

Notation Convention: Throughout this course, all **vector quantities are written in bold** (e.g., \mathbf{v} , \mathbf{a} , \mathbf{F}). Scalar quantities use regular (non-bold) type. When writing by hand, draw an arrow above the letter to denote a vector (\vec{a} , \vec{v} , \vec{F}).

Learning Goals

By the end of this lesson, you will be able to:

- Define and distinguish scalar and vector quantities with real-world examples.
- Define displacement, velocity, and acceleration, and connect them mathematically.
- Interpret and construct position-time (d-t), velocity-time (v-t), and acceleration-time (a-t) graphs.
- Extract kinematic quantities from the slope and area of motion graphs.
- Calculate instantaneous velocity using the derivative of a position function.

1 Scalars vs. Vectors

Scalar: A quantity with magnitude (size) only — no direction.

Examples: speed (60 km/h), temperature (25°C), mass (70 kg), distance (5 m)

Vector: A quantity with both magnitude and direction.

Examples: **velocity** (60 km/h North), **force** (40 N downward), **displacement** (5 m East)

Notation	Meaning	Example
Bold: \mathbf{v}	Single vector variable (print/type)	\mathbf{v} or \mathbf{a}
Arrow: \vec{v} (handwritten)	Single vector variable (written)	\vec{a} or \vec{v}
AB or \overrightarrow{AB}	Vector from point A (tail) to B (tip)	A = initial, B = terminal point
Magnitude: \mathbf{v}	Length/size (always a scalar)	$ \mathbf{AB} $ = length of segment AB

2 Key Definitions

Position (d): Location relative to a reference point. **Vector**. SI unit: metres (m).

Distance: Total path length travelled. **Scalar** — always ≥ 0 .

Displacement (Δd): Change in position. **Vector** — can be +, -, or 0.

$$\text{Formula: } \Delta d = d_f - d_i$$

Average Speed: $v_{\text{avg}} = \text{total distance} \div \Delta t$ (scalar)

Average Velocity: $v_{\text{avg}} = \Delta d \div \Delta t = (d_f - d_i) \div \Delta t$ (vector) SI unit: m/s

Acceleration (a): Rate of change of velocity. $a = \Delta v / \Delta t = (v_f - v_i) / \Delta t$ SI unit: m/s^2

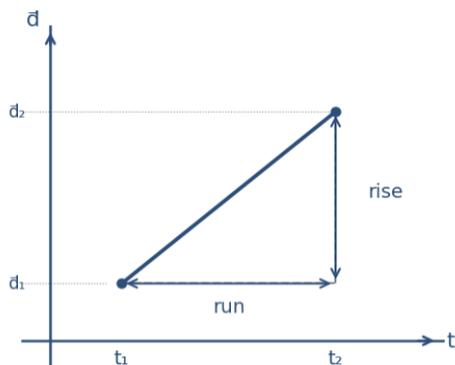
3 Motion Graphs

Position - Time Graphs

SPH4U · Unit 1: Kinematics · iascend.ca

Key Principle: The slope of a position-time graph equals the velocity.

$$\text{slope} = \text{rise/run} = (d_2 - d_1) / (t_2 - t_1) = \Delta d / \Delta t = v$$

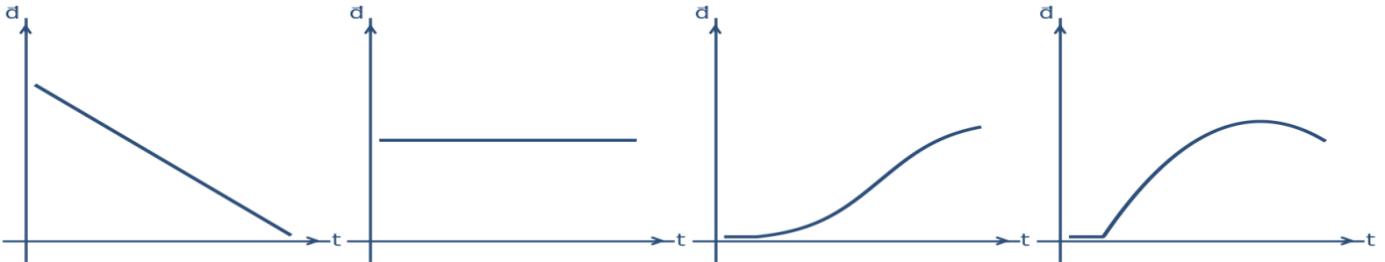


$$\begin{aligned} \text{slope} &= \text{rise} / \text{run} \\ \text{slope} &= (d_2 - d_1) / (t_2 - t_1) \\ \text{slope} &= \Delta d / \Delta t = v \end{aligned}$$

The slope of a position-time graph is equal to the velocity

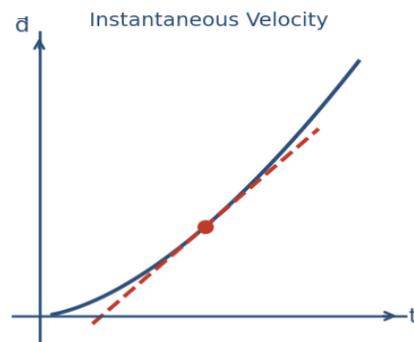
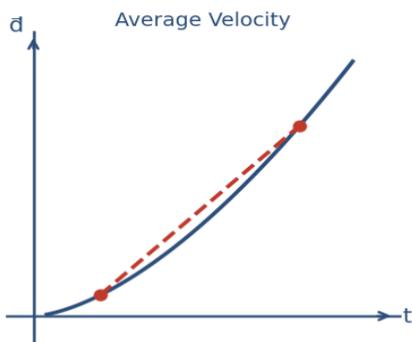
Other Position-Time Graphs

Under each graph shown below, briefly describe the motion of the object.



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Average Velocity vs. Instantaneous Velocity



Average Velocity

The slope of the line **connecting two points** on the position-time graph gives the average velocity between those two points.

$$\text{slope of secant line} = v_{\text{avg}}$$

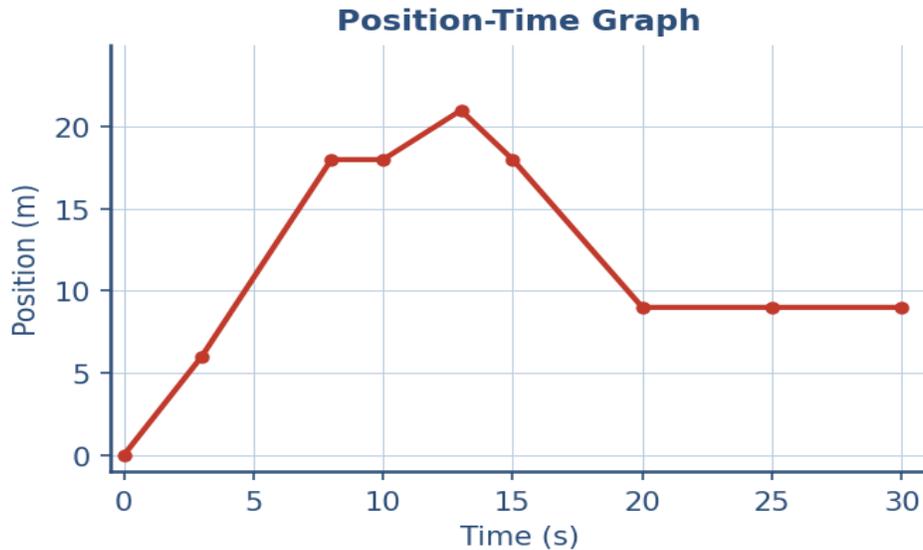
Instantaneous Velocity

The slope of the **tangent to the curve** at a specific point gives the instantaneous velocity at that moment.

$$\text{slope of tangent} = v_{\text{inst}}$$

Convert: Position-Time → Velocity-Time Graph

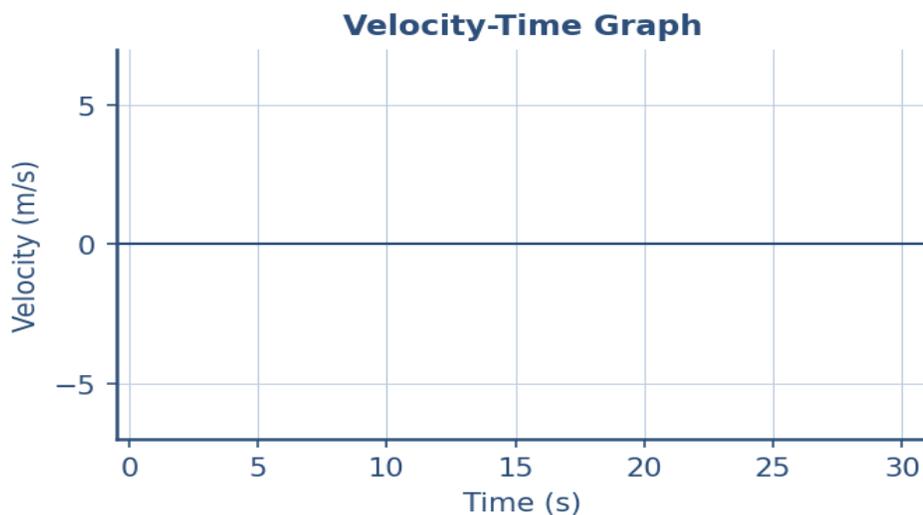
Convert the following position-time graph to a v-t graph. Calculate the slope in each segment.



Position-Time Graph

How to Convert:

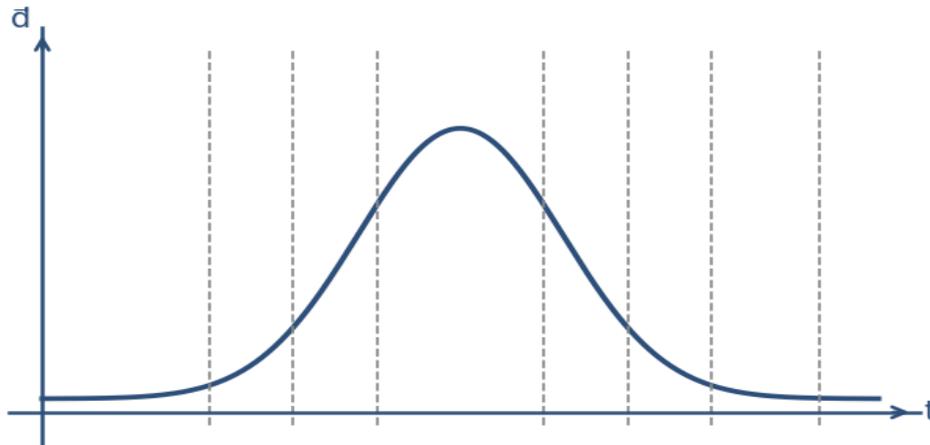
1. Identify each straight-line segment in the d-t graph.
2. Calculate the slope of each segment: $\text{slope} = \Delta d / \Delta t = \text{velocity}$.
3. Plot each calculated velocity as a horizontal line on the v-t graph for that time interval.
4. A steeper d-t slope → larger magnitude velocity. Downward slope → negative velocity.



Velocity-Time Graph (complete this graph)

More Position to Velocity-Time Graphs

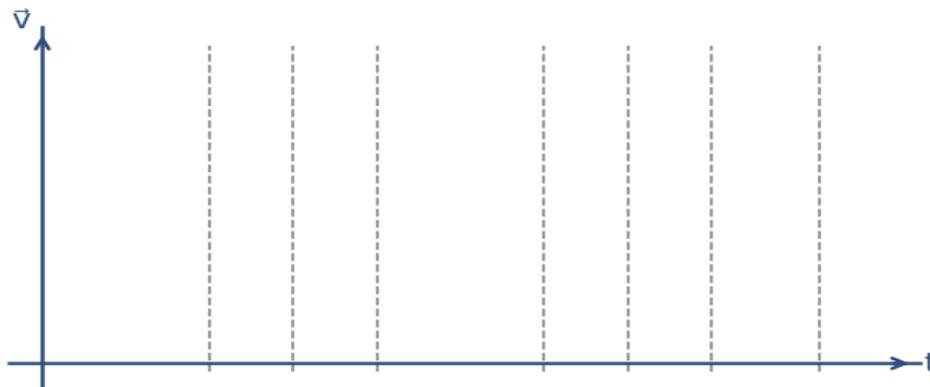
Convert the following position-time graph to a velocity-time graph. The dashed lines indicate key times where the slope changes.



Position-Time Graph

Hint: For a curved d - t graph, the slope (velocity) changes continuously.

- Rising curve with increasing slope \rightarrow positive, increasing velocity (positive acceleration)
- Flat curve (slope = 0) \rightarrow zero velocity (momentarily at rest or stopped)
- Falling curve \rightarrow negative velocity (moving backward)



Velocity-Time Graph (draw your answer here)

Acceleration

Any change in velocity is called acceleration.

Definition of Acceleration - the rate of change of velocity.

Formula: acceleration = change in velocity / time interval

$$\mathbf{a} = \Delta \mathbf{v} / \Delta t$$

The slope of a velocity-time graph is equal to acceleration.

Complete the following statements:

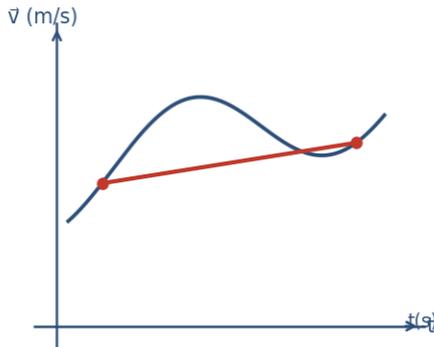
1. If $a = 0$, then the object is _____
2. If a is positive, then a forward-moving object will be _____
3. If a is positive, then a backward-moving object will be _____
4. If a is negative, then a forward-moving object will be _____
5. If a is negative, then a backward-moving object will be _____

An acceleration does not tell you which direction the object is traveling.

Sign of Acceleration	Direction of Motion	Speeding Up / Slowing Down
+	+	
+	-	
-	+	
-	-	

Changing Acceleration

The slope of a velocity-time graph is equal to acceleration.

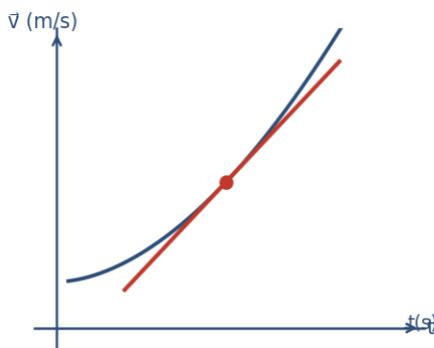


Average Acceleration

If asked to find the acceleration *between* two times, you are being asked for **average acceleration**.

Draw a **secant line** between the two points of interest. The slope of this line is the average acceleration.

$$a_{\text{avg}} = \Delta v / \Delta t$$



Instantaneous Acceleration

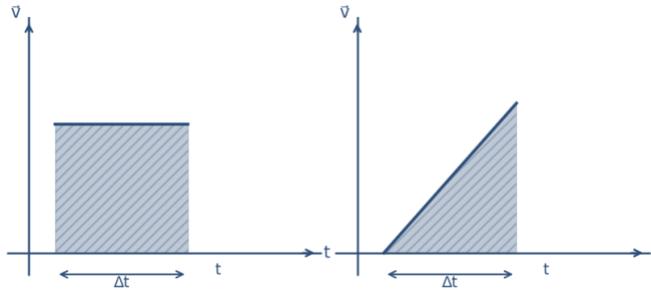
If asked for the acceleration *at* a specific time, you are being asked for **instantaneous acceleration**.

Draw a **tangent** to the curve at the point of interest. Its slope equals the instantaneous acceleration at that point.

$$a_{\text{inst}} = \text{slope of tangent to } v\text{-}t \text{ graph}$$

iAscend

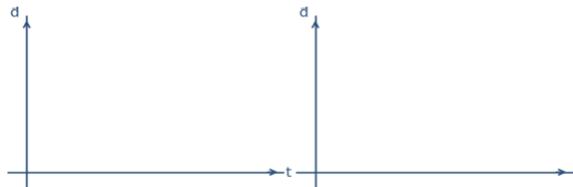
Area Under a v-t Graph



The area under a v-t graph is equal to displacement.

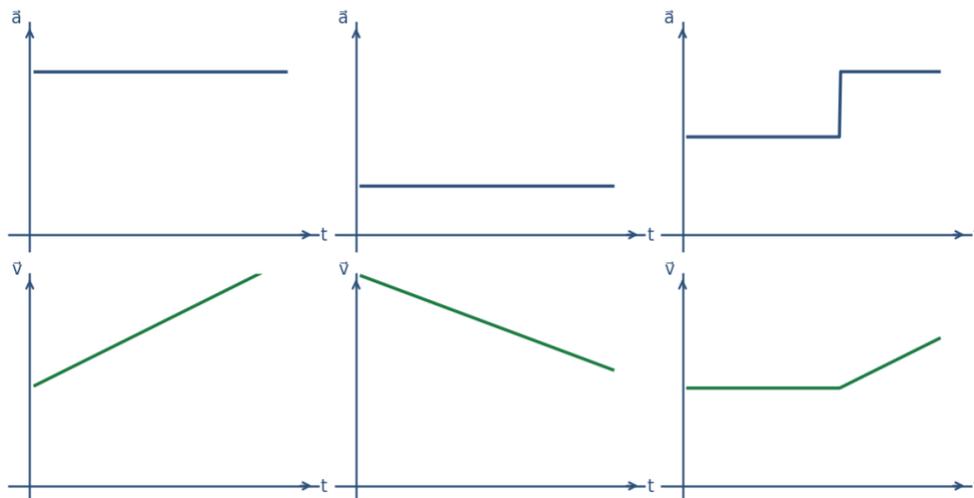
Area formulas (common shapes):

- Rectangle (constant velocity): $\Delta d = v \times \Delta t$
- Triangle (uniformly changing velocity): $\Delta d = \frac{1}{2} \times \text{base} \times \text{height}$
- Trapezoid: $\Delta d = \frac{1}{2} \times (v_1 + v_2) \times \Delta t$
- Area above t-axis = Δd is positive; Area below t-axis = Δd is negative



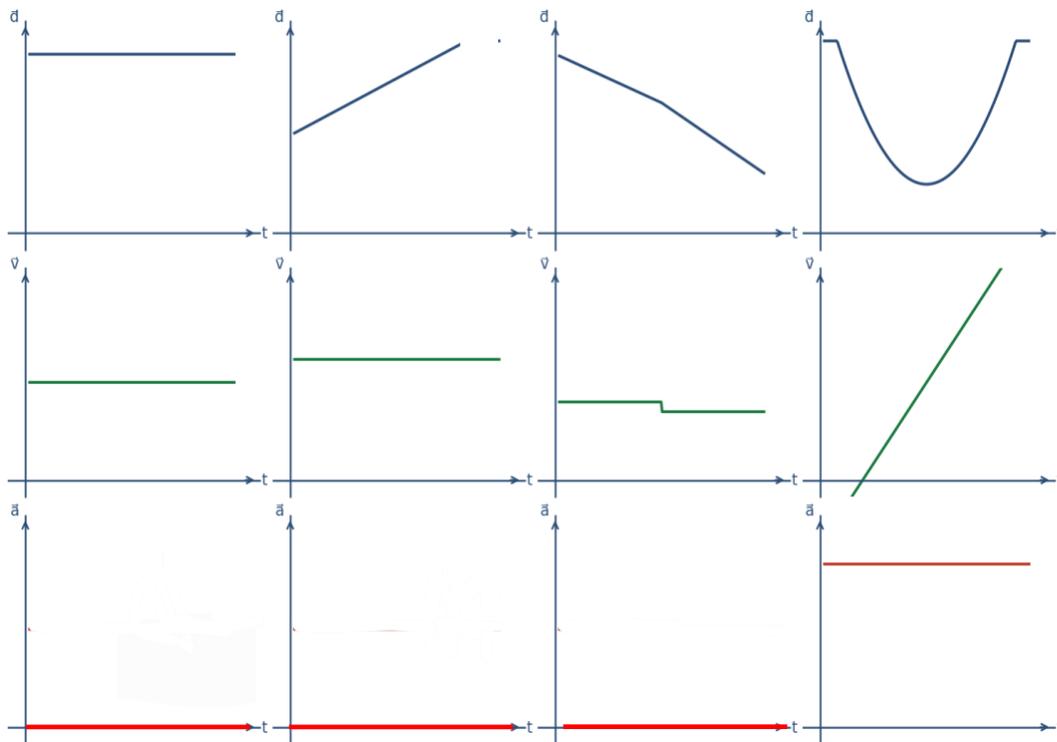
The corresponding d-t graphs (sketch below each v-t graph above)

Area under an a-t Graph



Sketch v-t and a-t Graphs from d-t Graphs

For each d-t graph shown in the top row, sketch the corresponding v-t graph (middle row) and a-t graph (bottom row).



Reminder:

- Constant slope on d-t → horizontal (constant) line on v-t → zero (flat at 0) on a-t
- Increasing slope on d-t → rising line on v-t → positive constant on a-t
- Decreasing slope on d-t → falling line on v-t → negative constant on a-t
- Curved d-t → straight line v-t → constant a-t (for uniform acceleration)

Kinematics: A Graphing Summary

Using Slope:

Quantity	How to Find It from a Graph
Velocity	Slope of a d vs t graph
Average Velocity	Slope of the secant line joining two points on a d vs t graph
Instantaneous Velocity	Slope of the tangent to a d vs t graph at the point of interest
Acceleration	Slope of a v vs t graph
Average Acceleration	Slope of the secant line joining two points on a v vs t graph
Instantaneous Acceleration	Slope of the tangent to a v vs t graph at the point of interest

Using Area:

Quantity	How to Find It from a Graph
Displacement (Δd)	Area under a v vs t graph
Change in Velocity (Δv)	Area under an a vs t graph

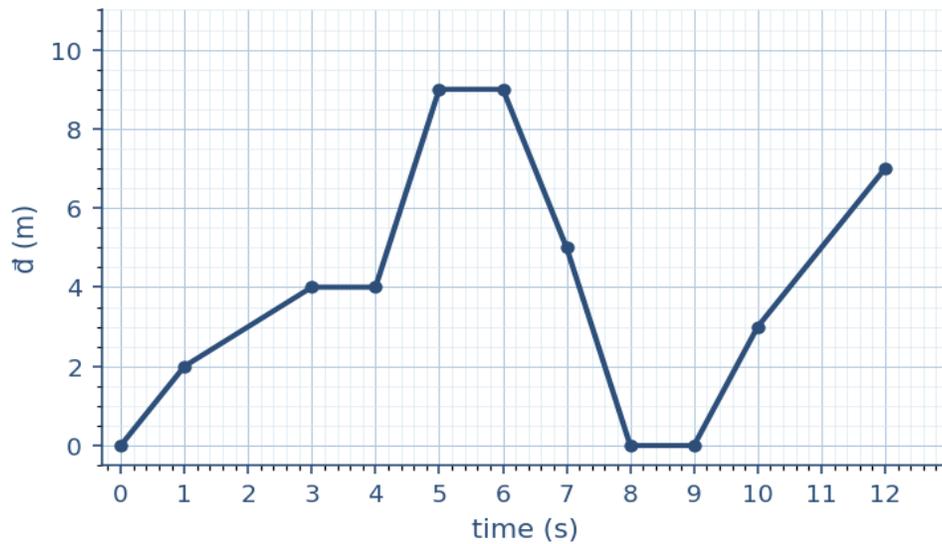
Connection Diagram:

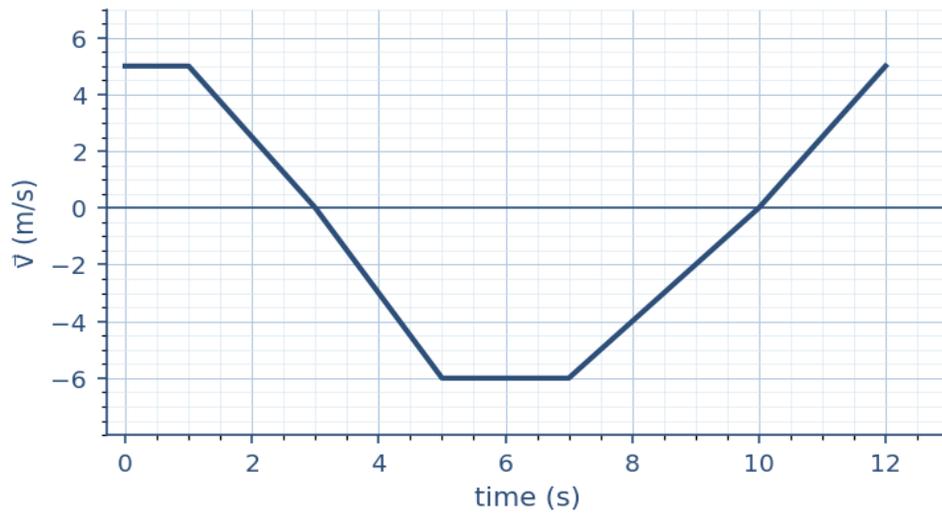
$d-t$ → slope $v-t$ → slope $a-t$
 $d-t$ ← Area $v-t$ ← Area $a-t$

x

Graphing Problems

Graph #1 - Position-Time Graph

**Find:**Velocity at $t = 2.0$ s _____Velocity at $t = 6.0$ s _____Velocity at $t = 9.5$ s _____Average velocity between $t = 1.0$ s and $t = 5.0$ s _____

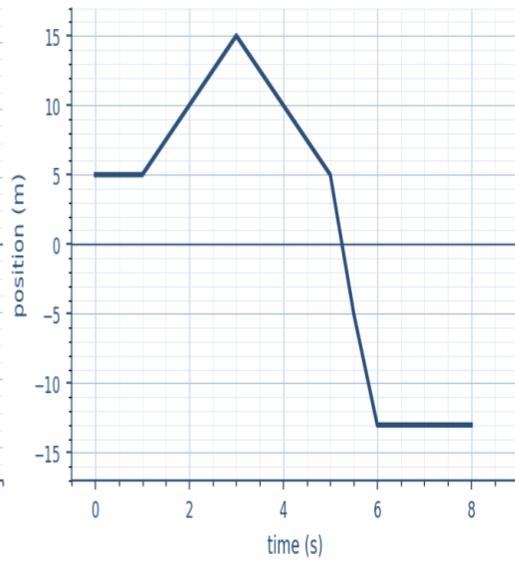
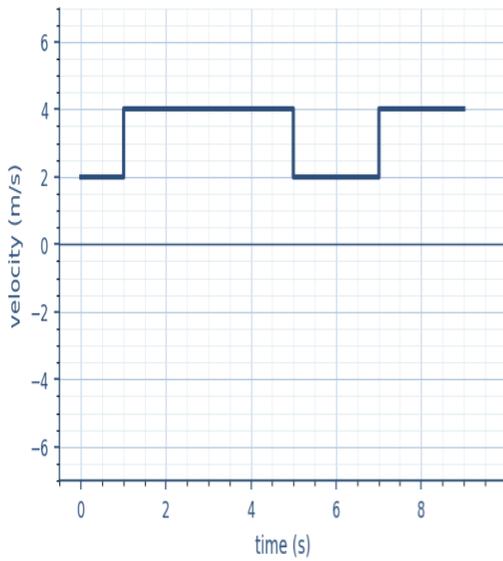
Graph #2 - Velocity-Time Graph**Find:**Acceleration at $t = 1.0$ s _____Acceleration at $t = 3.0$ s _____

Displacement during first two seconds _____

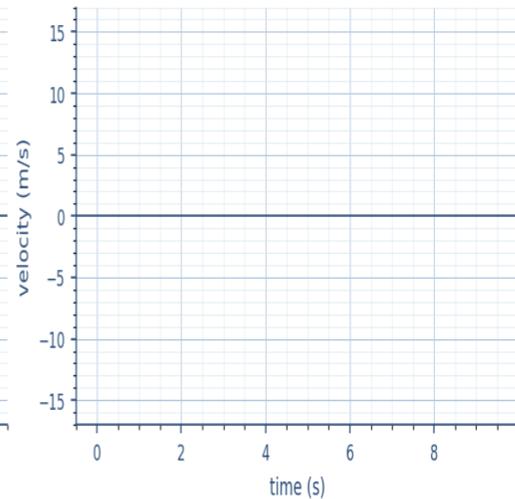
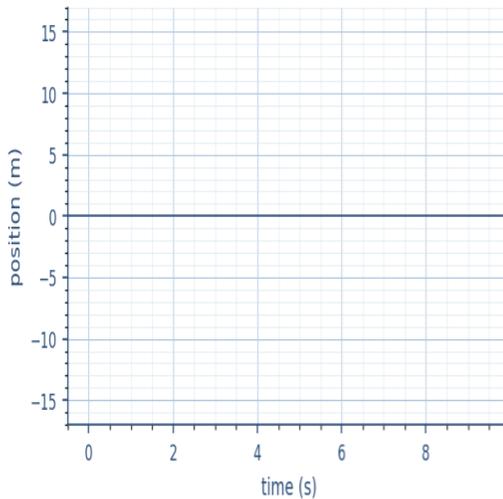
Displacement from $t = 4.0$ s to $t = 6.0$ s _____

Convert and Calculate

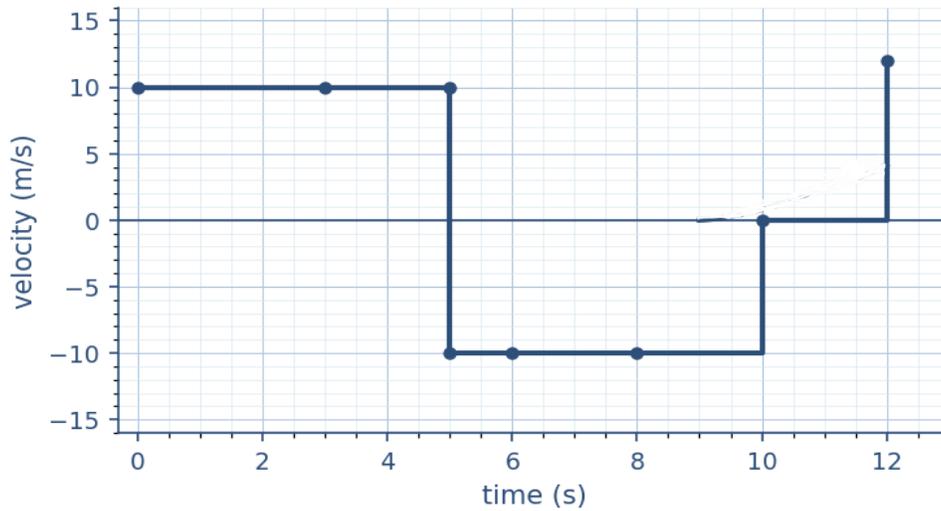
For each graph shown, draw the corresponding indicated graph directly in the blank axes. Then answer the questions.



Given graphs - convert to the graph type indicated below each



Answer graphs - draw here

Graph #3 - Velocity-Time Graph

Find:

- Calculate the acceleration at $t = 5.5 \text{ s}$ _____
- Calculate the average acceleration between $t = 1.0 \text{ s}$ and $t = 5.0 \text{ s}$ _____
- Calculate the displacement during the first 3.0 s _____
- Calculate the acceleration at $t = 9.0 \text{ s}$ _____

4 Worked Examples
Example 1 - Displacement and Average Velocity

A cyclist rides 5.0 km north, then 3.0 km south, then 2.0 km north in 0.50 h.

- a) Total distance? b) Displacement? c) Average speed? d) Average velocity?

Given:

$$d_1 = +5.0 \text{ km}, \quad d_2 = -3.0 \text{ km}, \quad d_3 = +2.0 \text{ km}$$

$$\Delta t = 0.50 \text{ h} = 1800 \text{ s}$$

Find: distance, Δd , speed, velocity

Solution:

a) Distance = $5.0 + 3.0 + 2.0 = 10.0 \text{ km}$

b) $\Delta d = 5.0 - 3.0 + 2.0 = +4.0 \text{ km [N]}$

c) $v_{\text{avg}} = 10.0 / 0.50 = 20 \text{ km/h}$

d) $\vec{v}_{\text{avg}} = 4.0 / 0.50 = 8.0 \text{ km/h [N]}$

Example 2 - Reading a d-t Graph

Object's motion: 0-4 s: d goes 0 → +20 m; 4-8 s: stays at +20 m; 8-12 s: d goes +20 m → -4 m.

a) Velocity in each segment? b) Total displacement? c) Total distance?

Slope = $\Delta d / \Delta t$:

Seg 1 (0-4 s): $v = (20-0)/4 = +5.0 \text{ m/s}$

Seg 2 (4-8 s): $v = 0 \text{ m/s}$

Seg 3 (8-12 s): $v = (-4-20)/4 = -6.0 \text{ m/s}$

a) $v_1 = +5.0 \text{ m/s}$, $v_2 = 0$, $v_3 = -6.0 \text{ m/s}$

b) $\Delta d = -4 - 0 = -4 \text{ m}$

c) Distance = $20 + 0 + 24 = 44 \text{ m}$

Example 3 - Displacement from Area Under v-t Graph

Car: 0→6 s: 0→+24 m/s; 6-10 s: +24 m/s (constant); 10-14 s: +24→0 m/s.

a) Acceleration each phase? b) Total displacement?

Acceleration (slope of v-t):

$a_1 = (24-0)/6 = +4.0 \text{ m/s}^2$

$a_2 = 0 \text{ m/s}^2$

$a_3 = (0-24)/4 = -6.0 \text{ m/s}^2$

Displacement (area under v-t):

$\Delta d_1 = \frac{1}{2}(6)(24) = 72 \text{ m}$

$\Delta d_2 = (4)(24) = 96 \text{ m}$

$\Delta d_3 = \frac{1}{2}(4)(24) = 48 \text{ m}$

Total: $\Delta d = 216 \text{ m}$

Example 4 - Instantaneous Velocity by Differentiation

A particle's position: $d(t) = 2t^2 - 4t + 1 \text{ (m, s)}$.

a) Average velocity between $t = 1 \text{ s}$ and $t = 3 \text{ s}$? b) Instantaneous velocity at $t = 2 \text{ s}$?

a) Average velocity:

$d(1) = 2(1)^2 - 4(1) + 1 = -1 \text{ m}$

$d(3) = 2(3)^2 - 4(3) + 1 = +7 \text{ m}$

$\vec{v}_{\text{avg}} = (7 - (-1))/(3-1) = +4.0 \text{ m/s}$

b) Instantaneous (derivative):

$v(t) = d'(t) = 4t - 4$

$v(2) = 4(2) - 4 = +4.0 \text{ m/s}$

Note: equal here only by coincidence of interval chosen.

5 Practice Problems

Show all steps. Include units and direction.

Problem 1

A student walks 120 m east, 50 m north, then 120 m west in 3.0 min.

- a) Total distance? b) Displacement (magnitude and direction)?
c) Average speed (m/s)? d) Average velocity magnitude (m/s)?

Problem 2

d-t graph data: $t=0$: $d=0$; $t=3$ s: $d=+15$ m; $t=7$ s: $d=+15$ m; $t=10$ s: $d=-6$ m.

- a) Velocity in each segment? b) Describe motion in words.
c) Total displacement and distance over 10 s? d) Sketch the v-t graph.

Problem 3

v-t graph: 0 to 10 s: $0 \rightarrow +30$ m/s (uniform); 10 to 15 s: $+30 \rightarrow -15$ m/s (uniform).

- a) Acceleration each phase? b) Displacement each phase?
c) Total distance over 15 s? d) What does negative velocity represent physically?

Problem 4

Position function: $d(t) = 3t^2 + 6t - 9$ (m, s).

- a) Find $v(t)$ by differentiation. b) Instantaneous velocity at $t = 0, 2, 4$ s?
c) When is the particle momentarily at rest? d) Sketch d-t and v-t graphs ($t = 0$ to 5 s).

Problem 5

Two cyclists start simultaneously from the same point. Cyclist A: constant 8.0 m/s east.
Cyclist B: accelerates from rest at 2.0 m/s² east.

- a) Position functions? b) When does B overtake A? c) Distance from start at overtaking?
d) Sketch both d-t curves on same axes. e) B's velocity at the moment of overtaking?

Problem 6

Velocity: $v(t) = -6t^2 + 24t - 18$ (m/s, s).

- a) Find $a(t)$? b) Times at rest? c) Interval(s) of positive-direction motion?
d) If $d(0)=0$, displacement $t=0$ to 4 s (by integration)? e) Total distance? f) When is $a=0$,
and what does this indicate?

Problem 7

v-t graph (trapezoid): +5 m/s at t=0; rises to +25 m/s at t=4 s; holds +25 m/s to t=9 s; falls to -10 m/s at t=14 s.

- Acceleration each phase; sketch a-t graph. b) Displacement each phase.
- At what time after t=9 s does the object return to its starting position? (Set $\Delta d_{\text{total}} = 0$)
- Velocity at the moment it returns to start?