COMMON P.G. ENTRANCE TEST - 2021 (CPET-2021)

Test Booklet No.:

HIGHER EDUCATION DEPARTMENT, GOVT. OF ODISHA

TEST BOOKLET

Subject Code : **33** Entrance Subject : **MATHEMATICS**

*Time Allowed: **90** Minutes* *Full Marks : **70***

INSTRUCTIONS TO CANDIDATES

1. **Please do not open this Question Booklet until asked to do so.**
2. Check the completeness of the Question Booklet immediately after opening.
3. Enter your **Hall Ticket No.** on the Test Booklet in the box provided alongside. **Do not** write anything else on the Test Booklet.
4. Fill up & darken Hall Ticket No. & Test Booklet No. in the Answer Sheet as well as fill up Test Booklet Serial No. & Answer Sheet Serial No. in the Attendance Sheet carefully. Wrongly filled up Answer Sheets are liable for rejection.
5. Each question has four answer options marked (A), (B), (C) & (D).
6. Answers are to be marked on the Answer Sheet, which is provided separately.
7. Choose the most appropriate answer option and darken the oval completely, corresponding to (A), (B), (C) or (D) against the relevant question number.
8. Use only **Blue/Black Ball Point Pen** to darken the oval for answering.
9. Please do not darken more than one oval against any question, as scanner will read such markings as wrong answer.
10. **Each question carries equal marks. There will be no negative marking for wrong answer.**
11. **Electronic items such as calculator, mobile, etc., are not permitted inside the examination hall.**
12. *Don't leave the examination hall until the test is over and permitted by the invigilator.*
13. **The candidate is required to handover the original OMR sheet to the invigilator and take the question booklet along with the candidate's copy of OMR sheet after completion of the test.**
14. Sheet for rough work is appended in the Test Booklet at the end.

1. If A = {0.2, 0.22, 0.222, ...} then

(A) $\sup A = \frac{3}{9}$

(B) $\sup A = \frac{2}{9}$

(C) $\sup A = \frac{4}{9}$

(D) $\sup A = \frac{5}{9}$

2. $\lim_{n\to\infty} \sqrt[n]{1^2 + 2^2 + 3^2 + ... + n^2}$ equals to

(A) −1

(B) 1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

3. Let $\{a_n\}$ be a sequence of real numbers. Suppose that $\lim_{n\to\infty} |an|^{\frac{1}{n}} = q$. Which of the following is true?

(A) If $q < 1$ then $\lim_{n\to\infty} a_n > 0$

(B) If $q > 1$ then $\lim_{n\to\infty} |a_n| = \infty$

(C) If $q < 1$ then $\lim_{n\to\infty} a_n < 0$

(D) If $q > 1$ then $\lim_{n\to\infty} |a_n| > \infty$

4. $\lim_{n\to\infty} (\sqrt{n} - 1)^n$ equals to

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) ∞

5. If $S = \left\{\frac{(n+1)^2}{2^2} : n \in \mathbb{N}\right\}$ then

(A) Least upper bound of S=2

(B) Least upper bound of S = $\frac{1}{2}$

(C) Least upper bound of S = $\frac{9}{4}$

(D) Least upper bound of S = $\frac{25}{16}$

6. $\lim_{n\to\infty} \prod_{k=2}^{n} \frac{k^3-1}{k^3+1}$ equals to

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) $\frac{5}{3}$

(D) $\frac{7}{3}$

7. Whic of the following is not true?

(A) Every sequence of real numbers has a monotone sub-sequence.

(B) Every bounded sequence of real numbers has a convergent sub-sequence.

(C) Every bounded sequence is convergent.

(D) Every convergent sequence is cauchy sequence.

8. $\lim_{n\to\infty} \frac{1}{n}(1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+....+n^{\frac{1}{n}})$ equals to

(A) 1

(B) 0

(C) e

(D) $\frac{1}{2}$

9. Assume that there is an integer n_0 such that $n \geq n_0$ the inequality $a_n \leq b_n$ *holds.* Which of the following is true?

(A) $\underline{\lim}_{n\to\infty} a_n \leq \underline{\lim}_{n\to\infty} b_n$

(B) $\underline{\lim}_{n\to\infty} a_n \geq \underline{\lim}_{n\to\infty} b_n$

(C) $\overline{\lim}_{n\to\infty} a_n \geq \overline{\lim}_{n\to\infty} b_n$

(D) $\overline{\lim}_{n\to\infty} b_n \leq \underline{\lim}_{n\to\infty} a_n$

10. $\lim_{n\to\infty} 2^{-n} n^2$ equals to

(A) 0

(B) ∞

(C) 2

(D) 1

11. The infinite series $\sum_{n=0}^{\infty}(\frac{1}{2})^n$ equals to

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) None

12. For what value of p, the infinite series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)p}$ is convergent.

(A) $p \le 1$ (B) $p > 1$ (C) $0<p \le 1$ (D) $p \le 0$

13. $\lim_{x\to 0} x\left[\frac{1}{x}\right]$ equals to

(A) 0 (B) 1 (C) ∞ (D) -1

14. Let $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then f(x) is continuous at

(A) $x = 0$ (B) $x = 1$ (C) $x = \pi$ (D) $x = \infty$

15. For what value of r, $\lim_{n\to\infty} r^n = 0$.

(A) $r > 1$ (B) $0 \le r < 1$ (C) $2 < r < 3$ (D) $|r|>|$

16. The point of maxima of $f(x) = x^2 e^{-x}, x > 0$ is

(A) 0 (B) 2 (C) -1 (D) 1

17. The points of continuity of f defined by $f(x) = \begin{cases} x^2 - 1, & \text{if } x \text{ is irrational} \\ 0 \quad , & \text{if } x \text{ is rational.} \end{cases}$ are

(A) $\sqrt{2}, 1$ (B) $1, \frac{1}{2}$ (C) $-\frac{1}{2}, \frac{1}{2}$ (D) $-1, 1$

18. Let $f(x) = \begin{cases} x\tan^{-1}\frac{1}{x}, & x \neq 0 \\ 0 & , x = 0. \end{cases}$

Then the left hand derivative and right hand derivative of f(x) at x = 0 are

(A) $\pi, -\pi$ (B) $\frac{-\pi}{3}, \frac{\pi}{3}$ (C) $\frac{\pi}{2}, \frac{\pi}{2}$ (D) None

19. The value of $\int_1^2\int_4^6 \frac{x}{y^2}\,dx\,dy$ is

(A) 8 (B) $\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{7}$

20. The value of $\int_1^2\int_0^1\int_0^1 24x^2y^3z\,dz\,dy\,dx$ equals to

(A) 6 (B) 1 (C) 7 (D) 8

21. What is the harmonic conjugate of $u(x, y) = 2x - x^3 + 3xy^2$?

(A) $2y - 3x^2y - y^3 + c$

(B) $2y - 3x^2y + y^3 + c$

(C) $2y + 3x^2y + y^3 + c$

(D) $-2y + 3x^2y - y^3 + c$

22. The real part of $(1+i)^8 + (1-i)^8$ is

(A) 2^4 (B) 2^6 (C) 2^5 (D) 2^8

23. The value of $\oint_c \frac{e^z}{z^2-9}dz$ where $C: |z-2| = 2$, is

(A) 0 (B) $\frac{\pi}{3}e^3$ (C) $\frac{1}{3}i\pi e^3$ (D) $\frac{\pi}{3}e^2$

24. If $z = e^{i\frac{2\pi}{n}}$ for an integer $n \geq 2$ then the value of $1 + z + z^2 + \ldots + z^{n-1}$ equals to

(A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$

25. The real and imaginary part of sin z, $z \in \mathbb{C}$ equals to

(A) cos y sin hx and sin y cos hx

(B) sinx cos hy and cosx sin hy

(C) $-$ sinx coshy and cosx cos hy

(D) sinx cos hx and cosy sin hy

26. Which of the following is not a vector space?

(A) $\mathbb{C}$ over $\mathbb{R}$ (B) $\mathbb{C}$ over $\mathbb{Q}$ (C) $\mathbb{C}$ over $\mathbb{Z}$ (D) $\mathbb{C}$ over $\mathbb{C}$

27. The characteristic polynomial of $A = \begin{pmatrix} 2 & 3 & -2 \\ 0 & 5 & 4 \\ 1 & 0 & -1 \end{pmatrix}$ equals to

(A) $\lambda^3 - 6\lambda^2 - 5\lambda + 12$ (B) $\lambda^3 - 6\lambda^2 + 5\lambda - 12$

(C) $\lambda^3 + 6\lambda^2 - 5\lambda - 12$ (D) $\lambda^3 - 6\lambda^2 - 5\lambda - 12$

28. Which of the following is a subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

(A) $W_1 = \{(a, b, c) \in \mathbb{R}^3 : a^2 + b^2 + c^2 \leq 1\}$ (B) $W_2 = \{(a, b, c) \in \mathbb{R}^3 : a \geq 0\}$

(C) $W_3 = \{(a, b, c) \in \mathbb{R}^3 : a + b + c = 0\}$ (D) $W_4 = \{(a, b, c) \in \mathbb{R}^3 : a, b, c \in Q\}$

29. For which value of K_1 will the vector $u = (1, -2, k)$ in $\mathbb{R}^3$ be a linear combinatin of the vectors $v = (3, 0, -2)$ and $\omega = (2, -1, -5)$?

(A) $K_1 = 8$ (B) $K_1 = -8$ (C) $K_1 = -7$ (D) $K_1 = -9$

30. The dimension of the subspace $\{(a, b, c) \in \mathbb{R}^3 : c = 3a\}$ of $\mathbb{R}^3$ over $\mathbb{R}$ is

(A) 1 (B) 3 (C) 2 (D) None

31. Let A and B be arbitrary matrices for which the product AB is defined. Which of the following is true?

(A) rank (AB) $\geq$ rank (B) (B) rank (AB) $\geq$ rank (A)

(C) rank (AB) $\leq$ rank (B) (D) rank (AB) $\leq$ rank (A)+rank(B)

32. Which of the following transformations from $\mathbb{R}^2$ over $\mathbb{R}$ to $\mathbb{R}^2$ over $\mathbb{R}$ is a linear transformation?

(A) $T(x, y) = (x + y, x)$

(B) $T(x, y) = (x^2, y^2)$

(C) $T(x, y) = (2x - y, x^2)$

(D) $T(x, y) = (x + 1, y+1)$

33. Let $A = \begin{pmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ Then the minimal polynomial of A is

(A) $(t-2)(t-1)$

(B) $(t-2)(t-1)^2$

(C) $(t-2)^2(t-1)$

(D) None.

34. The system of equations $x + 2y - 2z = -1$, $3x - y + 2z = 7$, $5x + 3y - 4z = 2$ has

(A) Unique solution

(B) No solution

(C) Infinitely many solutions

(D) Three solutions.

35. Let $B = \begin{pmatrix} 1 & 3 \\ 0 & -2 \\ 5 & -1 \\ -2 & 3 \end{pmatrix}$ Then the rank (B) is

(A) 0

(B) 2

(C) 1

(D) None

36. The determinant of the matrix $C = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 0 & -3 & 7 & -8 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 4 \end{pmatrix}$ is

(A) 0

(B) -26

(C) -120

(D) -14

37. The integrating factor of the differential equation $(2x \log x - xy)\, dy + 2y\, dx = 0$ is

(A) $\frac{1}{x^2}$

(B) x^2

(C) $\frac{1}{x}$

(D) $\frac{1}{xy}$

38. The particular integral of $y'' - 6y' + 9y = 6\,e^{3x}$ is

(A) $3x^2\,e^{3x}$ (B) $\frac{x^2}{2}e^{3x}$ (C) $3x\,e^{3x}$ (D) $\frac{x^2}{3}e^{3x}$

39. The differential equation $y''' - 8y = 0$ has the solution

(A) $y = c_1e^{-x} + e^{3x}(c_2\cos\sqrt{2}x + c_3\sin\sqrt{2}x)$

(B) $y = c_1e^{2x} + e^{-x}(c_2\cos\sqrt{3}x + c_2\sin\sqrt{3}x)$

(C) $y = c_1e^{3x} + e^{2x}(c_2\cos x + c_2\sin x)$

(D) None.

40. The Wronskian of the set {sin x, cos x} is

(A) –1 (B) 0 (C) 1 (D) 2

41. The partial differential equation $y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$ is hyperbolic in

(A) the second and fourth quadrants (B) the first and second quadrants

(C) the second and third quadrants (D) the first and third quadrants

42. The PDE $\frac{\partial^2 z}{\partial x \partial y} = xy^2$ has the solution

(A) $z = \frac{x^2y^3}{6} + F(y) + G(x)$ (B) $z = \frac{x^2}{6} + F(y) + G(y)$

(C) $z = \frac{xy^3}{6} + F(y)$ (D) $z = \frac{x^3y^2}{6} + F(y) + G(x)$

43. The subset {1, -1, i, -i} of complex numbers forms a group with respect to

(A) addition (B) subtraction (C) Multiplication (D) division

44. The number of generators of $\mathbb{Z}_{20}$ is

(A) 06 (B) 07 (C) 08 (D) 09

45. How many subgroups does $\mathbb{Z}_{20}$ have?

(A) 5 (B) 6 (C) 7 (D) 8

46. The number of elements of order 10 in $\mathbb{Z}_{30}$ is

(A) 2 (B) 3 (C) 4 (D) 5

47. If H is a subgroup of finite group G and order of H and G are respectively m and n, then

(A) $m | n$ (B) $n | m$ (C) $m \nmid n$ (D) None

48. If G is a group such that $a^2=e$, $\forall a \in G$, then G is

(A) Abelian group (B) Non abelian group

(C) Ring (D) Field

49. If H is a subgroup of G, then H is normal in G if

(A) H is a subgroup of index 2 in G (B) H is a subgroup of index 3 in G

(C) H is a subgroup of index 4 in G (D) None of the above.

50. If H is a subgroup of a group G, then G/H is a quotient group when

(A) H is not a normal subgroup (B) H is a normal subgroup of G

(C) H is not abelian of G (D) None of the above.

51. If G is a group, then for all a, b $\in$ G

(A) $(ab)^{-1} = a^{-1} b^{-1}$ (B) $(ab)^{-1} = b^{-1}.a^{-1}$ (C) $(ab)^{-1} = ab$ (D) $(ab)^{-1} = ba$

52. If H_1 and H_2 are two subgroups of G, then which of the following is a subgroup of G?

(A) $H_1 \cap H_2$ (B) $H_1 \cup H_2$ (C) H_1H_2 (D) None.

53. Order of convergence Newton Raphson method is

(A) 1.618 (B) 2 (C) 1 (D) 3

54. If $f(x) = \frac{1}{x}$ then the divided difference $f[x_0, x_1, x_2]$ is

(A) $\frac{1}{x_0 x_1 x_2}$ (B) $\frac{-1}{x_0 x_1 x_2}$ (C) $\frac{x_2 - x_0}{x_0 x_1 x_2}$ (D) None.

55. Given that f(0) = 1, f(1) = 3, f(3) = 55. The quadratic lagrange polynomial is

(A) $8x^2 - 6x - 1$ (B) $8x^2 - 6x + 1$ (C) $-8x^2 + 6x - 1$ (D) $8x^2 + 6x + 1$

56. The Newton Raphson scheme for finding the smallest positive root of the equation

$f(x) = x^3 - 5x + 1 = 0$ is

(A) $x_{k+1} = \frac{2x_k^3 - 1}{3x_k^2 - 5}$ (B) $x_{k+1} = \frac{3x_k^3 - 1}{2x_k^2 - 5}$ (C) $x_{k+1} = \frac{2x_k^2 - 1}{3x_k^3 - 5}$ (D) None.

57. Consider the Linear Programming Problem

Maximiz $Z = x_1 + x_2$

Subject to $x_1 - 2x_2 \leq 10$

$x_2 - 2x_1 \leq 10$

$x_1, x_2 \geq 0$

then,

(A) The LPP admits an optimal solution

(B) The LPP is unbounded

(C) The LPP admits no feasible solution

(D) The LPP admits unique feasible solution

58. Consider the LPP

Minimize $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

then

(A) $x_1 = \frac{21}{13}, x_2 \frac{10}{13}$

(B) $x_1 = \frac{10}{13}, x_2 \frac{21}{13}$

(C) $x_1 = \frac{20}{13}, x_2 \frac{5}{13}$

(D) None.

59. Which of the following is true?

(A) If primal has a feasible solution then its dual will also have a feasible solution.

(B) If primal has no feasible solution then its dual will have no feasible solution.

(C) If both, primal and dual, have feasible solution, then both will have bounded optimal solution.

(D) If primal has no feasible solution, then its dual will have an unbounded solution.

60. In linear Programming problem

(A) Objective function, constraints, and variables are all linear.

(B) Only objective function to be linear.

(C) Only constraints are to be linear.

(D) Only variables are to be linear.

61. If A and B are independent events, then

(A) $P(A/B) = P(A). P(B)$

(B) $P(A/B) = P(B)$

(C) $P(A/B) = P(A)$

(D) None of these.

62. If A and B are mutually exclusive events, then

(A) $P(A \cap B) = 1$

(B) $P(A \cap B) = 0$

(C) $P(A) = P(B)$

(D) $P(A \cap B) = P(A)\, P(B)$

63. Let X be a continuous random variable and f be a probability density function of X. Then which of the following is true?

(A) $f(x) \leq 0, -\infty < x < \infty$

(B) $\int_{-\infty}^{\infty} f(x)dx = 1$

(C) $P(x \leq x) \neq \int_{-\infty}^{x} f(x)dx$

(D) None.

64. The simultaneous limit

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2 + y^6}$$

(A) $\frac{1}{2}$

(B) 0

(C) does not exist

(D) None

65. Let $f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

Then $\int_{xy}(0, 0)$ is equal to

(A) -1

(B) 0

(C) 1

(D) None

66. If $a | b$ and $a | c$, then

(A) $a | bc$

(B) $c | a$

(C) $a | b+c$

(D) $b | a$

67. If a positive integer n is divided by 5, the remainder is 3. Which of the number below yields a remainder of 0 when it is divided by 5?

(A) $n + 3$

(B) $n + 2$

(C) $n - 1$

(D) $n + 1$

68. The asymptotes parallel to co-ordinate axes of the curve $4x^2 + 9y^2 = x^2y^2$ are

(A) $x = 3, x = -3, y = 2, y = -2$

(B) $x = 2, x = -2, y = 3, y = -3$

(C) $x = \frac{1}{3}, -\frac{1}{3}, y = -\frac{1}{2}, y = \frac{1}{2}$

(D) None

69. In the Taylor series expansion of $f(x) = \frac{x-1}{x+1}$ about the point $x = 0$, the coefficient of x^2 is

(A) 0 (B) 2 (C) -2 (D) -1

70. Let $f : A \to B$ be a map and X, Y be the subsets of A. Then which of the following is true?

(A) $f(X \cup Y) \neq f(x) \cup f(y)$

(B) $f(X \cap Y) \supseteq f(x) \cap f(y)$

(C) $f(X \cup Y) = f(x) \cup f(y)$

(D) None of the above.

□□□