3}\right)$

(D) f has a maximum at $\left(\frac{2}{3}, -\frac{4}{3}\right)$

53. If $\vec{F}(x, y) = y\hat{i} - x\hat{j}$, then what is the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the arc of the parabola $y = x^2$ from (0, 0) to (1, 1) ?

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{5}{6}$

54. The Gauss divergence theorem is used to convert :

(A) A line integral to a surface integral

(B) A line integral to a volume integral

(C) A surface integral to a line integral

(D) A surface integral to a volume integral

55. If $\vec{r} = x\hat{i} + y\hat{j} + \hat{k}$, S is a closed surface and V is the volume enclosed by the closed surface S, then the value of $\int_S \vec{r} \cdot \hat{n}\, dS$ is equal to ________.

(A) 6V

(B) 5V

(C) 4V

(D) 3V

56. The value of the integral $\int_0^1 \int_0^x \int_0^{x+y} (xy + 2yz)\,dz\,dy\,dx$ is equal to ________.

(A) $\frac{1}{20}$

(B) $\frac{9}{20}$

(C) $\frac{19}{30}$

(D) $\frac{7}{60}$

57. Which one of the following is a subspace of the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$ ?

(A) $W = \{(a, b, c) : a \geq 0\}$

(B) $W = \{(a, b, c) : a + b + c = 0\}$

(C) $W = \{(a, b, c) : a^2 + b^2 + c^2 \leq 1\}$

(D) $W = \{(a, b, c) : a, b, c \in \mathbb{Q}\}$

58. On an n-dimensional vector space V over the field $\mathbb{F}$, which one of the following is false ?

(A) Any subset of V with more than n vectors is linearly dependent over $\mathbb{F}$.

(B) Any linearly independent subset of V is a part of a basis of V.

(C) Any linearly independent subset of V with n elements is a basis of V

(D) Statements in (A), (B) and (C) are false.

59. If V, W are finite-dimensional vector spaces and $T : V \to W$ is a linear transformation, then which one of the following is false ?

(A) If dim(V) < dim(W), then T cannot be onto

(B) If dim(V) > dim(W), then T cannot be one-to-one

(C) dim(V) = rank(T) + nullity(T)

(D) If dim(V) = dim(W), then T is invertible

60. What is the rank of the matrix $P = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{pmatrix}$ ?

(A) 3

(B) 2

(C) 1

(D) 0

61. The matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (y, x)$, $(x, y) \in \mathbb{R}^2$ relative the standard ordered basis $\{(1, 0), (0, 1)\}$ is __________.

(A) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

62. If P is a 3 × 3 matrix such that $P^3 = P$, then its eigenvalues are __________.

(A) – 1, 1 and 2

(B) – 1, 0 and 1

(C) – 1, – 1 and 0

(D) 0, 1 and 2

63. Which one of the following matrices cannot be diagonalized ?

(A) $\begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$

(B) $\begin{pmatrix} 3 & 1 \\ -2 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$

64. If $f(z) = \dfrac{x^2 - y^2 + 2ixy}{x^2 + y^2}$ $(z = x + iy)$, then which one of the following is true ?

(A) $\lim_{z \to 0} f(z)$ exists along the line $y = x$.

(B) $\lim_{z \to 0} f(z)$ exists along the line $y = 2x$.

(C) $\lim_{z \to 0} f(z)$ exists along the line $y = x^2$.

(D) Statements in (A), (B) and (C) are false

65. For the power series : $\sum_{n=0}^{\infty} a_n(z-1)^n$, where $a_n = \begin{cases} \frac{1}{2^n}, & n = 0, 2, 4, \cdots \\ \frac{1}{3^n}, & n = 1, 3, 5, \cdots \end{cases}$, the radius of convergence is ________.

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) 3

66. Which one of the following is true for the function :

$f(z) = (x^3 + 3xy^2) + i(y^3 + 3x^2y)$ $(z = x + iy)$ ?

(A) Not differentiable at any point of $\mathbb{C}$.

(B) Differentiable at all the points of $\mathbb{C}$.

(C) Differentiable only at the points that lie on the X-axis .

(D) Differentiable only at the points that lie on both the co-ordinate axes.

67. $\int_1^i \cos(z)dz =$ ________.

$[\sin z]_1^i = \sin i - \sin 1$

(A) $-\sinh(1) + i\sin(1)$

(B) $\sin(1) - i\sinh(1)$

(C) $-\sin(1) + i\sinh(1)$

(D) $\sin(1) + i\sinh(1)$

68. Let $D = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disk. If $f : C \to D$ is analytic with $f(i) = 2i$, then what is the value of $f(-1)$ ?

(A) 2i

(B) – 2i

(C) 2

(D) – 2

69. In the Laurent series expansion of the funciton $f(z) = \frac{1}{z^2(1-z)}$ valid in the region $|z| > 1$, the coefficient of $\frac{1}{z^2}$ is ________.

(A) – 1

(B) 0

(C) $\frac{1}{2}$

(D) 1

70. For the function $f(z) = z^3 \exp\left(\frac{1}{z}\right)$, z = 0 is ________.

(A) a removal singularity

(B) a zero of order 2

(C) a simple pole

(D) an isolated essential singularity

71. $\int_{|z|=1} z^3 \cos\left(\frac{1}{z}\right) dz =$ ________.

(A) $i\pi$

(B) $\frac{i\pi}{2}$

(C) $\frac{i\pi}{6}$

(D) $\frac{i\pi}{12}$

72. Which one of the following collections of 2 × 2 matrices froms a group under matrix multiplication ?

(A) $\left\{\begin{pmatrix} a & b \\ b & c \end{pmatrix} : ac \neq b^2, a, b, c \in \mathbb{R}\right\}$

(B) $\left\{\begin{pmatrix} a & b \\ c & a \end{pmatrix} : a^2 \neq bc, a, b, c \in \mathbb{R}\right\}$

(C) $\left\{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : ac \neq 0, a, b, c \in \mathbb{R}\right\}$

(D) $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc \neq 0, a, b, c, d \in \mathbb{Z}\right\}$

73. Consider the following statements with regard to the subgroups H and K of a group G

P : If G is a commutative group, then HK is a subgroup of G.

Q : If either H or K is normal in G, then HK is a subgroup of G.

Which of the above statements is / are true ?

(A) Only P

(B) Only Q

(C) Neither P nor Q

(D) Both P and Q

74. Which one of the following statements is false for a group G ?

(A) If $(ab)^2 = a^2b^2$ for all a, b $\in$ G, then G is a commutative group.

(B) If $a^2 = e$ for all b $\in$ G, then G is a commutative group (e is the identity element of G).

(C) If the order of G is 2, then G is a commutative group.

(D) Statements in (A), (B), (C) are false

75. Consider the following statements :

P : If (G, *) is a group of order n and $x \in G$ is such that $\underbrace{(x * x * \cdots * x)}_{m \text{ times}} = e$(e is the identify element of G) for some $m \leq n$, then m divides n.

Q : If (G, *) is a group of even order, then there must be an element $x \in G$ such that $x \neq e$ and $(x * x) = e$.

Which of the above statements is / are true ?

(A) Only P

(B) Only Q

(C) Neither P nor Q

(D) Both P and Q

76. Which one of the following is not true ?

(A) A group of order 11 is cyclic.

(B) A group of order 15 is cyclic.

(C) A group of order 21 is cyclic.

(D) A group of order 51 is cyclic.

77. In $S_5$, the symmetric group of all permutations on the five symbols 1, 2, 3, 4, 5, what is the highest possible order of an element ?

(A) 5

(B) 6

(C) 10

(D) 15

78. Let $S_n$ denote the symmetric group of all permutations on n symbols and let $A_n$ be the subgroup of $S_n$ consisting of all even permutations of those n symbols. Then which one of the following is false ?

(A) $A_n$ is an Abelian group in $n \leq 3$

(B) $A_n$ is a normal subgroup of $S_n$

(C) $A_n$ is a simple group for each $n \geq 3$

(D) $Z(A_n) = \{e\}$, if $n \geq 4$ ($Z(A_n)$ denotes the center of $A_n$)

79. Let $\phi$ be a homomorphism from a group G onto a Group $G'$. If $O(G) = 225$ and $O(Ker(\phi)) = 5$, then $O(G')$ is __________.

(A) 15

(B) 25

(C) 45

(D) 75

80. The number of Sylow 5-subgroups in a group of order 80 is __________.

(A) 1

(B) 2

(C) 5

(D) 7

80 = 2*2*2*2*5

= .2^4

5k+1 / 16