Notation Convention: Throughout this course, all vector quantities are written in bold (e.g., **v**, **a**, **F**). Scalar quantities use regular (non-bold) type. When writing by hand, draw an arrow above the letter to denote a vector ($\vec{a}$, $\vec{v}$, $\vec{F}$).

Lesson Focus: Motion & Motion Graphs

By the end of this lesson, you will be able to:

Learning Goals	Success Criteria
☐ Define and distinguish scalar and vector quantities using real-world examples. ☐ Define displacement, velocity, and acceleration, and solve related problems. ☐ Interpret and construct position-time, velocity-time, and acceleration-time graphs. ☐ Analyze motion using slope and area relationships from graphs.	☐ Classify quantities as scalar or vector and use correct units and direction. ☐ Calculate displacement, velocity, and acceleration correctly using formulas and sign conventions. ☐ Construct and interpret d–t, v–t, and a–t graphs with correct slopes and areas. ☐ Solve and explain motion problems clearly using proper diagrams, units, and physics terminology.

1 - Scalars vs. Vectors

Scalar: A quantity with magnitude (size) only — no direction.

Examples: speed (60 km/h), temperature (25°C), mass (70 kg), distance (5 m)

Vector: A quantity with both magnitude and direction.

Examples: **velocity** (60 km/h North), **force** (40 N downward), **displacement** (5 m East)

Notation	Meaning	Example
Bold: **v**	Single vector variable (print/type)	**v** or **a**
Arrow: $\vec{v}$ (handwritten)	Single vector variable (written)	$\vec{a}$ or $\vec{v}$
AB or $\overrightarrow{AB}$	Vector from point A (tail) to B (tip)	A = initial, B = terminal point
Magnitude: |**v**|	Length/size (always a scalar)	|**AB**| = length of segment AB

2 - Key Definitions

Position (d): Location relative to a reference point. Vector. SI unit: metres (m).

Distance: Total path length travelled. **Scalar** — always ≥ 0.

Displacement (Δd): Change in position. **Vector** — can be +, −, or 0.

Formula: $\Delta \boldsymbol{d} = \boldsymbol{d}_f - \boldsymbol{d}_i$

Average Speed: v_{avg} = total distance ÷ Δt (scalar)

Average Velocity: $\mathbf{v}_{avg} = \Delta\mathbf{d} \div \Delta t = (\mathbf{d}_f - \mathbf{d}_i) \div \Delta t$ (vector) SI unit: m/s

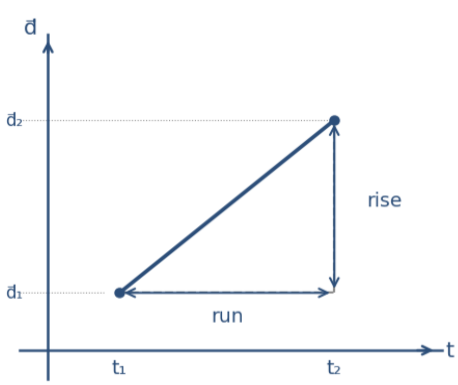
Acceleration (a): Rate of change of velocity. $\boldsymbol{a} = \Delta\mathbf{v} / \Delta t = (\mathbf{v}_f - \mathbf{v}_i) / \Delta t$ SI unit: m/s^2

3 - Motion Graphs

Position – Time Graphs

Key Principle: The slope of a position-time graph equals the velocity.

slope = rise/run = $(\mathbf{d}_2 - \mathbf{d}_1)/(t_2 - t_1) = \Delta\mathbf{d}/\Delta t = \mathbf{v}$



slope = rise / run

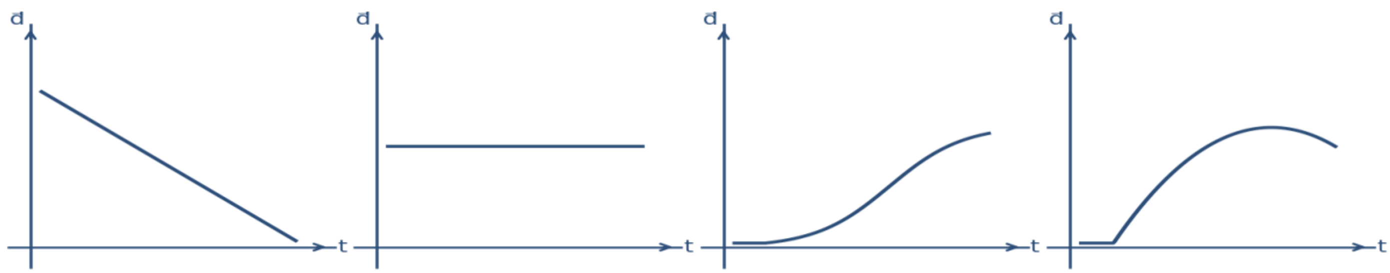
slope = $(\mathbf{d}_2 - \mathbf{d}_1) / (t_2 - t_1)$

slope = $\Delta\mathbf{d} / \Delta t = \mathbf{v}$

The slope of a position-time graph is equal to the velocity

Other Position-Time Graphs

Under each graph shown below, briefly describe the motion of the object.



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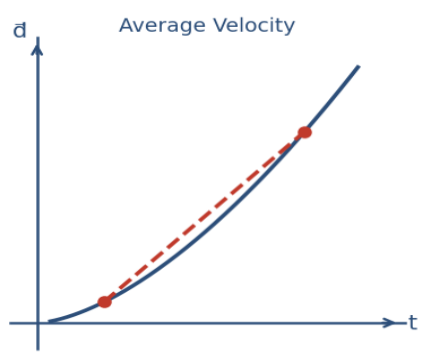
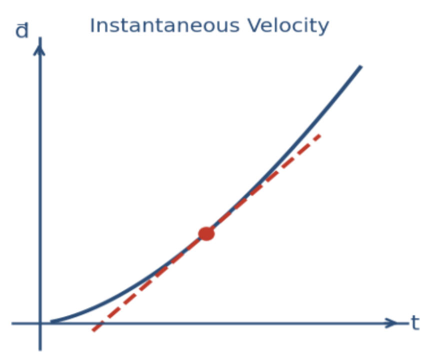
Average Velocity vs. Instantaneous Velocity





Average Velocity

The slope of the **line connecting two points** on the position-time graph gives the average velocity between those two points.

slope of secant line = v_{avg}

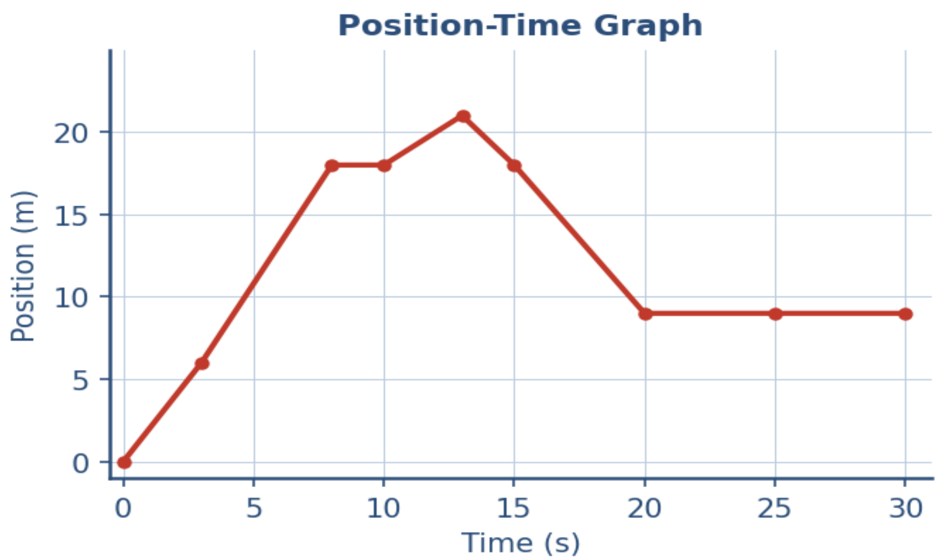
Instantaneous Velocity

The slope of the **tangent to the curve** at a specific point gives the instantaneous velocity at that moment.

slope of tangent = v_{inst}

Convert: Position-Time → Velocity-Time Graph

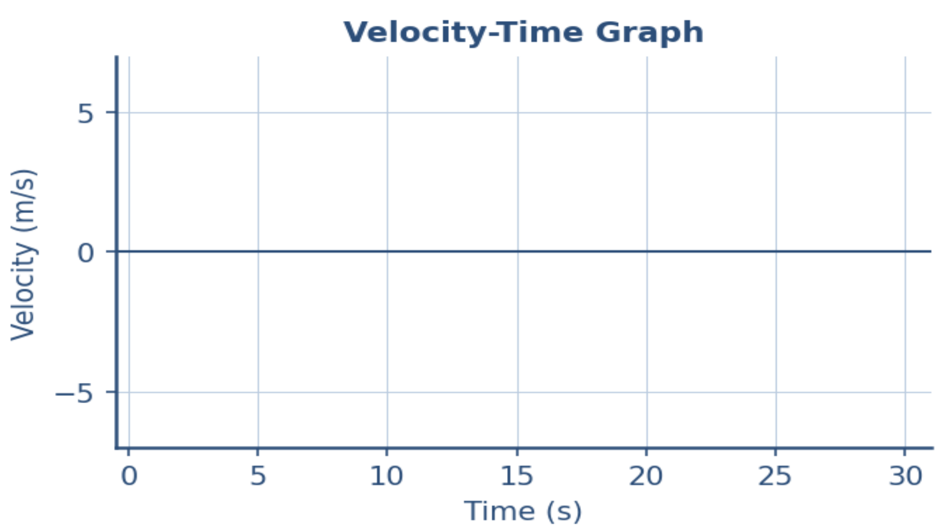
Convert the following position-time graph to a v-t graph. **Calculate the slope in each segment.**



Position-Time Graph

How to Convert:

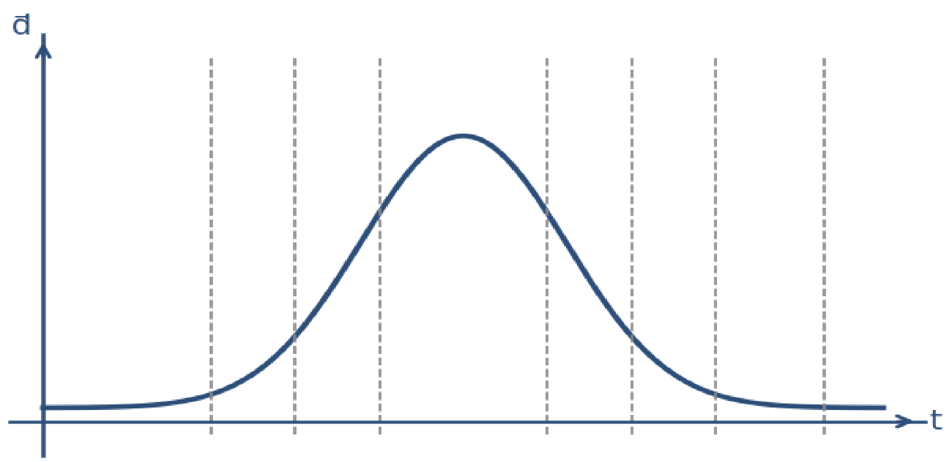
1. Identify each straight-line segment in the d-t graph.
2. Calculate the slope of each segment: slope = Δd/Δt = velocity.
3. Plot each calculated velocity as a horizontal line on the v-t graph for that time interval.
4. A steeper d-t slope → larger magnitude velocity. Downward slope → negative velocity.



Velocity-Time Graph (complete this graph)

More Position to Velocity-Time Graphs

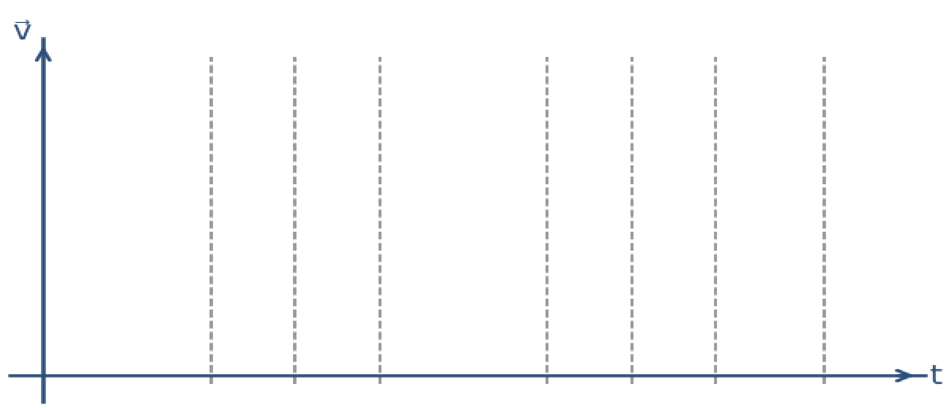
Convert the following position-time graph to a velocity-time graph. The dashed lines indicate key times where the slope changes.



Position-Time Graph

Hint: For a curved d-t graph, the slope (velocity) changes continuously.

- Rising curve with increasing slope → positive, increasing velocity (positive acceleration)
- Flat curve (slope = 0) → zero velocity (momentarily at rest or stopped)
- Falling curve → negative velocity (moving backward)



Velocity-Time Graph (draw your answer here)

Acceleration

Any change in velocity is called acceleration.

Definition of Acceleration – the rate of change of velocity.

Formula: acceleration = change in velocity / time interval

$$a = \Delta v / \Delta t$$

The slope of a velocity-time graph is equal to acceleration.

Complete the following statements:

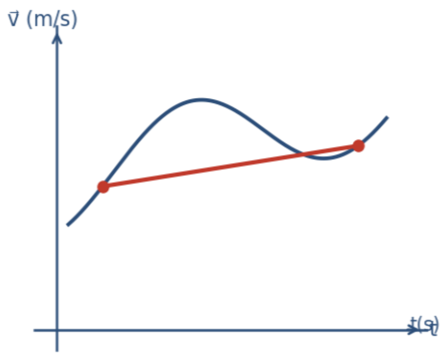
1. If a = 0, then the object is ____
2. If a is positive, then a forward-moving object will be __________________________________
3. If a is positive, then a backward-moving object will be _________________________________
4. If a is negative, then a forward-moving object will be __________________________________
5. If a is negative, then a backward-moving object will be _________________________________

An acceleration does not tell you which direction the object is traveling.

Sign of Acceleration	Direction of Motion	Speeding Up / Slowing Down
+	+	
+	–	
–	+	
–	–	

Changing Acceleration

The slope of a velocity-time graph is equal to acceleration.

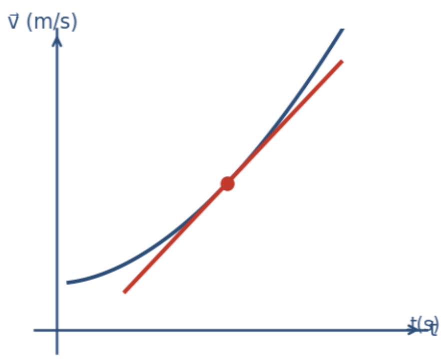


Average Acceleration

If asked to find the acceleration *between* two times, you are being asked for **average acceleration.**

Draw a **secant line** between the two points of interest. The slope of this line is the average acceleration.

$\mathbf{a}_{avg} = \Delta v / \Delta t$



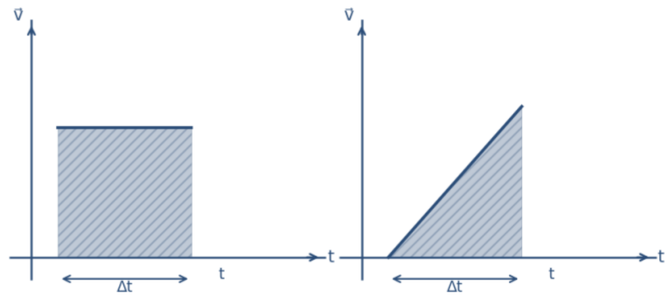
Instantaneous Acceleration

If asked for the acceleration *at* a specific time, you are being asked for **instantaneous acceleration.**

Draw a **tangent** to the curve at the point of interest. Its slope equals the instantaneous acceleration at that point.

$\mathbf{a}_{inst}$ = slope of tangent to v-t graph

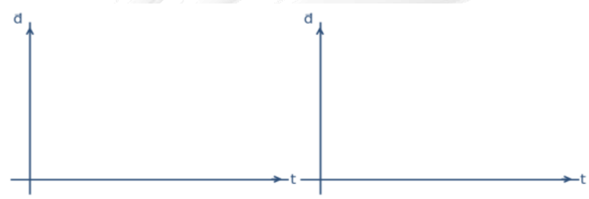
Area Under a v-t Graph



The area under a v-t graph is equal to displacement.

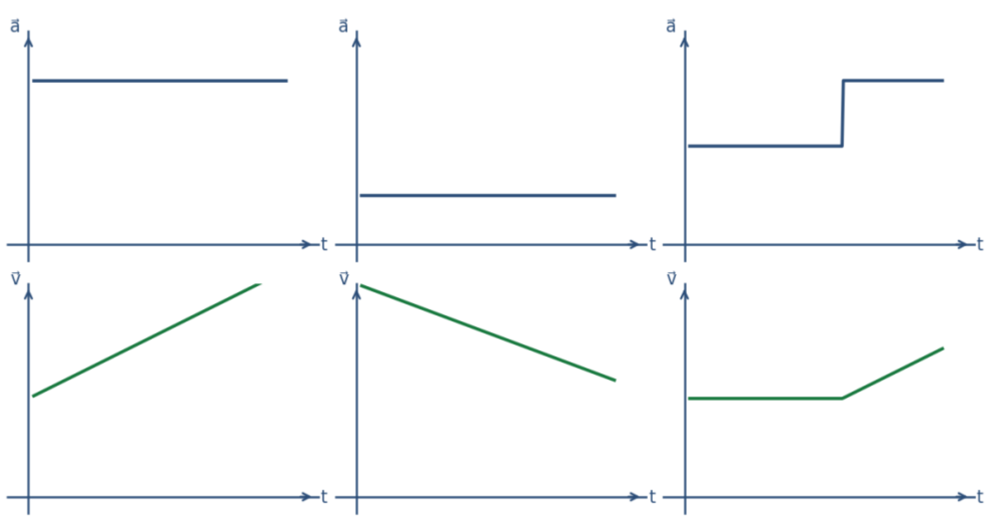
Area formulas (common shapes):

- Rectangle (constant velocity): $\Delta d = v \times \Delta t$
- Triangle (uniformly changing velocity): $\Delta d = \frac{1}{2} \times \text{base} \times \text{height}$
- Trapezoid: $\Delta d = \frac{1}{2} \times (v_1 + v_2) \times \Delta t$
- Area above t-axis = Δd is positive; Area below t-axis = Δd is negative



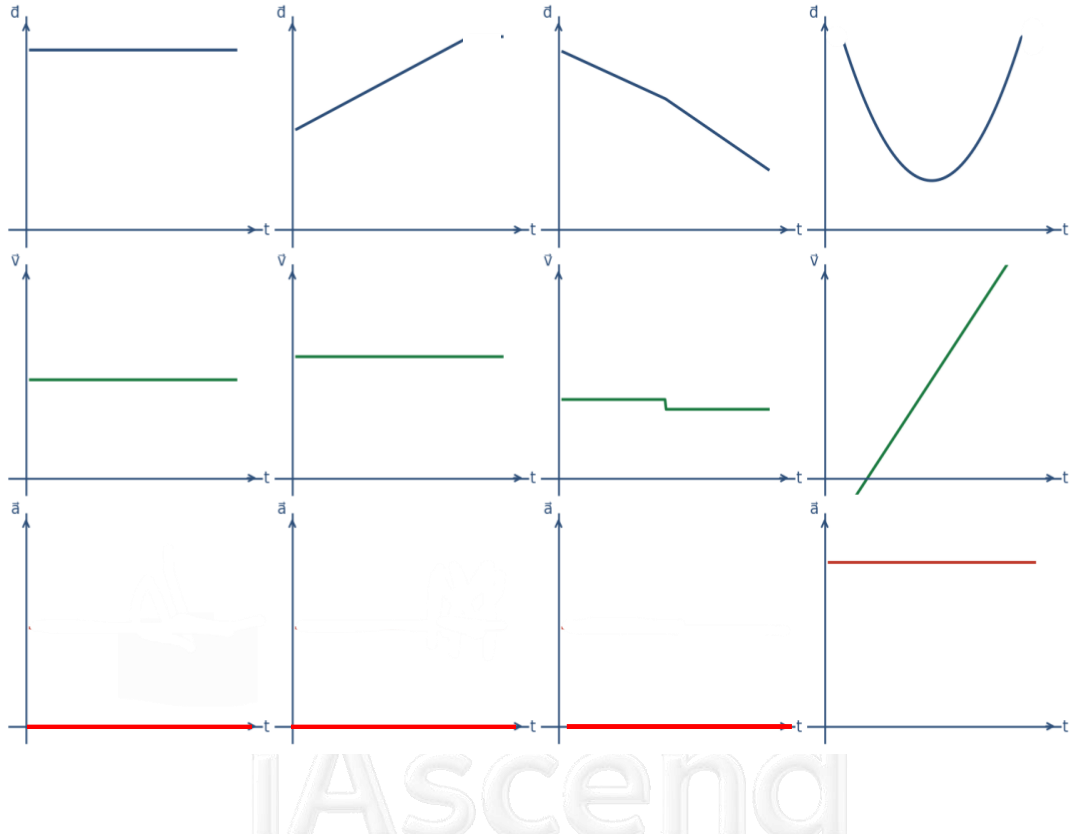
The corresponding d-t graphs (sketch below each v-t graph above)

Area under an a-t Graph



Sketch v-t and a-t Graphs from d-t Graphs

For each d-t graph shown in the top row, sketch the corresponding v-t graph (middle row) and a-t graph (bottom row).



Reminder:

- Constant slope on d-t → horizontal (constant) line on v-t → zero (flat at 0) on a-t
- Increasing slope on d-t → rising line on v-t → positive constant on a-t
- Decreasing slope on d-t → falling line on v-t → negative constant on a-t
- Curved d-t → straight line v-t → constant a-t (for uniform acceleration)

Kinematics: A Graphing Summary

Using Slope:

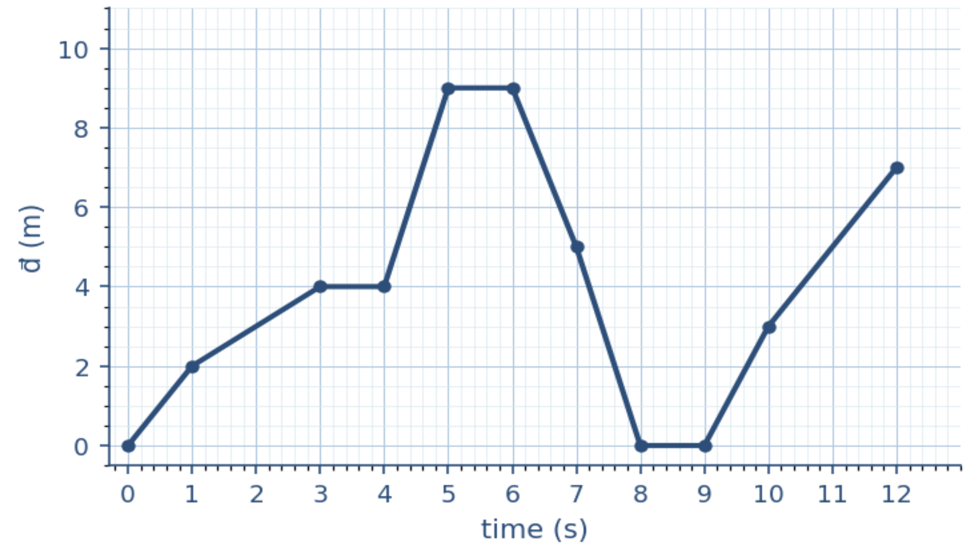
Quantity	How to Find It from a Graph
Velocity	Slope of a d vs t graph
Average Velocity	Slope of the secant line joining two points on a d vs t graph
Instantaneous Velocity	Slope of the tangent to a d vs t graph at the point of interest
Acceleration	Slope of a v vs t graph
Average Acceleration	Slope of the secant line joining two points on a v vs t graph
Instantaneous Acceleration	Slope of the tangent to a v vs t graph at the point of interest

Using Area:

Quantity	How to Find It from a Graph	
Displacement (Δd)	Area under a v vs t graph	**Connection Diagram:** d -t → slope → v-t → slope → a-t d -t ← Area ← v-t ← Area ← a-t
Change in Velocity (Δv)	Area under an a vs t graph	

Graphing Problems

Graph #1 – Position-Time Graph



Find:

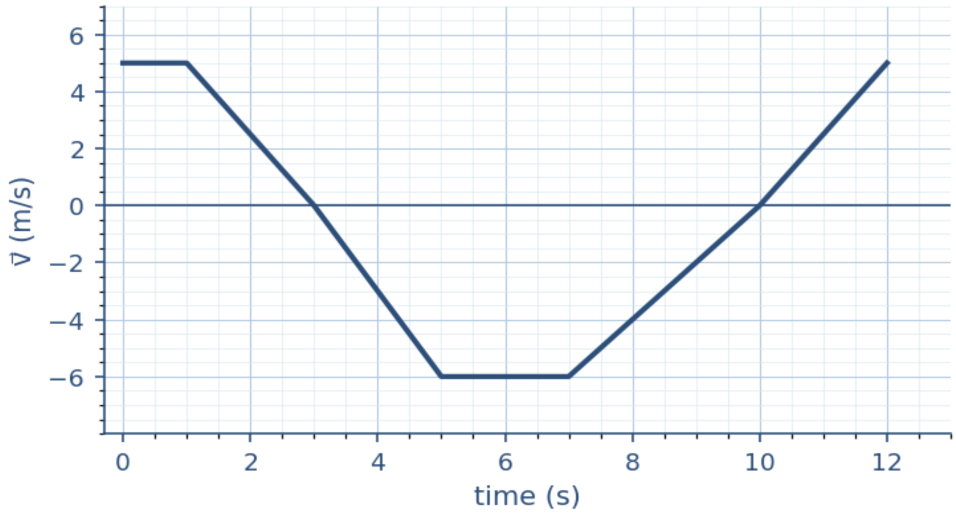
Velocity at t = 2.0 s ___________________________________

Velocity at t = 6.0 s ___________________________________

Velocity at t = 9.5 s ___________________________________

Average velocity between t = 1.0 s and t = 5.0 s _____________________

Graph #2 – Velocity-Time Graph



Find:

Acceleration at t = 1.0 s ______________________________________

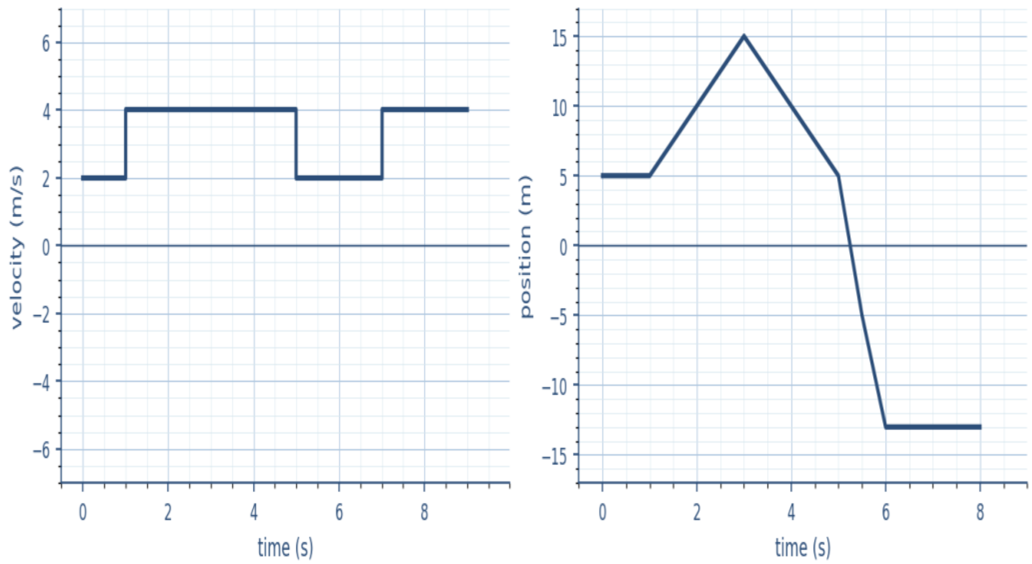
Acceleration at t = 3.0 s ______________________________________

Displacement during first two seconds ______________________________

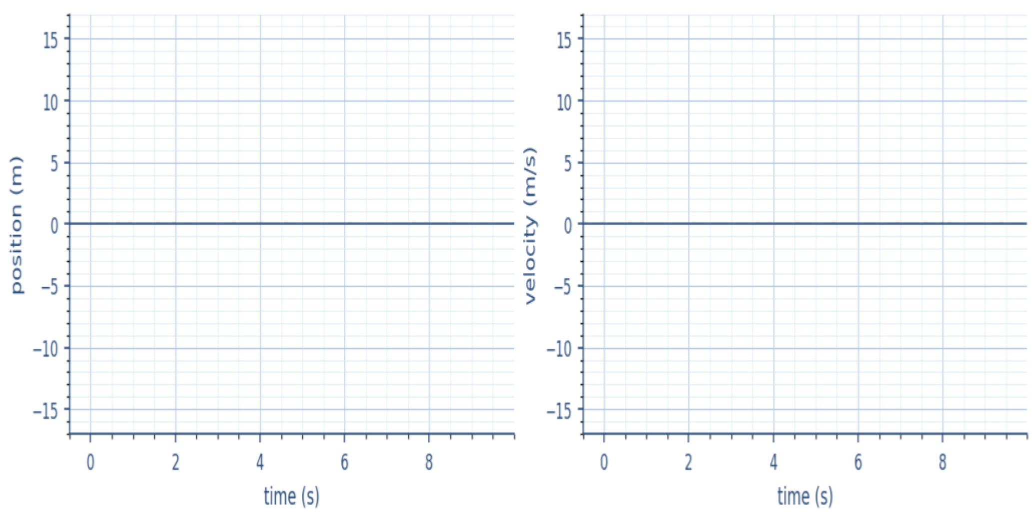
Displacement from t = 4.0 s to t = 6.0 s ___________________________

Convert and Calculate

For each graph shown, draw the corresponding indicated graph directly in the blank axes. Then answer the questions.

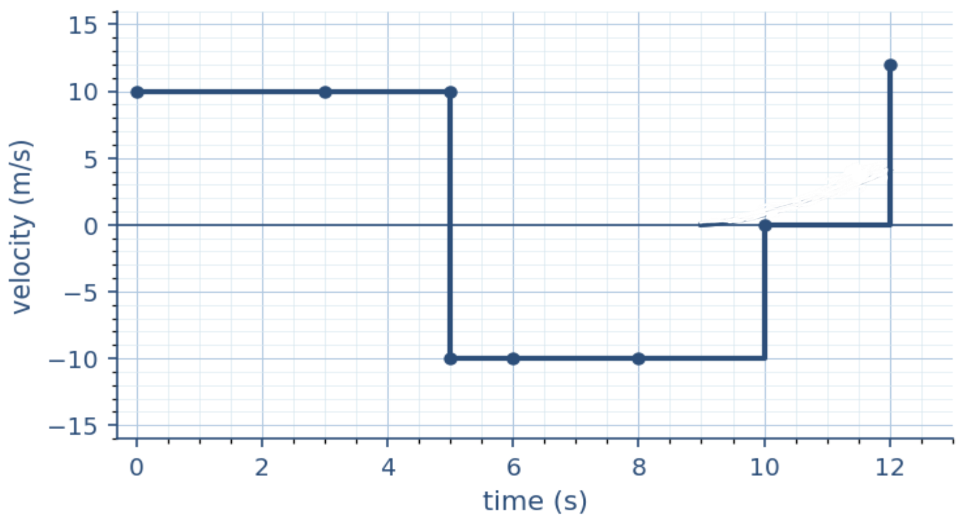


Given graphs – convert to the graph type indicated below each



Answer graphs – draw here

Graph #3 – Velocity-Time Graph



Find:

a) Calculate the acceleration at t = 5.5 s ________________________________

b) Calculate the average acceleration between t = 1.0 s and t = 5.0 s __________________

c) Calculate the displacement during the first 3.0 s ____________________________

d) Calculate the acceleration at t = 9.0 s ________________________________

4 - Worked Examples

Example 1 – Displacement and Average Velocity

A cyclist rides 5.0 km north, then 3.0 km south, then 2.0 km north in 0.50 h.

a) Total distance? b) Displacement? c) Average speed? d) Average velocity?

Given:

d_1 = +5.0 km, d_2 = −3.0 km, d_3 = +2.0 km

Δt = 0.50 h = 1800 s

Solution:

a) Distance = 5.0 + 3.0 + 2.0 = 10.0 km

b) Δd = 5.0 – 3.0 + 2.0 = +4.0 km [N]

c) v_{avg} = 10.0 / 0.50 = 20 km/h

d) $\vec{v}_{avg}$ = 4.0 / 0.50 = 8.0 km/h [N]

Example 2

Object's motion: 0–4 s: d goes 0 → +20 m; 4–8 s: stays at +20 m; 8–12 s: d goes +20 m → –4 m.

a) Velocity in each segment? b) Total displacement? c) Total distance?

Slope = Δd / Δt:

Seg 1 (0–4 s): v = (20–0)/4 = +5.0 m/s

Seg 2 (4–8 s): v = 0 m/s

Seg 3 (8–12 s): v = (–4–20)/4 = –6.0 m/s

a) v_1 = +5.0 m/s, v_2 = 0, v_3 = –6.0 m/s

b) Δd = –4 – 0 = –4 m

c) Distance = 20 + 0 + 24 = 44 m

Example 3 – Displacement from Area Under v-t Graph

Car: 0→6 s: 0→+24 m/s; 6–10 s: +24 m/s (constant); 10–14 s: +24→0 m/s.

a) Acceleration each phase? b) Total displacement?

Acceleration (slope of v-t):

a_1 = (24–0)/6 = +4.0 m/s²

a_2 = 0 m/s²

a_3 = (0–24)/4 = –6.0 m/s²

Displacement (area under v-t):

$\Delta d_1 = \frac{1}{2}(6)(24) = 72$ m

$\Delta d_2 = (4)(24) = 96$ m

$\Delta d_3 = \frac{1}{2}(4)(24) = 48$ m

Total: Δd = 216 m

Example 4 – Instantaneous Velocity by Differentiation

A particle's position: $d(t) = 2t^2 - 4t + 1$ (m, s).

a) Average velocity between t = 1 s and t = 3 s? b) Instantaneous velocity at t = 2 s?

a) Average velocity:

$d(1) = 2(1)^2 - 4(1) + 1 = -1$ m

$d(3) = 2(3)^2 - 4(3) + 1 = +7$ m

$\vec{v}_{avg} = (7-(-1))/(3-1) = +4.0$ m/s

b) Instantaneous (derivative):

$v(t) = d'(t) = 4t - 4$

$v(2) = 4(2) - 4 = +4.0$ m/s

Note: equal here only by coincidence of interval chosen.

5 - Practice Problems

Problem 1

A student walks 120 m east, 50 m north, then 120 m west in 3.0 min.

a) Total distance? b) Displacement (magnitude and direction)?

c) Average speed (m/s)? d) Average velocity magnitude (m/s)?

Problem 2

d-t graph data: t=0: d=0; t=3 s: d=+15 m; t=7 s: d=+15 m; t=10 s: d=−6 m.

a) Velocity in each segment? b) Describe motion in words.

c) Total displacement and distance over 10 s? d) Sketch the v-t graph.

Problem 3

v-t graph: 0 to 10 s: 0 → +30 m/s (uniform); 10 to 15 s: +30 → −15 m/s (uniform).

a) Acceleration each phase? b) Displacement each phase?

c) Total distance over 15 s? d) What does negative velocity represent physically?

Problem 4

Position function: $d(t) = 3t^2 + 6t - 9$ (m, s).

a) Find v(t) by differentiation. b) Instantaneous velocity at t = 0, 2, 4 s?

c) When is the particle momentarily at rest? d) Sketch d-t and v-t graphs (t = 0 to 5 s).

Problem 5

Two cyclists start simultaneously from the same point. Cyclist A: constant 8.0 m/s east. Cyclist B: accelerates from rest at 2.0 m/s^2 east.

a) Position functions? b) When does B overtake A? c) Distance from start at overtaking?

d) Sketch both d-t curves on same axes. e) B's velocity at the moment of overtaking?

Problem 6

Velocity: $v(t) = -6t^2 + 24t - 18$ (m/s, s).

a) Find a(t)? b) Times at rest? c) Interval(s) of positive-direction motion?

d) If d(0)=0, displacement t=0 to 4 s (by integration)? e) Total distance? f) When is a=0, and what does this indicate?

Problem 7

v-t graph (trapezoid): +5 m/s at t=0; rises to +25 m/s at t=4 s; holds +25 m/s to t=9 s; falls to −10 m/s at t=14 s.

a) Acceleration each phase; sketch a-t graph. b) Displacement each phase.

c) At what time after t=9 s does the object return to its starting position? (Set $\Delta d_{total} = 0$)

d) Velocity at the moment it returns to start?