

PART - A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*1. If the domain of the function $f(x) = \frac{\sqrt{x^2 - 25}}{(4 - x^2)} + \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then

$\alpha^2 + \beta^3$ is equal to:

- (1) 140 (2) 175
(3) 125 (4) 150

Ans. (4)

Sol. $x^2 - 25 \geq 0 \Rightarrow (x - 5)(x + 5) \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

$$4 - x^2 \neq 0 \Rightarrow x \neq -2, 2$$

$$x^2 + 2x - 15 > 0 \Rightarrow (x + 5)(x - 3) > 0 \Rightarrow x \in (-\infty, -5) \cup (3, \infty)$$

Taking intersection, we get $(-\infty, -5) \cup [5, \infty)$

$$\therefore \alpha = -5, \beta = 5 \Rightarrow \alpha^2 + \beta^2 = 25 + 125 = 150$$

*2. If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left| z + \frac{1}{2}(3 + 4i) \right|$ is:

- (1) 2 (2) $\frac{5}{2}$
(3) $\frac{3}{2}$ (4) 3

Ans. (3)

Sol. We know that $|z_1 + z_2| \geq ||z_1| - |z_2||$

$$\Rightarrow \left| z + \left(\frac{3}{2} + 2i \right) \right| \geq \left| |z| - \left| \frac{3}{2} + 2i \right| \right| \geq \left| 1 - \frac{5}{2} \right| \geq \frac{3}{2}$$

3. Consider a ΔABC where $A(1, 3, 2)$, $B(-2, 8, 0)$ and $C(3, 6, 7)$. If the angle bisector of $\angle BAC$ meets the line BC at D , then the length of the projection of the vector \overline{AD} on the vector \overline{AC} is:

- (1) $\frac{37}{2\sqrt{38}}$ (2) $\sqrt{19}$
(3) $\frac{39}{2\sqrt{38}}$ (4) $\frac{\sqrt{38}}{2}$

Ans. (1)

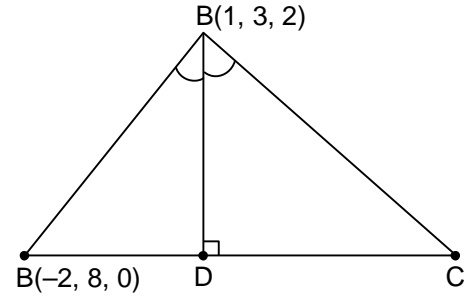
Sol. $|\overline{AB}| = |-3\hat{i} + 5\hat{j} - 2\hat{k}| = \sqrt{38}$

$|\overline{AC}| = |2\hat{i} + 3\hat{j} + 5\hat{k}| = \sqrt{38}$

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{1}{1} \Rightarrow BD = DC$

$\therefore D\left(\frac{1}{2}, 7, \frac{7}{2}\right) \Rightarrow \overline{AD} = -\frac{1}{2}\hat{i} + 4\hat{j} + \frac{3}{2}\hat{k}$

$\Rightarrow \overline{AD} \cdot \widehat{AC} = \left(-\frac{1}{2}\hat{i} + 4\hat{j} + \frac{3}{2}\hat{k}\right) \cdot \frac{(2\hat{i} + 3\hat{j} + 5\hat{k})}{\sqrt{38}} = \frac{37}{2\sqrt{38}}$



4. Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in \mathbb{R}$ and $(a, b)R_2(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$. Then
- (1) R_1 and R_2 both are equivalence relations
 - (2) only R_1 is an equivalence relation
 - (3) only R_2 is an equivalence relation
 - (4) neither R_1 nor R_2 is an equivalence relation

Ans. (3)

Sol. $a^2 + a^2 = 1 \Rightarrow a = \pm \frac{1}{\sqrt{2}} \therefore (a, a) \notin R_1 \forall a \in \mathbb{R} \therefore R_1$ is not reflexive

$\therefore R_1$ is not equivalence relation

$a + b = b + a \Rightarrow (a, b), (a, b) \in R_2 \therefore R_2$ is reflexive

If $(a, b) R_2 (c, d) \Rightarrow a + d = b + c$

\therefore If $a + d = b + c \Rightarrow c + b = d + a \Rightarrow (c, d) R_2 (a, b) \therefore R_2$ is symmetric

For transitive if we need to see it $(a, b) R_2 (c, d)$ and $(c, d) R_2 (e, f) \Rightarrow (a, b) R_2 (e, f)$

If $(a, b) R_2 (c, d) \Rightarrow a + d = b + c \dots (1)$

If $(c, d) R_2 (e, f) \Rightarrow c + f = d + e \dots (2)$

Adding equation (1) + (2), we get $a + f = b + e \Rightarrow (a, b) R_2 (e, f)$

$\therefore R_2$ is transitive $\Rightarrow R_2$ is equivalence relation

5. Let the system of equations $x + 2y + 3z = 5$, $2x + 3y + z = 9$, $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to:
- (1) 22
 - (2) 17
 - (3) 15
 - (4) 28

Ans. (2)

Sol. $D = 0 \Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$

$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$

$\therefore \lambda + 2\mu = 17$



6. If $\int_0^{\frac{\pi}{3}} \cos^4 x dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then $9a + 8b$ is equal to:

- (1) 2 (2) 1
 (3) 3 (4) $\frac{3}{2}$

Ans. (1)

Sol. $\int_0^{\frac{\pi}{3}} \left(\frac{1 + \cos 2x}{2}\right)^2 dx \Rightarrow \frac{1}{4} \int_0^{\frac{\pi}{3}} 1 + \cos^2 2x + 2\cos 2x dx$

$$= \frac{1}{4} \int_0^{\frac{\pi}{3}} \frac{3}{2} + \frac{\cos 4x}{2} + 2\cos 2x dx = \frac{3x}{8} + \frac{\sin 4x}{32} + \frac{\sin 2x}{4} \Big|_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{8} + \frac{7}{64}\sqrt{3} \Rightarrow a = \frac{1}{8}, b = \frac{7}{64}$$

$$\Rightarrow 9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$$

*7. Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non constant G.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is:

- (1) 8 (2) 9
 (3) $\frac{20}{3}$ (4) $\frac{80}{9}$

Ans. (4)

Sol. Let common ratio of G.P. be R

$$\therefore \alpha, \beta \text{ are roots of } px^2 + pRx - pR^2 = 0$$

$$\therefore \alpha + \beta = -R; \alpha\beta = -R^2$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4} \Rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4} \Rightarrow \frac{1}{R} = \frac{3}{4} \Rightarrow R = \frac{4}{3}$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = R^2 + 4R^2 = 5R^2 = 5 \times \frac{15}{9} = \frac{80}{9}$$

8. Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is:

- (1) $\frac{9}{35}$ (2) $\frac{3}{35}$
 (3) $\frac{24}{35}$ (4) $\frac{18}{35}$

Ans. (4)

Sol. Let A denote the event of AJAY appearing in JEE exam and B denote the event of VIJAY appearing in JEE exam



Given $P(\bar{A}) = \frac{2}{7} \Rightarrow P(A \cap B) = \frac{1}{5}$

We need to find $P(A \cap \bar{B})$

$P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{5}{7} - \frac{1}{5} \Rightarrow \frac{18}{35}$

*9. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R and PQ such that $PR : RQ = 4 : 3$ as P moves on the ellipse, is:

- (1) $\frac{13}{21}$ (2) $\frac{\sqrt{139}}{23}$
 (3) $\frac{\sqrt{13}}{7}$ (4) $\frac{11}{19}$

Ans. (3)

Sol. $P(3 \cos \theta, 2 \sin \theta)$

Line passing through P and parallel to y-axis is $x = 3 \cos \theta$

$\therefore Q(3 \cos \theta, 3 \sin \theta)$

Let R be (h, k)

$\Rightarrow h = \frac{12 \cos \theta + 9 \cos \theta}{7} \Rightarrow k = \frac{12 \sin \theta + 6 \sin \theta}{7}$

\therefore Locus $\frac{7x^2}{21^2} + \frac{7y^2}{18^2} = 1$

$\therefore 18^2 = 21^2 (1 - e^2) \Rightarrow e^2 = \frac{117}{21^2} \Rightarrow e = \frac{\sqrt{117}}{21} = \frac{\sqrt{13}}{7}$

*10. Consider 10 observations x_1, x_2, \dots, x_{10} such that $\sum_{i=1}^{10} (x_i - \alpha) = 2$ and $\sum_{i=1}^{10} (x_i - \beta)^2 = 40$,

where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to:

- (1) 2 (2) 1
 (3) $\frac{5}{2}$ (4) $\frac{3}{2}$

Ans. (1)

Sol. $\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{2 + 10\alpha}{10} = 0.2 + \alpha$

$\Rightarrow 0.2 + \alpha = 1.2 \Rightarrow \alpha = 1$

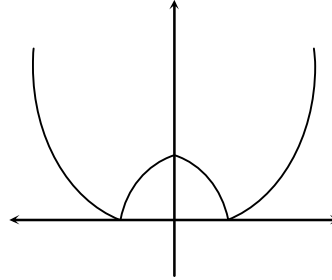
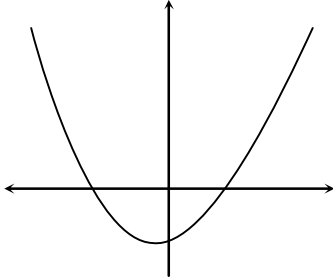
$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{10} - (\bar{x})^2 \Rightarrow \frac{84}{25} = \frac{40 - 10\beta^2 + 2\beta(2 + 10\alpha)}{10} - \frac{36}{25}$

$\Rightarrow 5\beta^2 - 12\beta + 4 = 0 \Rightarrow \beta = 2$

11. Let $f(x) = |2x^2 + 5|x| - 3|$, $x \in \mathbb{R}$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m + n$ is equal to:
- (1) 5 (2) 3
(3) 2 (4) 0

Ans. (2)

Sol. Let $g(x) = 2x^2 + 5x - 3 = (2x - 1)(x + 3)$ \therefore Graph of $y = f(x) = |g(|x|)|$ is
Graph of $y = g(x)$ is



It is continuous everywhere

$$\therefore m = 0$$

It is non differentiable at $x = -\frac{1}{2}$, $x = \frac{1}{2}$ and $x = 0$

$$\therefore n = 3$$

- *12. The number of solutions of the equation $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is:
- (1) 0 (2) 3
(3) 1 (4) 2

Ans. (1)

Sol. Let $\cos x = t \Rightarrow 4t^3 + 4t^2 + 4t - 13 = 0$ where $t \in [-1, 1]$

$$\text{Let } f(t) = 4t^3 + 4t^2 + 4t - 13$$

$$f'(t) = 12t^2 + 8t + 4 > 0 \quad \forall t \in \mathbb{R}$$

$\therefore f(t)$ is always increasing

$$f(-1) = -4 + 4 - 4 - 13 = -17$$

$$f(1) = 4 + 4 + 4 - 13 = -1$$

\therefore For $t \in [-1, 1]$ $f(t)$ is never 0

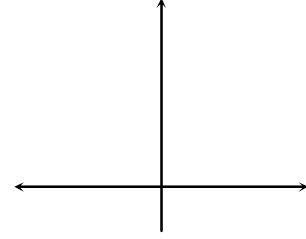
- *13. Let the locus of the midpoints of the chords of the circle $x^2 + (y - 1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q . Then, the length of PQ is:

- (1) $\frac{1}{2}$ (2) 1
(3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Ans. (3)



Sol. Let the midpoint of chord be (h, k)
 \therefore Equation of chord is $T = S_1$
 $\Rightarrow hx + ky - (y + k) = h^2 + k^2 - 2k$
 \therefore It passes through $(0, 0) \therefore -k = h^2 + k^2 - 2k$
 \therefore Locus is $x^2 + y^2 - y = 0$
 For P and Q $(1 - y)^2 + y^2 - y = 0$
 $\Rightarrow 2y^2 - 3y + 1 = 0 \Rightarrow y = 1, \frac{1}{2}$
 $\therefore x = 0, x = \frac{1}{2}$
 $\therefore P(0, 1)$ and $Q = \left(\frac{1}{2}, \frac{1}{2}\right) \therefore PQ = \frac{1}{\sqrt{2}}$



14. Let α be a non-zero real number. Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \rightarrow -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in \mathbb{R}$, then $f(-\log_e 2)$ is equal to _____.
- (1) 7 (2) 9
 (3) 3 (4) 5

Ans. (2)

Sol. $\frac{dy}{dx} = \alpha y + 3 \Rightarrow \int \frac{dy}{\alpha y + 3} = \int dx \Rightarrow \frac{1}{\alpha} \ln|2\alpha + 3| = x + c$
 $\Rightarrow \ln|\alpha y + 3| = \alpha(x + c)$,
 at $x = 0, y = 2$
 $\Rightarrow \ln|2\alpha + 3| = \alpha c \Rightarrow c = \frac{1}{\alpha} \ln|2\alpha + 3|$
 $\Rightarrow \frac{1}{\alpha} \ln|\alpha y + 3| = x + \frac{1}{\alpha} \ln|2\alpha + 3|$
 $\Rightarrow \ln|\alpha y + 3| = \alpha x + \ln|2\alpha + 3| \Rightarrow \frac{\alpha y + 3}{2\alpha + 3} = e^{\alpha x}$
 $\Rightarrow \alpha y + 3 = (2\alpha + 3)e^{\alpha x}$ as $x \rightarrow -\infty, y = 1 \Rightarrow \alpha + 3 = 0 \Rightarrow \alpha = -3$
 $\Rightarrow -3y + 3 = -3e^{-3x} \Rightarrow -y + 1 = -e^{-3x} \Rightarrow y = 1 + e^{-3x}$
 At $x = -\log_e 2; y = 1 + e^{3 \log_e 2} = 9$

15. Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point $R(1, 2, 3)$. If the centroid of the triangle PQR is (α, β, γ) , then $\alpha^2 + \beta^2 + \gamma^2$ is:
- (1) 18 (2) 24
 (3) 26 (4) 36

Ans. (1)

Sol. Any point on the line is $(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$
 Its distance from $R(1, 2, 3)$ is 6 units
 $\sqrt{(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2} = 6$
 $\Rightarrow 72\lambda^2 - 72\lambda + 36 = 36 \Rightarrow \lambda = 0, 1$



$\therefore P(-3, 4, -1)$ and $Q(5, 6, 1)$
Centroid $(1, 4, 1) \therefore \alpha^2 + \beta^2 + \gamma^2 = 1 + 16 + 1 = 18$

16. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$ is equal to:
- (1) -1 (2) 2
(3) 0 (4) 1

Ans. (3)

Sol. Let $I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$

Using the property $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$\Rightarrow I = \int_a^1 2(1-x)^3 - 3(1-x)^2 - ((1-x)+1)^{\frac{1}{3}} dx = \int_a^1 (-2x^3 + 3x^2 + x - 1)^{\frac{1}{3}} dx = -I$$

$$\Rightarrow I = 0$$

17. If the mirror image of the point $P(3, 4, 9)$ in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ) , then $14(\alpha + \beta + \gamma)$ is:
- (1) 102 (2) 138
(3) 132 (4) 108

Ans. (4)

Sol. Any point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is

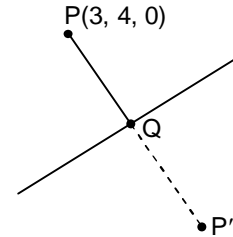
$$Q(3\lambda + 1, 2\lambda - 1, \lambda + 2)$$

$$\text{D.R of PQ } (3\lambda - 2, 2\lambda - 5, \lambda - 7)$$

$$\text{PQ is perpendicular to line } 3(3\lambda - 2) + 2(2\lambda - 5) + \lambda - 7 = 0$$

$$\Rightarrow 14\lambda - 23 = 0 \Rightarrow \lambda = \frac{23}{14}$$

$$\therefore Q\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right) \therefore \text{Image } P'\left(\frac{62}{7}, \frac{4}{7}, -\frac{12}{7}\right) \therefore 14(\alpha + \beta + \gamma) = 108$$



- *18. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is $15 : 7$, then $S_{15} - S_5$ is equal to:
- (1) 800 (2) 890
(3) 790 (4) 690

Ans. (3)

Sol. $S[2a + 9d] = 390 \Rightarrow 2a + 9d = 78$

$$\frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a - 3d = 0 \Rightarrow a = 3 \Rightarrow d = 8$$

$$\therefore S_{15} - S_5 = \frac{15}{2}[6 + 112] - \frac{5}{2}[6 + 32] = 790$$



*19. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^3}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is:

- (1) $\frac{1}{9}$ (2) $\frac{1}{4}$
 (3) $\frac{4}{9}$ (4) $\frac{9}{4}$

Ans. (4)

Sol. $n = {}^{18}C_{12} \left(\frac{1}{3}\right)^6 \left(\frac{1}{2}\right)^{12}$; $m = {}^{18}C_6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{2}\right)^6$

$$\therefore \frac{n}{m} = \frac{3^6}{2^6} \Rightarrow \left(\frac{n}{m}\right)^{\frac{1}{3}} = \frac{9}{4}$$

20. Let $f(x) = \begin{cases} x-1, & x \text{ is even,} \\ 2x, & x \text{ is odd,} \end{cases} x \in \mathbb{N}$. If for some $a \in \mathbb{N}$, $f(f(f(a))) = 21$, then $\lim_{x \rightarrow a^-} \left\{ \frac{|x|^3}{a} - \left[\frac{x}{a} \right] \right\}$,

where $[t]$ denotes the greatest integer less than or equal to t , is equal to:

- (1) 169 (2) 121
 (3) 225 (4) 144

Ans. (4)

Sol. $f(x) = \begin{cases} x-1 & ; x \text{ is even} \\ 2x & ; x \text{ is odd} \end{cases}$

If a is even $f(f(a)) = f(a-1) = 2(a-1)$

$$\therefore f(f(f(a))) = 2(a-1) - 1 \Rightarrow 2a - 3$$

$$\Rightarrow 2a - 3 = 21 \Rightarrow a = 12$$

If a is odd $f(f(a)) = f(2a) = 2a - 1$

$f(f(f(a))) = 2(2a - 1)$ thus not possible

$$\therefore \lim_{x \rightarrow 12^-} \frac{|x|^3}{a} - \left[\frac{x}{a} \right] = 144$$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*21. Three points $O(0, 0)$, $P(a, a^2)$, $Q(-b, b^2)$, $a > 0$, $b > 0$, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ . If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

Ans. 7

Sol. Equation of PQ = $y - a^2 = \frac{a^2 - b^2}{a + b}(x - a)$

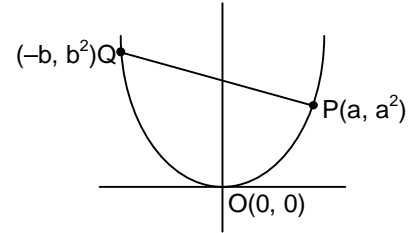
$\Rightarrow (a - b)x - y + ab = 0$

$\therefore S_1 = \int_{-b}^a ((a - b)x + ab - x^2) dx$
 $= (a + b) \left(\frac{(a - b)^2}{2} + ab - \frac{(a^2 + b^2 - ab)}{3} \right)$

$S_2 = \frac{1}{2} ab(a + b)$

$\therefore \frac{S_1}{S_2} = \frac{\frac{(a - b)^2}{2} + ab - \frac{(a^2 + b^2 - ab)}{3}}{ab/2} = \frac{(a + b)^2}{3ab} = \frac{1}{3} \left(\frac{a}{b} + \frac{b}{a} + 2 \right)$

\therefore minimum value of $\frac{S_1}{S_2}$ is $\frac{4}{3}$ ($\because \frac{a}{b} + \frac{b}{a} \geq 2$) $\therefore m + n = 7$



22. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to _____.

Ans. 8

Sol. $y^2 = \frac{k}{2}x$ and $\left(y - \frac{1}{k}\right)^2 = \frac{-2}{k}\left(x - \frac{1}{2k}\right)$

For point of intersection

$ky^2 = 2\left(y - \frac{2}{k}y^2\right) \Rightarrow y = 0$ and $y = \frac{2k}{k^2 + 4}$

\therefore Area bounded $\int_0^{\frac{2k}{k^2+4}} \left(\frac{2y - ky^2}{2} - \frac{2y^2}{k} \right) dy = \frac{2k^2}{3(k^2 + 4)^2} = \frac{2}{3\left(k^2 + \frac{16}{k^2} + 8\right)}$

For maximum area we have to minimize $k^2 + \frac{16}{k^2} + 8$

$\frac{k^2 + \frac{16}{k^2}}{2} \geq 4 \Rightarrow k^2 + \frac{16}{k^2} \geq 8$

It holds for $k = \pm 2$

23. If $y = \frac{(\sqrt{x} + 1)(x^2 - \sqrt{x})}{x\sqrt{x} + x + \sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$, then $96y' \left(\frac{\pi}{6} \right)$ is equal to _____.

Ans. 105

Sol. $y = \frac{x^2\sqrt{x} - x + x^2 - \sqrt{x}}{x\sqrt{x} + x + \sqrt{x}} + \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}$

$y = \frac{(x - 1)(x\sqrt{x} + x + \sqrt{x})}{x\sqrt{x} + x + \sqrt{x}} + \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}$

$\frac{dy}{dx} = 1 - \cos^4 x \sin x + \cos^2 x \sin x$



$$\therefore \text{ at } x = \frac{\pi}{6}, \frac{dy}{dx} = 1 - \frac{9}{32} + \frac{3}{8} = \frac{35}{32}$$

$$\therefore 96y' \left(\frac{\pi}{6} \right) = 105$$

24. If $\frac{dx}{dy} = \frac{1+x-y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to _____.

Ans. 5

Sol. $\frac{dx}{dy} - \frac{x}{y} = \frac{1}{y} - y$

Integrating factor = $e^{\int -\frac{1}{y} dy} = \frac{1}{y}$

$$\therefore \frac{x}{y} = \int \left(\frac{1}{y^2} - 1 \right) dy \Rightarrow x = -1 - y^2 + cy$$

at $y = 1, x = 1 \therefore c = 3 \therefore x = -1 - y^2 + 3y$

at $y = 2, x = -1 - 4 + 6 = 1$

$$\therefore 5x(2) = 5$$

25. Let $f: (0, \infty) \rightarrow \mathbb{R}$ and $F(x) = \int_0^x tf(t)dt$. If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to _____.

Ans. 219

Sol. $F(x^2) = \int_0^{x^2} tf(t)dt$

Differentiating both sides w.r.t. x

$$F'(x^2) \times 2x = x^2 f(x^2) \times 2x$$

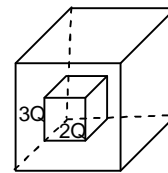
$$\Rightarrow f(x^2) = 2 + \frac{5}{2}x$$

$$\therefore \sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12} 2 + \frac{5}{2}r = 24 + \frac{5}{2} \times \frac{12 \times 13}{2} = 219$$



29. C_1 and C_2 are two hollow concentric cubes enclosing charges $2Q$ and $3Q$ respectively as shown in figure. The ratio of electric flux passing through C_1 and C_2 is:

- (1) 3 : 2
 (2) 5 : 2
 (3) 2 : 5
 (4) 2 : 3

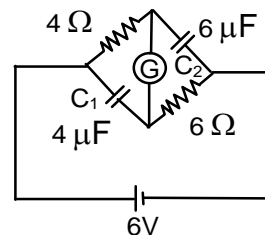


Ans. (3)

Sol.
$$\frac{Q_{c_1}}{Q_{c_2}} = \frac{2Q}{2Q + 3Q} = \frac{2}{5}$$

30. A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is:

- (1) 1
 (2) $\frac{2}{3}$
 (3) $\frac{3}{2}$
 (4) $\frac{1}{2}$



Ans. (4)

Sol.
$$\frac{Q_1}{Q_2} = \frac{C_1 V_1}{C_2 V_2} = \frac{4(3)}{6(4)} = \frac{1}{2}$$

31. Match List-I with List-II.

List-I (Number)	List-II (Significant figure)
(A) 1001	(I) 3
(B) 010.1	(II) 4
(C) 100.100	(III) 5
(D) 0.0010010	(IV) 6

Choose the correct answer from the options given below:

- (1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III) (2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
 (3) (A)-(I), (B)-(II), (C)-(III), (D)-(IV) (4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. (1)

Sol. Theoretical

- *32. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become:

- (1) $\frac{1}{100}$ th (2) $\frac{1}{10}$ th
 (3) 100 times (4) 10 times

Ans. (2)

Sol. $\frac{E_2}{E_1} = \frac{4\pi(10R)^2 \times T}{1000(4\pi)R^2 \times T} = \frac{1}{10}$

*33. A cricket player catches a ball of mass 120g moving with 25 m/s speed. If the catching process is completed in 0.1 s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit):

- (1) 30 (2) 24
(3) 12 (4) 25

Ans. (1)

Sol. $F = \frac{\Delta P}{\Delta t} = 30\text{N}$

34. In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2Ω , the balance length changes by:

- (1) 62.5 cm (2) 22.5 cm
(3) 20 cm (4) 65 cm

Ans. (2)

Sol. $\frac{2}{40} = \frac{X}{60}$
 $X = 3\Omega$
 $\frac{2}{\ell'} = \frac{6/5}{100 - \ell'}$, $\ell' = 62.5\text{ cm}$, $\Delta\ell = 22.5\text{ cm}$.

*35. A diatomic gas ($\gamma = 1.4$) does 200 J of work when it is expanded isobarically. The heat given to the gas in the process is:

- (1) 800 J (2) 600 J
(3) 700 J (4) 850 J

Ans. (3)

Sol. $\frac{Q}{w} = \frac{nC_p\Delta T}{nR\Delta T} = \frac{C_p}{R} = \frac{7}{2}$
 $Q = 700\text{ J}$

*36. Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1}):

- (1) -50 and -30 (2) -50 and 30
(3) -30 and 50 (4) 50 and -30

Ans. (2)

Sol. $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$
 $v_{BA} = -30 - 20$ (taking north direction as positive)



$$= -50 \text{ m/s.}$$

$$\vec{V}_{GB} = -\vec{V}_{BG} = 30 \text{ m/s}$$

*37. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T . If the force of attraction between planet and star is proportional to $R^{-\frac{3}{2}}$ then choose the correct option:

- (1) $T^2 \propto R^{\frac{3}{2}}$ (2) $T^2 \propto R^3$
 (3) $T^2 \propto R^{\frac{5}{2}}$ (4) $T^2 \propto R^{\frac{7}{2}}$

Ans. (3)

Sol. $\frac{mv^2}{R} = \frac{GMm}{R^{3/2}}$

$$V = \sqrt{\frac{GM}{R^{1/2}}}$$

$$T = \frac{2\pi R}{V} = \frac{2\pi R^{5/4}}{\sqrt{GM}}$$

38. A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit, will be:

- (1) 60° (2) 45°
 (3) 15° (4) 30°

Ans. (1)

Sol. $a \sin \theta = \lambda$

$$\theta = 30^\circ$$

$$\text{Angular spread} = 2\theta = 60^\circ$$

*39. If the root mean square velocity of hydrogen molecule at a given temperature and pressure is 2 km/s, the root mean square velocity of oxygen at the same condition in km/s is:

- (1) 1.0 (2) 1.5
 (3) 2.0 (4) 0.5

Ans. (4)

Sol. $V_{\text{rms}} = \sqrt{\frac{3RT}{M_0}}$

$$V_{\text{rms}O_2} = \frac{(V_{\text{rms}})_{H_2}}{4} = 0.5 \text{ km/s}$$

40. If frequency of electromagnetic wave is 60 MHz and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is :

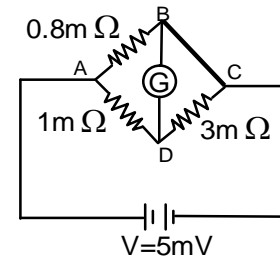
- (1) 2.5 (2) 5
 (3) 10 (4) 2

Ans. (2)



Sol. $C = \gamma \lambda$
 $\lambda = 5\text{m}$

41. To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement show in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at 25°C and resistance of the semiconductor arm is $3\text{m}\Omega$. Arm BC is cooled at a constant rate of 2°C/s . If the galvanometer G shows no deflection after 10 s, then α is:



- (1) $-1 \times 10^{-2} \text{ }^\circ\text{C}^{-1}$ (2) $-2 \times 10^{-2} \text{ }^\circ\text{C}^{-1}$
 (3) $-2.5 \times 10^{-2} \text{ }^\circ\text{C}^{-1}$ (4) $-1.5 \times 10^{-2} \text{ }^\circ\text{C}^{-1}$

Ans. (1)

Sol. $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$\Delta R = R\alpha\Delta T$

$\frac{0.8}{1} = \frac{x}{3}$

$x = 2.4 \text{ m}\Omega$

$\Delta x = -0.6 \text{ m}\Omega$

$\frac{\Delta x}{x} = \alpha\Delta T$

$\alpha = -1 \times 10^{-2} \text{ }^\circ\text{C}^{-1}$

42. Conductivity of a photodiode starts changing only if the wavelength of incident light is less than 660 nm . The band gap of photodiode is found to be $\left(\frac{X}{8}\right) \text{ eV}$. The value of X is:

(Given, $h = 6.6 \times 10^{-34} \text{ Js}$, $e = 1.6 \times 10^{-19} \text{ C}$).

- (1) 11 (2) 13
 (3) 15 (4) 21

Ans. (3)

Sol. $\frac{hc}{\lambda} = \frac{Xe}{8}$

$X = 15$

43. A transformer has an efficiency of 80% and work at 10 V and 4 kW. If the secondary voltage is 240 V, then the current in the secondary coil is:

- (1) 1.33 A (2) 13.33 A
 (3) 1.59 A (4) 15.1 A

Ans. (2)

Sol. $P \times 0.8 = V_s I_s$

$I_s = 13.33 \text{ A}$

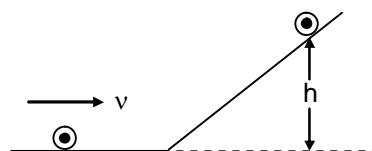


44. In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G , the resistance of ammeter will be:
- (1) $199 G$ (2) $200 G$
 (3) $\frac{G}{200}$ (4) $\frac{G}{199}$

Ans. (none)

Sol. $\frac{G}{X} = \frac{95}{9}$
 $X = G/19$
 $R = \frac{GX}{G+X} = \frac{G}{20}$

- *45. A disc of radius R and mass M is rolling horizontally without slipping with speed v , it then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is:



- (1) $\frac{3 v^2}{4 g}$ (2) $\frac{v^2}{g}$
 (3) $\frac{2 v^2}{3 g}$ (4) $\frac{1 v^2}{2 g}$

Ans. (4)

Sol. $\frac{1}{2}mv^2 = mgh$ (Rotational K.E. remains constant)
 $H = \frac{v^2}{2g}$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

- *46. A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1 . The frequency of oscillation if a mass $9 m$ is suspended from the same spring is f_2 . The value of $\frac{f_1}{f_2}$ is_____.

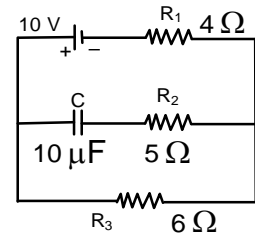
Ans. 3

Sol. $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$



$$\frac{f_1}{f_2} = \sqrt{\frac{9m}{m}} = 3$$

47. In an electrical circuit drawn below the amount of charge stored in the capacitor is _____ μC .



Ans. 60

Sol. At steady state

$$I = \frac{10}{4+6} = 1\text{A}$$

$$V = 6\text{V}$$

$$Q = 60 \mu\text{C}$$

- *48. A particle initially at rest starts moving from reference point $x = 0$ along $x -$ axis, with velocity v that varies as $v = 4\sqrt{x}$ m/s. The acceleration of the particle is _____ ms^{-2}

Ans. 8

Sol. $a = \frac{Vdv}{dx}$

$$= (4\sqrt{x}) \left(4 \times \frac{1}{2\sqrt{x}} \right)$$

$$= 8 \text{ m/s}^2$$

49. Suppose a uniformly charged wall provides a uniform electric field of 2×10^4 N/C normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance of 10 cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}} \mu\text{C}$ where $x =$ _____. [use $g = 10 \text{ m/s}^2$].

Ans. 3

Sol. $\tan \theta = \frac{qE}{mg}$

$$Q = \frac{1}{\sqrt{3}} \frac{mg}{E} = \frac{1}{\sqrt{3}} \frac{2 \times 10^{-3} \times 10}{2 \times 10^4} = \frac{1}{\sqrt{3}} \mu\text{C}$$

50. A moving coil galvanometer has 100 turns and each turn has an area of 2.0 cm^2 . The magnetic field produced by the magnet is 0.01 T and the deflection in the coil is 0.05 radian when a current of 10 mA is passed through it. The torsional constant of the suspension wire is $x \times 10^{-5} \text{ N-m/rad}$. The value of x is _____

Ans. 4

Sol. $G\theta = nIAB$

$$N \times 10^{-5} \times 0.05 = 100 \times 10^{-2} \times 2 \times 10^{-4} \times 0.01$$

$$N = 4$$



In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Statement I is false but Statement II is true
- (2) Both Statement I and Statement II are false
- (3) Both Statement I and Statement II are true
- (4) Statement I is true but Statement II is false

Ans. (4)

*54. The set of meta directing functional groups from the following sets is:

- (1) $-\text{CN}$, $-\text{NH}_2$, $-\text{NHR}$, $-\text{OCH}_3$
- (2) $-\text{CN}$, $-\text{CHO}$, $-\text{NHCOCH}_3$, $-\text{COOR}$
- (3) $-\text{NO}_2$, $-\text{NH}_2$, $-\text{COOH}$, $-\text{COOR}$
- (4) $-\text{NO}_2$, $-\text{CHO}$, $-\text{SO}_3\text{H}$, $-\text{COR}$

Ans. (4)

55. $[\text{Co}(\text{NH}_3)_6]^{3+}$ and $[\text{CoF}_6]^{3-}$ are respectively known as:

- (1) Inner orbital complex, spin paired complex
- (2) Spin paired complex, spin free complex
- (3) Spin free complex, spin paired complex
- (4) Outer orbital complex, inner orbital complex

Ans. (2)

Sol. NH_3 is strong ligand while F^- is weak ligand.

56. The transition metal having highest 3rd ionisation enthalpy is:

- (1) Mn
- (2) Fe
- (3) Cr
- (4) V

Ans. (1)

Sol. $\text{Mn}(25) = [\text{Ar}], 4s^2, 3d^5$

$\text{Mn}^{++} = [\text{Ar}]4s^0, 3d^5, d^5$ is stable due to half filled subshell.

57. Match List-I with List-II

List-I Compound		List-II Use	
(A)	Carbon tetrachloride	(I)	Paint remover
(B)	Methylene chloride	(II)	Refrigerators and air conditioners
(C)	DDT	(III)	Fire extinguisher
(D)	Freons	(IV)	Non Biodegradable insecticide

Choose the correct answer from the options given below:

- (1) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)
- (2) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)
- (3) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)
- (4) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

Ans. (2)



63. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R)
Assertion (A): In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidising in nature.
Reason (R): Extra stability to half filled electronic configuration is observed than incompletely filled electronic configurations.
 In the light of the above statements, choose the most appropriate answer from the options given below.
 (1) (A) is true but (R) is false
 (2) Both (A) and (R) are true and (R) is the correct explanation of (A)
 (3) Both (A) and (R) are true but (R) is not the correct explanation of (A)
 (4) (A) is false but (R) is true

Ans. (2)

- *64. The functional group that shows negative resonance effect is:
 (1) $-\text{OH}$ (2) $-\text{OR}$
 (3) $-\text{COOH}$ (4) $-\text{NH}_2$

Ans. (3)

- *65. The number of radial node/s for 3p orbital is:
 (1) 3 (2) 2
 (3) 1 (4) 4

Ans. (3)

Sol. Radial node $= (n - \ell - 1) = (3 - 1 - 1) = 1$

- *66. Which among the following has highest boiling point?
 (1) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2 - \text{OH}$ (2) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3$
 (3) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHO}$ (4) $\text{H}_5\text{C}_2 - \text{O} - \text{C}_2\text{H}_5$

Ans. (1)

Sol. Due to H-bonding

- *67. Given below are two statements:
Statement (I): A π bonding MO has lower electron density above and below the inter-nuclear axis.
Statement (II): The π^* antibonding MO has a node between the nuclei.
 In the light of the above statements, choose the most appropriate answer from the options given below:
 (1) Both Statement I and Statement II are true
 (2) Both Statement I and Statement II are false
 (3) Statement I is true but Statement II is false
 (4) Statement I is false but Statement II is true

Ans. (4)



68. Given below are two statements:

Statement (I): Both metals and non-metals exist in p and d-block elements.

Statement (II): Non-metals have higher ionisation enthalpy and higher electronegativity than the metals.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (1) Both statement I and statement II are false
- (2) Both statement I and statement II are true
- (3) Statement I is false but statement II is true
- (4) Statement I is true but statement II is false

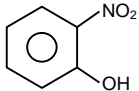
Ans. (3)

69. Which of the following compounds show colour due to d-d transition?

- (1) $K_2Cr_2O_7$
- (2) $CuSO_4 \cdot 5H_2O$
- (3) $KMnO_4$
- (4) K_2CrO_4

Ans. (2)

*70. Select the compound from the following that will show intramolecular hydrogen bonding.

- (1) 
- (2) H_2O
- (3) C_2H_5OH
- (4) NH_3

Ans. (1)

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

71. The number of tripeptides formed by three different amino acids using each amino acid once is ____.

Ans. 6

72. Mass of ethylene glycol (antifreeze) to be added to 18.6 kg of water to protect the freezing point at $-24^\circ C$ is _____ kg (Molar mass in $g\ mol^{-1}$ for ethylene glycol 62, K_f of water = $1.86\ K\ kg\ mol^{-1}$)

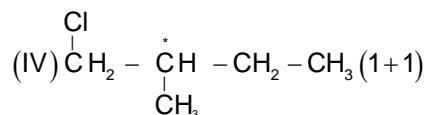
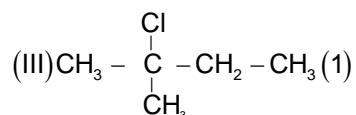
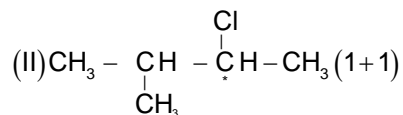
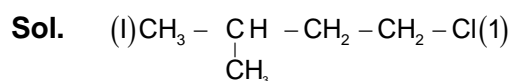
Ans. 15

Sol. $\Delta T = \frac{K_f \times w \times 1000}{MW}, 24 = \frac{1.86 \times w \times 1000}{62 \times 18.6 \times 1000}, w = 14.88\ kg$

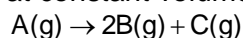
*73. Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is _____.

Ans. 6





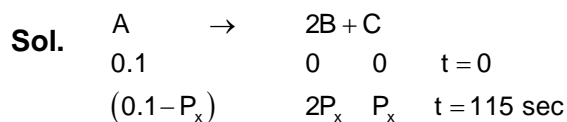
74. The following data were obtained during the first order thermal decomposition of a gas A at constant volume:



S. No.	Time/s	Total pressure/(atm)
(1)	0	0.1
(2)	115	0.28

The rate constant of the reaction is _____ $\times 10^{-2} \text{ s}^{-1}$ (nearest integer)

Ans. 2



According to question

$$0.28 = 0.1 - P_x + 2P_x + P_x$$

$$P_x = \frac{(0.28 - 0.1)}{2} = 0.09$$

$$K = \frac{2.303}{115} \log \frac{0.1}{(0.1 - 0.09)} = 2 \times 10^{-2}$$

*75. For a certain reaction at 300K, K = 10, then ΔG° for the same reaction is _____ $\times 10^{-1} \text{ kJ mol}^{-1}$. (Given R = 8.314 $\text{JK}^{-1} \text{ mol}^{-1}$)

Ans. 57

Sol. $\Delta G^\circ = -2.303RT \log K = -\frac{2.303 \times 8.314 \times 300 \times \log 10}{1000} = 5.74 = 57.4 \times 10^{-1} \text{ kJ/mol}$

