JEE MAIN - 13 Full Syllabus Date : 20-01-2025

PART : MATHEMATICS

1. Let A (–1,1) and B (2,3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is ax + by = 15, then 5a + 2b is :

(1) 4 (2) $-\frac{12}{5}$ (3) $-\frac{6}{5}$ (4) 6

Ans. **NTA (2)**
RESO(2)

Sol. Equation of Line AB $\equiv$ 2x– 3y + 5= 0 AB $\equiv \sqrt{(2+1)^2+(3-1)^2} = \sqrt{13}$ As point P is on other side of origin

$\Delta PAB = 10 \Rightarrow \frac{1}{2}\sqrt{13}\frac{(-2h+3k-5)}{\sqrt{13}} = 10$

– 2x + 3y = 25

$\frac{3(-2x+3y)}{5} = 15$

2. Let the circle $C_1 : x^2 + y^2 - 2(x+y)+1=0$ and C_2 be a circle having centre at (–1,0) and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

(1) 6 (2) 1 (3) 2 (4) 4

Ans. **NTA (3)**
RESO(3)

Sol. $C_1(1,1)$, r_1= 1, $(x+1)^2 + y^2 = 4$

Radical axis $4x + 2y - 4 = 0 \Rightarrow P(0,2)$

Distance = $\sqrt{(1-0)^2+(1-2)^2}$

Square of distance = 2

3. 60 word can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is :

(1) JBBOH (2) OBBJH (3) HBBJO (4) OBBHJ

Ans. **NTA (2)**
RESO(2)

Sol. B, B, H, J, O

B				

4!

H				

$\frac{4!}{2} = 12$

J				

$\frac{4!}{2} = 12$

Cumulative sum = 48
OBBHJ 1
OBBJH 1
50 th word is OBBJH

4. Let $f, g : R \to R$ be defined as :

$$f(x) = |x-1| \text{ and } g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0. \end{cases}$$

Then the function $f(g(x))$ is

(1) onto but not one-one. (2) one-one but not onto

(3) both one-one and onto. (4) neither one-one nor onto.

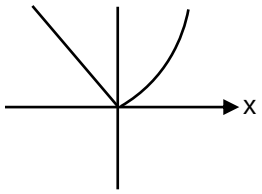
Ans. **NTA (4)**

RESO(4)

Sol. $f(x) = \begin{cases} -x+1 : x < 1 \\ x-1 : x \geq 1 \end{cases}$, $f(g(x)) \begin{cases} -g(x)+1+ : g(x) < 1 \\ g(x)-1 : g(x) \geq 1 \end{cases}$

$$f(g(x)) = \begin{cases} -(x+1)+1 : x+1 < 1 : x \leq 0 \\ x+1-1 : x+1 \geq 1 : x \leq 0 \\ -e^x+1 : e^x < 1 : x \geq 0 \\ e^x-1 : e^x \geq 1 : x \geq 0 \end{cases}$$

$$f(g(x)) = \begin{cases} -x : & x \leq 0 \\ e^x-1 : & x \geq 0 \end{cases}$$



5. Let $f : [-1,2] \to R$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where [t] denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is:

(1) 4 (2) 3 (3) 6 (4) 5

Ans. **NTA (1)**

RESO(1)

Sol. $$f(x) = \begin{cases} 2x^2 + x + \left[x^2\right] - (-1) : & -1 \leq x < 0 \\ 2x^2 + x + \left[x^2\right] - 0 : & 0 \leq x < 0 \\ 2x^2 + x + \left[x^2\right] - 0 : & 1 \leq x < 2 \\ 12 & x = 2 \end{cases}$$

$1 \leq x < 2 \Rightarrow 1 \leq x^2 < 4 \quad \Rightarrow 1 \leq x^2 < 2,\ 2 \leq x^2 < 3,\ 3 \leq x^2 < 4$

$\Rightarrow 1 \leq x < \sqrt{2},\ \sqrt{2} \leq x < \sqrt{3},\ \sqrt{3} \leq x < 2$

$$f(x) = \begin{cases} 3 & : & x = -1 \\ 2x^2 + x + 1 & : & -1 < x < 0 \\ 2x^2 + x & : & 0 \leq x < 1 \\ 2x^2 + x & : & 1 \leq x < \sqrt{2} \\ 2x^2 + x + 1 & : & \sqrt{2} \leq x < \sqrt{3} \\ 2x^2 + x + 2 & : & \sqrt{3} \leq x < 2 \\ 12 & : & x = 2 \end{cases}$$

Not continuous at x = –1 , 0, $\sqrt{2}$, $\sqrt{3}$

6. If $y(\theta)=\dfrac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}$, then at $\theta=\dfrac{\pi}{2}$, $y''+y'+y$ is equal to :

(1) 1 (2) 2 (3) $\frac{3}{2}$ (4) $\frac{1}{2}$

Ans. **NTA (2)**
RESO(2)

Sol. $y(\theta)=\dfrac{2\cos^2\theta+2\cos\theta-1}{4\cos^3\theta+8\cos^2\theta+2\cos\theta-2}$

Let $t=\cos\theta$ $\quad y=\dfrac{2t^2+2t-1}{2(2t^3+4t^2+t-1)}$

$$2t^2+2t-1\overline{)\,2t^3+4t^2+t-1\,(t+1}$$
$$\underline{2t^3+2t^2-t}$$
$$2t^2+2t-1$$
$$\underline{2t^2+2t-1}$$
$$0$$

$y=\dfrac{1}{2(1+t)}=\dfrac{1}{2(1+\cos\theta)}$

$y=\dfrac{1}{4}\sec^2\left(\dfrac{\theta}{2}\right) \Rightarrow y'=\dfrac{1}{4}\sec^2\left(\dfrac{\theta}{2}\right)\tan\left(\dfrac{\theta}{2}\right)$

$y''=\dfrac{1}{4}\left(2\sec\left(\dfrac{\theta}{2}\right)\sec\left(\dfrac{\theta}{2}\right)\tan\left(\dfrac{\theta}{2}\right)\dfrac{1}{2}\tan\left(\dfrac{\theta}{2}\right)+\dfrac{1}{4}\sec^2\left(\dfrac{\theta}{2}\right).\sec^2\left(\dfrac{\theta}{2}\right).\dfrac{1}{2}\right.$

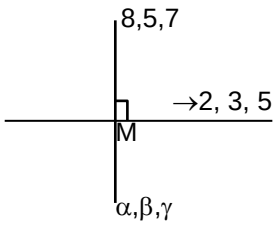
At $\theta=\dfrac{\pi}{2}$ $\quad y=\dfrac{1}{2}$, $\quad y'=\dfrac{1}{2}$, $\quad y''=1$

7. Let (α,β,γ) be the image of the point (8,5,7) in the line $\dfrac{x-1}{2}=\dfrac{y+1}{3}=\dfrac{z-2}{5}$. Then $\alpha+\beta+\gamma$ is equal to :

(1) 16 (2) 20 (3) 18 (4) 14

Ans. **NTA (4)**
RESO(4)

Sol.



M=(1+2λ, – 1 + 3λ , 2 + 5λ)

2λ –7, 3λ – 6, 5λ – 5 is perpendicular to 2, 3, 5

$\Rightarrow \lambda=\dfrac{3}{2}\Rightarrow M\left(4,\dfrac{7}{2},\dfrac{19}{2}\right)$

$\Rightarrow \alpha+8=8, \quad \beta+5=7, \quad \gamma+7=19$

$\alpha+\beta+\gamma=0+2+12=14$

8. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$. is the matrix of cofactors of the elements of A, then det(AB) is equal to :

(1) 64 (2) 216 (3) 343 (4) 125

Ans. **NTA (2)**
RESO(2)

Sol. cofactor of 1 –1 element = $2\alpha^2 - \alpha\beta = 3\alpha$

$\Rightarrow 2\alpha - \beta = 3$

Cofactor of 2–1 element = $-(2\alpha^2 - 3\alpha) = -\alpha$

$\Rightarrow 2\alpha - 3 = 1 \Rightarrow \alpha = 2$

$\Rightarrow \beta = 1$ $\Rightarrow |A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 6$

$|AB| = |A|\,|B| = |A|\,|A|^2 = |A|^3 = 216$

9. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies :

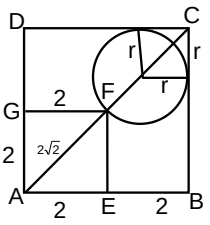
(1) r = 1 (2) $r^2 - 8r + 8 = 0$ (3) $2r^2 - 4r + 1 = 0$ (4) $2r^2 - 8r + 7 = 0$

Ans. **NTA (2)**
RESO(2)

Sol. $r + r\sqrt{2} = 2\sqrt{2}$

$r = 4 - 2\sqrt{2}$

$(r-4)^2 = 8 \Rightarrow r^2 - 8r + 8 = 0$



10. Let $S_1 = \{z \in C : |z| \leq 5\}, S_2 = \left\{z \in C : \text{Im}\left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right) \geq 0\right\}$ and $S_3 = \{z \in C : \text{Re}(z) \geq 0\}$. Then the area of the region $S_1 \cap S_2 \cap S_3$ is :

(1) $\frac{125\pi}{24}$ (2) $\frac{125\pi}{4}$ (3) $\frac{125\pi}{6}$ (4) $\frac{125\pi}{12}$

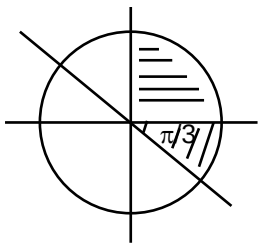
Ans. **NTA (4)**
RESO(4)

Sol. $S_1 : x^2 + y^2 \leq 5^2$; $S_2 : \sqrt{3}x + y \geq 0$; $S_3 : x \geq 0$

$\frac{z+1-i\sqrt{3}}{1-\sqrt{3}i} = \frac{\left((x+1)+i\left(y-\sqrt{3}\right)\left(1+i\sqrt{3}\right)\right)}{1+3}$

$I_m(z) = (x+1)\sqrt{3} + \left(y-\sqrt{3}\right) = \sqrt{3}x + y$

$S_1 \cap S_2 \cap S_3 = \frac{1}{4}\pi 5^2 + \frac{1}{2}5^2\frac{\pi}{3} = \frac{125\pi}{12}$



11. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x}+\frac{2x}{\sqrt[3]{5}}\right)^{12}, x \neq 0,$ is $\alpha \times 2^8 \times \sqrt[5]{3},$ then 25α is equal to :

(1) 693 (2) 639 (3) 742 (4) 724

Ans. **NTA (1)**
RESO(1)

Sol. $^{12}C_r\left(\frac{\sqrt[5]{3}}{x}\right)^{12-r}\left(\frac{2x}{\sqrt[3]{5}}\right)^{r} = {}^{12}C_r \frac{3^{\frac{12-r}{5}}.2^r}{5^{r/3}}x^{2r-12}$

$2r - 12 = 0 \Rightarrow r = 6$

$\alpha \times 2^8 \times \sqrt[5]{3} = {}^{12}C_6 \frac{3^{\frac{6}{5}}2^6}{5^2} = \frac{11.3.4.7.3.3^{\frac{1}{5}}2^6}{5^2}$

$\alpha = \frac{3^2.7.11}{5^2} \Rightarrow 25\alpha = 693$

12. The coefficients a, b, c in the quadratic equation $ax^2+bx+c=0$ are from the set {1, 2, 3, 4, 5, 6}. If the probability of this equation having one real root bigger than the other is p, then 216p equals :

(1) 19 (2) 38 (3) 76 (4) 57

Ans. **NTA (2)**
RESO(2)

Sol. $D > 0$ $b^2 > 4ac \Rightarrow b = 1, 2$ not possible
$b = 3 \Rightarrow 9 > 4ac \Rightarrow ac = 1, 2 \Rightarrow (1, 1), (1, 2), (2, 1)$: 3 ways
$b = 4 \Rightarrow 4 > ac \Rightarrow ac = 1, 2, 3$: 5 ways
$b = 5 \Rightarrow 25 > 4\ ac \Rightarrow ac = 1, 2, 3,4, 5, 6$
(1,1), (1,2), (2,1), (1,3), (3,1), (1,4), (2,2), (4,1), (1,5), (5,1), (1,6), (2,3), (3,2), (6,1) : 14 ways
$b = 6 \Rightarrow 9 > ac \Rightarrow ac =1, 2, 3,4, 5, 6, 8$
$ac = 8 \Rightarrow (2, 4), (4, 2)$: 16 ways
Favourable = 3+5 +14+16 = 38 ways

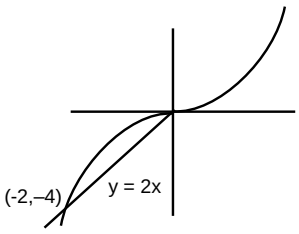
$P = \frac{38}{6^3}$

13. The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is :

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$ (3) 1 (4) $\frac{4}{3}$

Ans. **NTA (4)**
RESO(4)

Sol. $y = \begin{cases} -x^2 : x < 0 \\ x^2 : x \geq 0 \end{cases}$, $y = \begin{cases} x-(-x) : x < 0 \\ x - x : x \geq 0 \end{cases}$



$\text{Area} = \int_{-2}^{0}\left(-x^2-2x\right)dx = \frac{4}{3}$

14. The values of m, n, for which the system of equations

$x+y+z=4,$

$2x+5y+5z=17,$

$x+2y+mz=n$

has infinitely many solutions, satisfy the equation :

(1) $m^2+n^2-mn=39$ (2) $m^2+n^2+m+n=64$

(3) $m^2+n^2+mn=68$ (4) $m^2+n^2-m-n=46$

Ans. **NTA (1)**
RESO(1)

Sol. Augmented matrix is

$$\begin{bmatrix}1 & 1 & 1 & : & 4\\ 2 & 5 & 5 & : & 17\\ 1 & 2 & m & : & n\end{bmatrix} R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1$$

$$\Rightarrow \begin{bmatrix}1 & 1 & 1 & : & 4\\ 0 & 3 & 3 & : & 9\\ 0 & 1 & m-1 & : & n-4\end{bmatrix}$$

After dividing R_2 by 3 we obtain

m-1 =1, n – 4 = 3

m = 2, n = 7

$m^2 + n^2 – mn = 4+49 – 14 = 49 – 10 = 39$

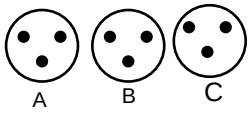
15. Let the set S = {2,4,8,16,…,512} be partitioned into 3 sets A,B,C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to :

(1) 1520 (2) 1640 (3) 1710 (4) 1680

Ans. **NTA (4)**
RESO(4)

Sol. All three partitions are different, with some cardinal number (equal to 3).

Number of ways = $\left(\frac{9!}{3!3!3!}\frac{1}{3!}\right)3!$



16. Let $\vec{a}=2\hat{i}+5\hat{j}-\hat{k}, \vec{b}=2\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{c}$ be three vectors such that $\left(\vec{c}+\hat{i}\right)\times\left(\vec{a}+\vec{b}+\hat{i}\right)=\vec{a}\times\left(\vec{c}+\hat{i}\right)$.

If $\vec{a}\cdot\vec{c}=-29$, then $\vec{c}\cdot\left(-2\hat{i}+\hat{j}+\hat{k}\right)$ is equal to :

(1) 12 (2) 5 (3) 10 (4) 15

Ans. **NTA (2)**
RESO(2)

Sol. $\left(\bar{c}+\hat{i}\right)\times\left(2\bar{a}+\bar{b}+\hat{i}\right)=\bar{0}\Rightarrow\bar{c}+\hat{i}=\lambda\left(2\bar{a}+\bar{b}+\hat{i}\right)$

$2\bar{a}+\bar{b}+\hat{i}=7\hat{i}+8\hat{j}$

$x + 1 = 7\lambda,\quad y = 8\lambda,\ z = 0$

$\bar{a}.\bar{c}=-29\Rightarrow 2x+5y-z=-29\Rightarrow\lambda=-\frac{1}{2}$

$\bar{c}=\left(-1-\frac{7}{2},-4,0\right)$

$\bar{c}.\left(-2\hat{i}+\hat{j}+\hat{k}\right)=5$

17. Let $\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx, m,n>0$. If $\int_0^1 (1-x^{10})^{20}dx = a\times\beta(b,c)$, then 100 (a + b + c) equals_____.

(1) 2012 (2) 2120 (3) 1021 (4) 1120

Ans. **NTA (2)**
RESO(2)

Sol. $\int_0^1 \left(1-x^{10}\right)^{20}dx$

Put $t = x^{10}$, $x = t^{1/10}$

$$dx = \frac{1}{10}t^{\frac{-9}{10}}dt$$

$$= \int_0^1 (1-t)^{20}\,\frac{1}{10}t^{\frac{-9}{10}}dt = \frac{1}{10}\beta(b,c)$$

$$a=\frac{1}{10},\ m-1=\frac{-9}{10},\ n-1=20$$

$$m=\frac{1}{10},\ n=21$$

$$b=\frac{1}{10},\ c=21$$

100(a+b+c) = 2120

18. For $x \ge 0$, the least value of K, for which $4^{1+x}+4^{1-x}, \frac{K}{2}, 16^{x}+16^{-x}$ are three consecutive terms of an A.P., is equal to :

(1) 8 (2) 16 (3) 10 (4) 4

Ans. **NTA (3)**
RESO(3)

Sol. $2b = a + c \Rightarrow k = 4\left(4^x+\frac{1}{4^x}\right)+\left(16^x+\frac{1}{16^x}\right)$

A.M $\ge$ G.M

K $\ge$ 4(2) + (2) = 10

Least value of k = 10

19. The differential equation of the family of circle passing through the origin and having center at line y=x is :

(1) $(x^2+y^2-2xy)dx=(x^2+y^2+2xy)dy$

(2) $(x^2-y^2+2xy)dx=(x^2-y^2+2xy)dy$

(3) $(x^2-y^2+2xy)dx=(x^2-y^2-2xy)dy$

(4) $(x^2+y^2+2xy)dx=(x^2+y^2-2xy)dy$

Ans. **NTA (3)**
RESO(3)

Sol. Let equation of circle is:

$(x - r)^2 + (y - r)^2 = 2r^2$

$x^2 + y^2 - 2rx - 2ry = 0$ ….(1)

Differentiate with respect to x

$2x + 2y\frac{dy}{dx} - -2r\left(1+\frac{dy}{dx}\right) = 0$ ....(2)

By (1) and (2)

$(x^2 + y^2) = (x + y)\left(\frac{2x+2y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)$

$(x^2 + y^2)\left(1+\frac{dy}{dx}\right) = 2x^2 + 2xy + 2(x + y)y\ \frac{dy}{dx}$

$(x^2 + y^2 - 2x^2 - 2xy) = (2xy + 2y^2 - x^2 - y^2)\ \frac{dy}{dx}$

$(y^2 - x^2 - 2xy)dx = (y^2 - x^2 + 2xy)dy$

$(x^2 - y^2 + 2xy)\ dx = (x^2 - y^2 - 2xy)dy$

20. Consider three vector $\vec{a},\vec{b},\vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} = \vec{b}\times\vec{c}$. If $a \in \left[0,\frac{\pi}{3}\right]$ is the angle between the vector $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to :

(1) 105 (2) 110 (3) 124 (4) 121

Ans. **NTA (3)**
RESO(3)

Sol. $\vec{a} = \vec{b}\times\vec{c} \quad \Rightarrow \quad \vec{a}.\vec{c} = 0$ ....(1)

$|\vec{a}|^2 = |\vec{b}|^2|\vec{c}|^2 \sin^2\alpha$

Now, $|\vec{c}|^2 = \frac{4}{9}\text{cosec}^2\alpha, \quad \alpha \in \left[0,\frac{\pi}{3}\right]$ ....(2)

Now, $27\ |\vec{c} - \vec{a}|^2 = 27\ \{|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{c}\}$

$= 27\left\{\frac{4}{9}\text{cosec}^2\alpha + 4 - 0\right\} = 12\ \text{cosec}^2\alpha + 108$

Now, $\alpha \in \left[0,\frac{\pi}{3}\right]$ So, $27\ |c - a|^2$ must be minimum if $\text{cosec}^2\alpha$ is minimum.

$\Rightarrow \quad \text{cosec}^2\alpha = \frac{4}{3}$

$\Rightarrow \quad 27\ |c - a|^2 = 12\left(\frac{4}{3}\right) + 108 = 16 + 108 = 124$

21. If $1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+\ldots\ldots$ upto $\infty = 2+\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with gcd(a,b) = 1, then 11a + 18b is equal to …….

Ans. **NTA (76)**
RESO(76)

Sol. Let $S = 1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+\ldots\ldots$ upto ∞

$S = 1+\frac{1}{2}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)+\frac{1}{6}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)^2+\frac{1}{12}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)^3+\frac{1}{20}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)^4+\ldots..$

$= 1 + \left(1-\frac{1}{2}\right)x + \left(\frac{1}{2}-\frac{1}{3}\right)x^2 + \left(\frac{1}{3}-\frac{1}{4}\right)x^3 + \left(\frac{1}{4}-\frac{1}{5}\right)x^4 + \ldots\ldots$where $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}$

$= \left(1+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots.\right) - \left(\frac{1}{2}x+\frac{1}{3}x^2+\frac{1}{4}x^3+\frac{1}{5}x^4+\ldots.\right)$

$= 1 - \ln(1-x) + \frac{1}{x}(x + \ln(1-x))$

$= 2 + \frac{1}{x}\ln(1-x) - \ln(1-x)$

$= 2 + \left(\frac{1-x}{x}\right)\ln(1-x)$

$= 2 + \left\{\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}-1\right\}\ln\left(1-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$

$= 2 + \frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\ln\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$

$= 2 + \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}}\right)\ln\left(\frac{2}{3}\right)$

$= 2 + \left(\sqrt{\frac{3}{2}}+1\right)\ln\left(\frac{2}{3}\right)$

Hence, a = 2, b = 3
So, 11a + 18b = 22 + 54 = 76.

22. Let the point (–1, α, β) lie on the line of the shortest distance between the lines $\frac{x+2}{-3}=\frac{y-2}{4}=\frac{z-5}{2}$ and $\frac{x+2}{-1}=\frac{y+6}{2}=\frac{z-1}{0}$, then $(\alpha-\beta)^2$ is equal to……

Ans. **NTA (25)**
RESO(25)

Sol. $L_1 = \frac{x+2}{-3}=\frac{y-2}{4}=\frac{z-5}{2}$

$L_2 = \frac{x+2}{-1}=\frac{y+6}{2}=\frac{z-1}{0}$

Let P $(-3t_1-2, 4t_1+2, 2t_1+5)$ and Q $(-t_2-2, 2t_2-6, 1)$ lies on L_1 and L_2 respectively and PQ is line of shortest distance.

D'ratio of PQ are $< 3t_1 - t_2, -4t_1 + 2t_2 - 8, -2t_1 - 4 >$

$PQ \perp L_1 \Rightarrow -3(3t_1 - t_2) + 4(-4t_1 + 2t_2 - 8) + 2(-2t_1 - 4) = 0$

$\Rightarrow -29t_1 + 11t_2 = 40$ ….(1)

$PQ \perp L_2 \Rightarrow -1(3t_1 - t_2) + 2(-4t_1 + 2t_2 - 8) + 0(-2t_1 - 4) = 0$

$\Rightarrow -11t_1 + 5t_2 = 16$ ….(2)

By (1) and (2) we have $t_1 = -1$, $t_2 = 1$

$\Rightarrow$ P (1, – 2, 3) and Q (–3, –4, 1)

Equation of PQ is $\frac{x-1}{4}=\frac{y+2}{2}=\frac{z-3}{2}$

Point (–1, α, β) lies on it.

$\Rightarrow \quad -\frac{1}{2} = \frac{\alpha+2}{2} = \frac{\beta-3}{2}$

$\Rightarrow \quad \alpha = -3$ and $\beta = 2$

$\Rightarrow \quad (\alpha-\beta)^2 = 25$

23. Let y = y(x) be the solution of the differential equation

$$\frac{dy}{dx} + \frac{2x}{\left(1+x^2\right)^2} y = xe^{\frac{1}{\left(1+x^2\right)}}; y(0) = 0.$$

Then the area enclosed by the curve $f(x) = y(x)e^{-\frac{1}{(1+x^2)}}$ and the line y – x = 4 is …….

Ans. **NTA (18)**
RESO(18)

Sol. $\frac{dy}{dx} + \frac{2x}{(1+x^2)^2 y} = xe^{\frac{1}{1+x^2}}$ is a linear differential equation so integral factor $e^{\int \frac{2x}{(1+x^2)^2} dx} = e^{-\left(\frac{1}{1+x^2}\right)}$ and the solution of the differential equation is given by

$$y.\ e^{-\left(\frac{1}{1+x^2}\right)} = \int x.e^{\frac{1}{1+x^2}}.e^{-\frac{1}{1+x^2}} dx + C$$

$$y.\ e^{-\left(\frac{1}{1+x^2}\right)} = \frac{x^2}{2} + C \quad \ldots.(1)$$

Now, given y(0) = 0 $\Rightarrow$ C = 0

$\Rightarrow \quad y = \frac{x^2}{2} e^{\frac{1}{1+x^2}}$

Now, equation of curve $f(x) = y(x)\ e^{-\left(\frac{1}{1+x^2}\right)}$ $\Rightarrow$ $f(x) = \frac{x^2}{2}$

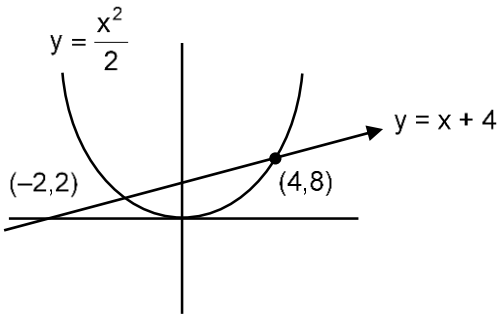
Point of Intersection of $y = \frac{x^2}{2}$ and y = x + 4 are P (–2, 2) and Q (4, 8)

area bounded by curves

$$= \int_{-2}^{4} \left(x + 4 - \frac{x^2}{2}\right) dx = \left(\frac{x^2}{2} + 4x - \frac{x^3}{6}\right)\Bigg|_{-2}^{4}$$

$$= \left(8 + 16 - \frac{32}{3}\right) - \left(2 - 8 + \frac{4}{3}\right)$$

= 30 – 12 = 18



24. The Number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x-1)^2 = 0$, where $-\pi \le x \le \pi$, is

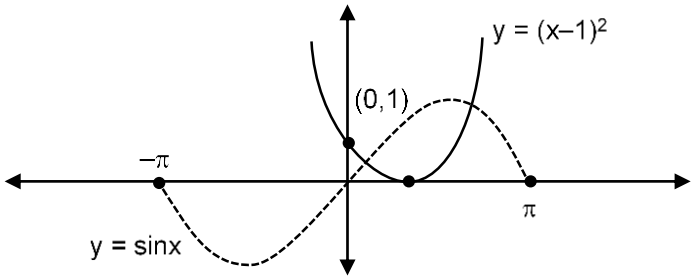
Ans. **NTA (2)**
RESO(2)

Sol. $\sin^2 x + (2 + 2x - x^2) \sin x - 3(x - 1)^2 = 0, \quad x \in [-\pi, \pi]$

$\Rightarrow \quad \sin^2 x - (x - 1)^2 \sin x + 3\sin x - 3(x - 1)^2 = 0$

$\Rightarrow \quad (x - 1)^2 (\sin x + 3) = \sin x (\sin x + 3)$

$\sin x \neq -3$ so, $\quad (x - 1)^2 = \sin x$



Hence only 2 solutions.

25. Let a > 0 be a root of the equation $2x^2 + x - 2 = 0$, If $\lim\limits_{x \to \frac{1}{a}} \dfrac{16(1 - \cos(2 + x - 2x^2))}{(1 - ax)^2} = \alpha + \beta\sqrt{17}$, Where $\alpha, \beta \in Z$ then $\alpha + \beta$ is equal to ……

Ans. **NTA (170)**
RESO(170)

Sol. $2x^2 + x - 2 = 0, \quad \Rightarrow \quad x = \dfrac{-1 \pm \sqrt{1+16}}{4}$

As a > 0 is a root of quadratic equation

$\Rightarrow \quad a = \dfrac{-1 + \sqrt{17}}{4} \quad \Rightarrow \quad \dfrac{1}{a} = \dfrac{4}{\sqrt{17} - 1} = \left(\dfrac{\sqrt{17} + 1}{4}\right)$

Now let other root is b

$\Rightarrow \quad 2x^2 + x - 2 = 2 (x - a) (x - b)$

Now $\quad \lim\limits_{x \to \frac{1}{a}} \dfrac{16(1 - \cos(2 + x - 2x^2))}{(1 - ax)^2} = \lim\limits_{x \to \frac{1}{a}} \dfrac{32\sin^2\left(\frac{2 + x - 2x^2}{2}\right)}{(1 - ax)^2}$

$$= \lim_{x \to \frac{1}{a}} 32\left\{\frac{\sin\left(\frac{2 + x - 2x^2}{2}\right)}{\left(\frac{2 + x - 2x^2}{2}\right)}\right\}^2 . \frac{(2 + x - 2x^2)^2}{4(1 - ax)^2}$$

$$= \lim_{x \to \frac{1}{a}} 32.1\frac{\left(-2\left(x - \frac{1}{a}\right)\left(x - \frac{1}{b}\right)\right)^2}{4(1 - ax)^2}$$

$$= \lim_{x \to \frac{1}{a}} 32.1 \frac{\left(-2\left(x-\frac{1}{a}\right)\left(x-\frac{1}{b}\right)\right)^2}{4(1-ax)^2}$$

$$= \lim_{x \to \frac{1}{a}} \frac{32(ax-1)^2(bx-1)^2}{a^2b^2(1-ax)^2} = \frac{32(b-a)^2}{a^4b^2} = \frac{32\left(-\frac{\sqrt{17}}{2}\right)^2}{\left(\frac{\sqrt{17}-1}{4}\right)^2} = \frac{32\times 17}{4\left(\frac{18-2\sqrt{17}}{16}\right)} = \frac{64\times 17}{9-\sqrt{17}}$$

$$= 17\left(9+\sqrt{17}\right) = 153 + 17\sqrt{17}$$

$$\Rightarrow \qquad \alpha + \beta = 170$$

PART : PHYSICS

26. The vehicles carrying inflammable fluids usually have metallic chains touching the ground :
(1) To conduct excess charge due to air friction to ground and prevent sparking
(2) To protect tyres from catching dirt from ground
(3) It is a custom
(4) To alert other vehicles
ज्वलनशील द्रवों को ले जाने वाले वाहनों में सामान्यतः धरती को छूती हुई एक धात्वित जंजीर होती है :
(1) वायु घर्षण के कारण आधिक्य आवेश को धरती की ओर चालन करने तथा जलाने से बचने में।
(2) टायरों को धरती से धूल को पकड़ने से बचाने के लिए।
(3) यह एक परम्परा है।
(4) दूसरे वाहनों को सचेत करने के लिए।

Ans. **(1)**

Sol. Developed charge due to friction will move to ground by metallic chains to prevent sparking.

27. Given below are two statements :
Statement-I : When the white light passed through a prism, the red light bends lesser than yellow and violet.
Statement-II : The refractive indices are different for different wavelengths in dispersive medium. In the light of the above statements, choose the correct answer from the options given below :
(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are false
(3) Both Statement I and Statement II are true
(4) Statement I is false but Statement II is true
नीचे दो कथन दिये गये है :
कथन-I : जब श्वेत प्रकाश एक प्रिज्म से गुजरता है तो पीली व बैंगनी की तुलना में लाल रंग का प्रकाश कम विचलित होता है।
कथन-II : परिक्षेपी माध्यम में विभिन्न तरंगदैर्ध्ये के लिए आवर्तनांक भिन्न–भिन्न होता है।
उपरोक्त कथनों के आधार पर, नीचे दिये गये विकल्पों में से सबसे उचित उत्तर चुनिए :
(1) कथन I सही है परन्तु कथन II गलत हैं।
(2) दोनों कथन I व कथन II गलत हैं।
(3) दोनों कथन I व कथन II सही हैं।
(4) कथन I गलत है परन्तु कथन II सही हैं।

Ans. **(3)**

Sol. As $\lambda_{red} > \lambda_{yellow} > \lambda_{violet}$
Light ray with longer wavelength bends less.

28. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to :
एक नियत शक्ति स्त्रोत के अन्तर्गत एक पिण्ड एक दिशीय गति कर रहा है। t समय में इसका विस्थापन किसके समानुपाती है :
(1) $t^{2/3}$ (2) t (3) t^2 (4) $t^{3/2}$

Ans. **(4)**

Sol. $P \times t = \frac{1}{2} mv^2$

P = constant

$V = k\sqrt{t}$

where k is constant

$\frac{dx}{dt} = k\sqrt{t}$

$\int_0^x dx = k\int_0^x \sqrt{t}\ dt$

$x = k\ t^{3/2}$ $\quad x \propto t^{3/2}$

29. A vernier callipers has 20 divisions on the vernier scale, which coincides with 19th division on the main scale. The least count of the instrument is 0.1 mm. One main scale division is equal to _____ mm.
एक वर्नियर कैलीपर्स के वर्नियर पैमाने पर 20 भाग हैं जो इसे मुख्य पैमाने के 19 भागों साथ संपाती हैं। यंत्र की अल्पतमांक 0.1 mm है। मुख्य पैमाने के एक भाग का मान (मिमी में) किसके बराबर है _____ mm.

(1) 2 (2) 1 (3) 5 (4) 0.5

Ans. **(1)**

Sol. 20 VSD = 19 MSD

1 VSD = $\frac{19}{20}$ MSD

LC = 1 MSD – 1 VSD

LC = 1 MSD – $\frac{19}{20}$ MSD

$0.1 = \frac{MSD}{20}$

MSD = 2 mm

30. A series LCR circuit is subjected to an ac signal of 200 V, 50 Hz. If the voltage across the inductor (L = 10 mH) is 31.4 V, then the current in this circuit is _____.

(1) 63 A (2) 68 A (3) 10 mA (4) 10 A

एक श्रेणीबद्ध LCR परिपथ 200 V, 50 Hz के एक प्रत्यावर्ती सिग्नल से जोड़ा गया है। यदि प्रेरक (L = 10 mH) के सिरों के बीच वोल्टेज 31.4 V हो तो इस परिपथ में धारा है _____।

Ans. **(4)**

Sol. $V_L = i\ x_L = i\omega L$

$31.4 = (i) \times 2\pi fL$

$i = \frac{31.4}{2\pi \times 50 \times 10^{-2}} = \frac{31.4}{3.14}$

i = 10 A

31. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e. mean free path) is represented by :
यदि n संख्या घनत्व व d अणु का व्यास हो तब दो लगातार टक्करों के बीच एक अणु द्वारा तय की गई औसत दूरी (औसत मुक्त पथ) है :

(1) $\frac{1}{\sqrt{2}n\pi d^2}$ (2) $\frac{1}{\sqrt{2n\pi d^2}}$ (3) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$ (4) $\sqrt{2}n\pi d^2$

Ans. **(1)**

Sol. mean free path = $\frac{1}{\sqrt{2}n\pi d^2}$

32. A heavy box of mass 50 kg is moving on a horizontal surface. If co-efficient of kinetic friction between the box and horizontal surface is 0.3 then force of kinetic friction is :
50 kg का एक भारी बॉक्स एक क्षैतिज तल पर गति कर रहा है। यदि बॉक्स तथा क्षैतिज तल के मध्य गजि घर्षण गुणांक 0.3 है। गतिज घर्षण बल है :

(1) 1.47 N (2) 14.7 N (3) 147 N (4) 1470 N

Ans. **(3)**

Sol. $f_k = \mu_k N = 0.3 \times 50 \times 9.8 = 147$ N

33. Match List-I with List-II :

List-I		**List-II**
(A) A force that restores an elastic body of unit area to its original state	(I)	Bulk modulus
(B) Two equal and opposite forces parallel to opposite faces	(II)	Young's modulus
(C) Forces perpendicular everywhere to the surface per unit area same everywhere	(III)	Stress
(D) two equal and opposite forces perpendicular to opposite faces	(IV)	Shear modulus

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(II), (D)-(IV) (2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II) (4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

सूची-I का सूची-II से मिलान कीजिए।

सूची-I		**सूची-II**
(A) एक प्रत्यास्थ पिण्ड के एकांक क्षेत्र का वह बल जो उसको मूल अवस्था में बनाये रखता है।	(I)	आयतन प्रत्यास्थता गुणांक
(B) विपरीत सतहों के दो बराबर व विपरीत समान्तर बल	(II)	यंग प्रत्यास्थता गुणांक
(C) सतह के प्रति एकांक क्षेत्रफल पर लम्बवत बल सभी जगह समान होते है	(III)	प्रतिबल
(D) विपरीत सतहों के दो बराबर व विपरीत लम्बवत बल	(IV)	दृड़ता गुणांक

नीचे दिये गये विकल्पों से सही उत्तर चुनिए :

(1) (A)-(III), (B)-(I), (C)-(II), (D)-(IV) (2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II) (4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Ans. **(3)**

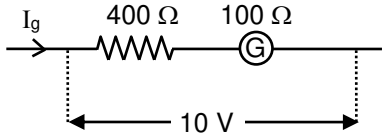
Sol. Based on Theory

34. A galvanometer of resistance 100 Ω when connected in series with 400 Ω measures a voltage of upto 10 V. The value of resistance required to convert the galvanometer into ammeter to read upto 10 A is x × 10^{-2} Ω. The value of x is :
100 Ω प्रतिरोध के एक धारामापी को जब 400 Ω प्रतिरोध के साथ श्रेणीक्रम में जोड़ा जाता है तो 10 V तब का वोल्टेज मापता है। धारामापी को अमीटर में बदलने के लिए आवश्क प्रतिरोध का मान x × 10^{-2} Ω है जब यह 10 A तक धारा पढ़ सकता है। x का मान है :

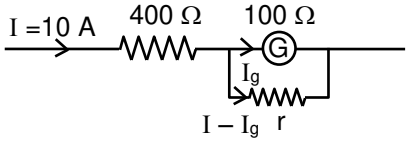
(1) 20 (2) 200 (3) 2 (4) 800

Ans. **(1)**

Sol.



$$I_g = \frac{10}{500} = \frac{1}{50} \text{ A}$$



$$(I - I_g)r = I_g \times 100$$

$$r = \left(\frac{I_g}{I - I_g} \times 100\right)$$

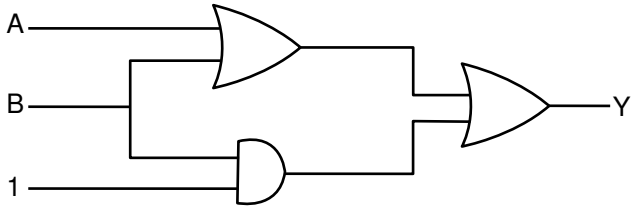
$$r = 0.2\ \Omega = 20 \times 10^{-2}\ \Omega$$

35. Which of the following statement is not true about stopping potential (V_0) ?
(1) It is 1/e times the maximum kinetic energy of electrons emitted.
(2) It increases with increase in intensity of the incident light.
(3) It depends on the nature of emitter material.
(4) It depends upon frequency of the incident light.
निरोधी विभव (V_0) के लिए निम्नलिखित में से कौन सा कथन सत्य नहीं है ?
(1) यह उत्सर्जित इलैक्ट्रॉनों की अधिकतम गतिज ऊर्जा का 1/e गुना होता है।
(2) यह आपतित प्रकाश की तीव्रता में वृद्धि के साथ बढ़ता है।
(3) यह उत्सर्जित पदार्थ की प्रकृति पर निर्भर करता है।
(4) यह आपतित प्रकाश की आवृत्ति पर निर्भर करता है।

Ans. **(2)**

Sol. $KE_{maximum} = hf - \phi = eV$

36. The output (Y) of logic circuit given below is 0 only when :
दिये गये लॉजिक परिपथ का निर्गत (Y) केवल 0 (शून्य) है जब :



(1) A = 0, B = 1 (2) A = 1, B = 0 (3) A = 1, B = 1 (4) A = 0, B = 0

Ans. **(4)**

Sol. y = (A + B) + B = A + B

y = 0 only when A = 0, B = 0

37. What is the dimensional formula of ab^{-1} in the equation $\left(P + \frac{a}{V^2}\right)$ (V – b) = RT, where letters have their usual meaning.
समीकरण $\left(P + \frac{a}{V^2}\right)$ (V – b) = RT में ab^{-1} की विमा क्या है जबकि वर्णों का प्रचलित अर्थ लिया गया है :

(1) $[M^{-1}L^5T^3]$ (2) $[M^6L^7T^4]$ (3) $[M^0L^3T^{-2}]$ (4) $[ML^2T^{-2}]$

Ans. **(4)**

Sol. $[P] = \left[\frac{a}{V^2}\right]$

$[M_1 L^{-1} T^{-2}] = \frac{a}{[L^6]}$

$a = [M^1 L^5 T^{-2}]$

and $[V] = [b] = [L^3]$

$\frac{a}{b} = \frac{[M^1L^5T^{-2}]}{[L^3]} = [M^1 L^2T^{-2}]$

38. The angular momentum of an electron in a hydrogen atom is proportional to :
(Where r is the radius of orbit of electron)
हाइड्रोजन परमाणु में एक इलैक्ट्रॉन का कोणीय संवेग किसके समानुपाती है (जहॉ r इलैक्ट्रॉन की कक्षा की त्रिज्या है) :

(1) $\sqrt{r}$ (2) $\frac{1}{r}$ (3) $\frac{1}{\sqrt{r}}$ (4) r

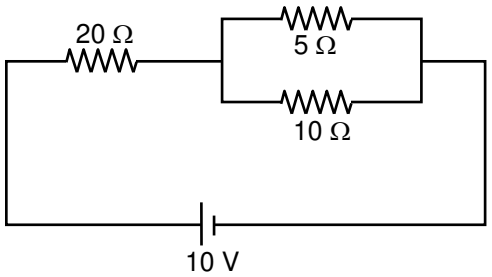
Ans. **(1)**

Sol. $L = \frac{nh}{2\pi}$

$r = \frac{n^2}{z} r_0$

$L \propto \sqrt{r}$

39. The ratio of heat dissipated per second through the resistance 5 Ω and 10 Ω in the circuit given below is :
दिये गये परिपथ में 5 Ω तथा 10 Ω में प्रति सेकंड ऊष्मा क्षय का अनुपात है :



(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

Ans. **(1)**

Sol. $P = \frac{V^2}{R}$

$\therefore$ voltage across 5Ω and 10 Ω is same

$P \propto \frac{1}{R}$

$\frac{P_1}{P_2} = \frac{R_2}{R_1} \Rightarrow P_1 : P_2 = 10 : 5$

$P_1 : P_2 = 2 : 1$

40. A particle moves in x-y plane under the influence of a force F such that its linear momentum is $\vec{p}(t) = \hat{i}\cos(kt) - \hat{j}\sin(kt)$. If k is constant, the angle between $\vec{F}$ and $\vec{P}$ will be :

$\vec{F}$ बल के अन्तर्गत एक कण x-y तल में इस प्रकार गति करता है कि इसका रेखीय संवेग $\vec{p}(t) = \hat{i}\cos(kt) - \hat{j}\sin(kt)$ है। यदि k नियतांक हो तो $\vec{F}$ व $\vec{P}$ के बीच का कोण होगा :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. **(3)**

Sol. $\vec{F} = \frac{d\vec{P}}{dt} = (-k \sin kt)\ \hat{i} - (k \cos kt)\ \hat{j}$

$\cos\theta = \frac{\vec{P} \cdot \vec{F}}{|\vec{P}||\vec{F}|}$

$\cos\theta = 0$

$\theta = \pi/2$

41. A man carrying a monkey on his shoulder does cycling smoothly on a circular track of radius 9 m and completes 120 resolutions in 3 minutes. The magnitude of centripetal acceleration of monkey is (in m/s^2) :

(1) zero (2) $57600\pi^2$ ms^{-2} (3) $16\pi^2$ ms^{-2} (4) $4\pi^2$ ms^{-2}

एक व्यक्ति एक बन्दर को अपने कंधो पर बैठाकर 9 m त्रिज्या के वृत्तीय पथ पर सुविधाजनक तरीके से साईकिल चला रहा है तथा 3 मिनट में 120 चक्कर पूरे करता है। बन्दर के अभिकेन्द्र त्वरण का परिमाण है : (m/s^2 में)

(1) शून्य (2) $57600\pi^2$ ms^{-2} (3) $16\pi^2$ ms^{-2} (4) $4\pi^2$ ms^{-2}

Ans. **(3)**

Sol. $\omega = \frac{\Delta\theta}{\Delta t} = \frac{120 \times 2\pi}{3 \times 60} = \frac{4\pi}{3}$ rad/sec.

$a_c = \omega^2 r$

$= \left(\frac{16}{9}\pi^2\right) \times 9$

$= 16\ \pi^2$ m/sec^2

42. Match List-I with List-II :

List-I **EM-Wave**	**List-II** **Wavelength Range**
(A) Infra-red	(I) $< 10^{-3}$ nm
(B) Ultraviolet	(II) 400 nm to 1 nm
(C) X-rays	(III) 1 mm to 700 nm
(D) Gamma rays	(IV) 1 nm to 1^{-3} nm

Choose the correct answer from the options given below :

(1) (A)-(IV), (B)-(III), (C)-(II), (D)-(I) (2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(3) (A)-(III), (B)-(II), (C)-(IV), (D)-(I) (4) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

सूची I का सूची II से मिलान कीजिए।

सूची-I (विद्युतचुंबकीय तरंगे)	**सूची-II (तरंगदैर्ध्य परास)**
(A) अवरक्त	(I) $< 10^{-3}$ nm
(B) पराबैंगनी	(II) 400 nm to 1 nm
(C) X-किरण	(III) 1 mm to 700 nm
(D) गामा किरणें	(IV) 1 nm to 1^{-3} nm

नीचे दिये गये विकल्पों से सही उत्तर चुनिए :

(1) (A)-(IV), (B)-(III), (C)-(II), (D)-(I) (2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(3) (A)-(III), (B)-(II), (C)-(IV), (D)-(I) (4) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Ans. **(3)**

Sol. Wavelength is increasing as we go from gamma rays to infra-red.

43. During an adiabatic process, if the pressure of a gas is found to be proportional to the cube of its absolute temperature, then the ratio of $\frac{C_P}{C_V}$ for the gas is :

रूद्धोष्म प्रक्रम के दौरान किसी गैस का दाब यदि इसके परमताप के घन के अनुक्रमानुपाती हो तब इस गैस के लिए $\frac{C_P}{C_V}$ अनुपात है :

(1) $\frac{9}{7}$ (2) $\frac{5}{3}$ (3) $\frac{7}{5}$ (4) $\frac{3}{2}$

Ans. **(4)**

Sol. $P \propto T^3$

$$\frac{T^3}{P} = \text{constant}$$

$$\Rightarrow \frac{P^3V^3}{P} = P^2V^3 = PV^{3/2} = PV^{\gamma} = \text{constant}$$

$\gamma = 3/2$

44. The electrostatic force $\left(\vec{F}_1\right)$ and magnetic force $\left(\vec{F}_2\right)$ acting on a charge q moving with velocity v can be written :

v वेग से गतिमान एक आवेश q पर लगने वाले स्थिरवैद्युत बल $\left(\vec{F}_1\right)$ तथा चुम्बकीय बल $\left(\vec{F}_2\right)$ को लिखा जा सकता है :

(1) $\vec{F}_1 = q\vec{E}, \vec{F}_2 = q\left(\vec{V}\times\vec{B}\right)$ (2) $\vec{F}_1 = q\vec{B}, \vec{F}_2 = q\left(\vec{B}\times\vec{V}\right)$

(3) $\vec{F}_1 = q\vec{V}.\vec{E}, \vec{F}_2 = q\left(\vec{B}.\vec{V}\right)$ (4) $\vec{F}_1 = q\vec{E}, \vec{F}_2 = q\left(\vec{B}\times\vec{V}\right)$

Ans. **(1)**

Sol. $\vec{F}_1 = q\vec{E}$

$\vec{F}_2 = q\,(\vec{V}\times\vec{B})$

45. A satellite revolving around a planet in stationary orbit has time period 6 hours. The mass of planet is one-fourth the mass of earth. The radius orbit of planet is :
(Given = Radius of geo-stationary orbit of r earth is 4.2×10^4 km)

एक ग्रह के परितः किसी स्थायी कक्षा में परिक्रमण करते उपग्रह का परिक्रमण काल 6 घंटा है। ग्रह का द्रव्यमान पृथ्वी के द्रव्यमान का एक चौथाई है। ग्रह की कक्षा की त्रिज्या है (पृथ्वी की भूस्थरीय कक्षा की त्रिज्या 4.2×10^4 km) :

(1) 8.4×10^4 km (2) 1.05×10^4 km (3) 1.4×10^4 km (4) 1.68×10^5 km

Ans. **(2)**

Sol. $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}\left(\frac{m_2}{m_1}\right)^{1/2}$$

$$\frac{6}{24} = \frac{(r_1)^{3/2}}{(4.2\times10^4)^{3/2}}\left(\frac{m}{m/4}\right)^{1/2}$$

$r_1 = 1.05 \times 10^4$ km

46. The current in an inductor is given by $I = (3t + 8)$ where t is in second. The magnitude of induced emf produced in the inductor is 12 mV. The self-inductance of the inductor _____ mH.
एक प्रेरक में धारा $I = (3t + 8)$ A है, जहाँ t सेकंड में है। प्रेरक में उत्पन्न विद्युत वाहब बल 12 mV है। प्रेरक का स्व–प्रेरकत्व _____ mH है।

Ans. **4**

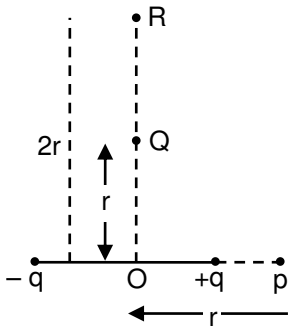
Sol. $\varepsilon = \left|L\frac{di}{dt}\right|$

$12 \times 10^3 = L\,(3)$

$L = 4$ mH

47. The electric field at point P due to an electric dipole is E. The electric field at point R on equatorial line will be $\frac{E}{x}$. The value of x :

वैद्युत द्विध्रुव के कारण बिन्दु P पर वैद्युत क्षेत्र E है। लम्बार्धक रेखा के बिन्दु R पर वैद्युत क्षेत्र $\frac{E}{x}$ है। x का मान :



Ans. **16**

Sol. $|E_P| = \frac{2kP}{r^3}$

$$|E_R| = \frac{kP}{(2r)^3}$$

$$\frac{|E_R|}{|E_P|} = \frac{1}{16} \Rightarrow E_R = \frac{E_P}{16}$$

$x = 16$

48. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 'm' number of turns. It carries a current of 5 A. If the magnitude of the magnetic field inside the solenoid is 6.28×10^{-3} T then the value of m is _____.
0.5 m लम्बाई तथा 1 cm त्रिज्या की एक परिनालिका को 'm' फेरों से बनाया गया है। इसमें प्रवाहित धारा 5 A है। यदि परिनालिका के अन्दर चुम्बकीय क्षेत्र का परिमाण 6.28×10^{-3} T हो तब m का मान _____ है।

Ans. **500**

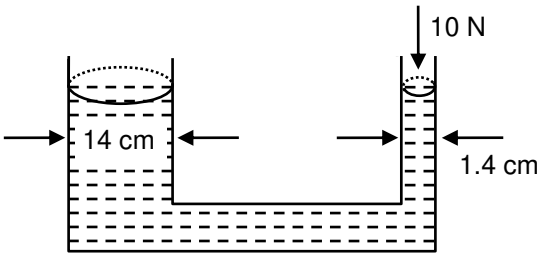
Sol. $\mu_0 ni = 6.28 \times 10^{-3}$

$$\frac{\mu_0 mi}{\ell} = 6.28 \times 10^{-3}$$

$$\frac{4\pi \times 10^{-7} \times m \times 5}{0.5} = 6.28 \times 10^{-3}$$

$\Rightarrow m = 500$

49. A hydraulic press containing water has two arms with diameters as mentioned in the figure. A force of 10 N is applied on the surface of water in the thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is _____ N.
जल युक्त एक हाइड्रोलिक प्रेस में प्रदर्शित चित्र के अनुसार अलग–अलग व्यास की दो भुजाएँ ली गई हैं। पतली भुजा में जल की सतह पर एक 10 N का बल आरोपित किया जाता है। पानी की साम्यावस्था बनाये रखने के लिए चौड़ी भुजा पर आवश्यक आरोपित बल _____ N है।



Ans. **1000**

Sol. $P_1 = P_2$

$$\frac{10}{\pi\left(\frac{1.4}{2}\right)^2} = \frac{F}{\pi\left(\frac{14}{2}\right)^2}$$

$F = 1000$ N

50. The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is _____ m.
किसी स्थान पर एक प्रक्षेप्य की अधिकतम ऊँचाई 64 m है। यदि प्रारम्भिक वेग को आधा कर दिया जाये तो प्रक्षेप्य की नई अधिकतम ऊँचाई _____ m है।

Ans. **16**

Sol. $H_{max.} = \frac{u^2}{2g} = 64$ m

$$H'_{max.} = \frac{(u/2)^2}{2g} = \frac{u^2}{4(2g)} = \frac{64}{4} = 16 \text{ m}$$

PART : CHEMISTRY

51. Match List - I with List - II.

	List - I		**List - II**
(A)	ICl	(I)	T-shape
(B)	ICl_3	(II)	Square pyramidal
(C)	ClF_5	(III)	Pentagonal bipyramidal
(D)	IF_7	(IV)	Linear

Choose the correct answer from the options given below :

(1) (A) – (IV), (B) – (I), (C) – (II), (D) – (III) (2) (A) – (I), (B) – (IV), (C) – (III), (D) – (II)

(3) (A) – (I), (B) – (III), (C) – (II), (D) – (IV) (4) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

Ans. **NTA (1)**

Sol. ICl $\rightarrow$ Linear (diatomic) IV

$ICl_3 \rightarrow$ T-shape (sp^3d 3BP + 2LP) I

$ClF_5 \rightarrow$ Square pyramidal (sp^3d^2 4BP + 2LP) II

$IF_7 \rightarrow$ Pantagonal bipyramidal (sp^3d^2 7BP) III

52. Given below are two statements :

Statement I : The metallic radius of Na is 1.86 A° and the ionic radius of Na^+ is lesser than 1.86 A°.

Statement II : Ions are always smaller ins size than the corresponding elements.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is incorrect but Statement II is true.

(2) Statement I is correct but Statement II is false.

(3) Both Statement I and Statement II are false.

(4) Both Statement I and Statement II are true.

Ans. **NTA (2)**

Sol. Anion is bigger and cation is smaller than corresponding atom.

53. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : NH_3 and NF_3 molecule have pyramidal shape with a lone pair of electrons on nitrogen atom. The resultant dipole moment of NH_3 is greater than that of NF_3.

Reason (R) : In NH_3, the orbital dipole due to lone pair is in the same direction as the resultant dipole moment of the N–H bonds. F is the most electronegative element.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A).

(2) Both (A) and (R) are true but (R) is the correct explanation of (A).

(3) (A) is true but (R) is false.

(4) (A) is false but (R) is true.

Ans. **NTA (2)**

Sol. Theory Based.

54. For the electro cell

$M \mid M^2 \mid\mid X \mid X^{2-}$

If $E^0_{(M^{2+}/M)} = 0.46$ and $E^0_{(X/X^{2-})} = 0.34$ V

Which of the following is correct ?

(1) $M + X \rightarrow M^{2+} + X^{2-}$ is a spontaneous reaction

(2) $E_{cell} = -0.80$ V

(3) $E_{cell} = 0.80$ V

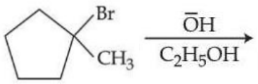
(4) $M^2 = X^{2-} \rightarrow M + X$ is a spontaneous reaction

Ans. **NTA (4)**

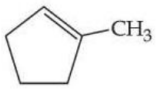
Sol. $E^o_{cell} = E^o_{OP\ M/M^{+2}} + E^o_{RP\ X/X^{-2}}$

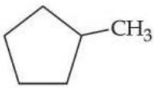
$= -0.46 + 0.34 = -0.12$ V

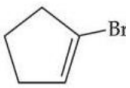
55. Identify the major product in the following reaction.

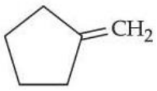


Major Product

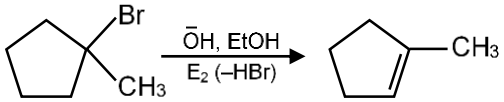
(1) 

(2) 

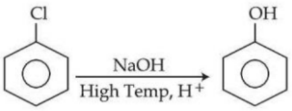
(3) 

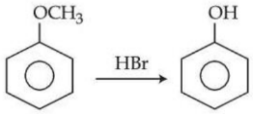
(4) 

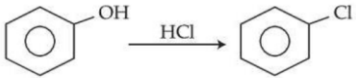
Ans. **NTA (1)**

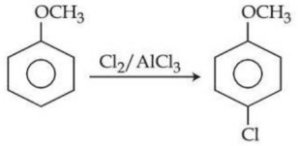
Sol. 

56. Which one of the following reactions is NOT possible ?

(1) 

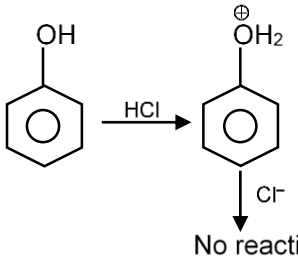
(2) 

(3) 

(4) 

Ans. **NTA (3)**

Sol.



as C–O bond is stronger due to partial double bond character.

57. The metal atom present in the complex MABXL (where A, B, X and L are unidentate ligand and M is metal) involves sp^3 hybridization. The number of geometrical isomers exhibited by the complex is :

(1) 0 (2) 2 (3) 4 (4) 3

Ans. **NTA (1)**

Sol. Tetrahedral complex does not show geomtric isomerism.

58. The number of complexes from the following with no electrons in the t_2 orbital is _____ .

$TiCl_4$, $[MnO_4]^-$, $[FeO_4]^{2-}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$

(1) 3 (2) 4 (3) 1 (4) 2

Ans. **NTA (1)**

Sol. In tetrahedral complex splitting will be invert to octahedral with less energy gap

59. Match list - I with LIst - II.

	List I (Pair of Compounds)	**List II (Isomerism)**
(A)	n-propanol and Isopropanol	(I) Metamarism
(B)	Methaoxypropane and ethoxyethane	(II) Chain Isomerism
(C)	Propanone and Isopentane	(III) Position Isomerism
(D)	Neopentane and Isopentane	(IV) Functional Isomerism

Choose the correct answer from the options given below :

(1) (A) – (III), (B)–(I), (C)–(II), (D)–(IV) (2) (A) – (II), (B)–(I), (C)–(IV), (D)–(IV)

(3) (A) – (I), (B)–(III), (C)–(IV), (D)–(II) (4) (A) – (III), (B)–(I), (C)–(IV), (D)–(II)

Ans. **NTA (4)**

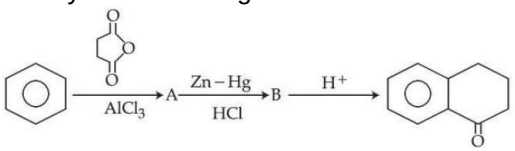
Sol. (A) n-propanol & Isopropanol are position isomers

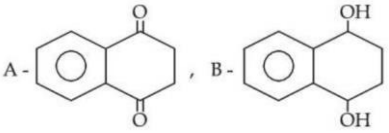
(B) Methoxy propane & ethoxyethane are metamers

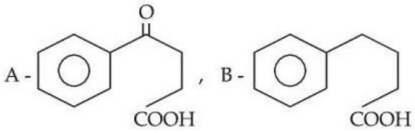
(C) Propanone & propanal are functional isomers

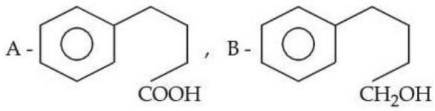
(D) Neopentane & Isopentane are chain isomers

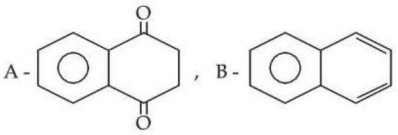
60. Identify and B in the given chemical reaction sequence :



(1) 

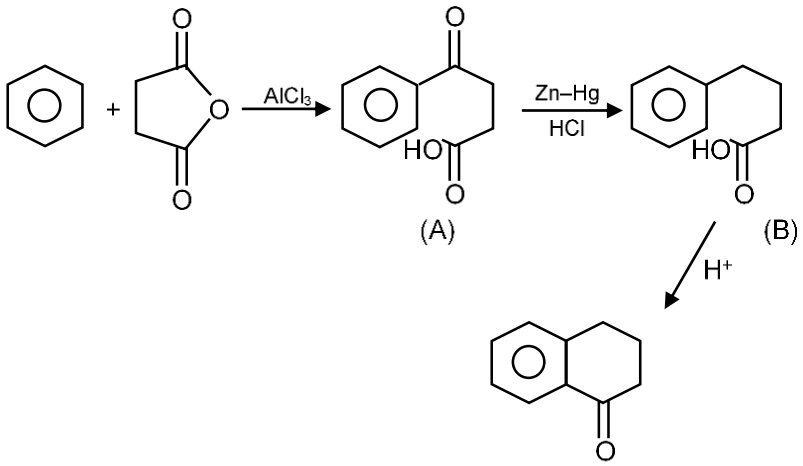
(2) 

(3) 

(4) 

Ans. **NTA (2)**

Sol.



61. While preparing crystals of Mohr's salt, dil H_2SO_4 is added to a mixture of ferrous sulphate and ammonium sulphate, before dissolving this mixture in water, dil H_2SO_4 is added here to :

(1) increase the rate of formation of crystals

(2) prevent the hydrolysis of ferrous sulphate

(3) prevent the hydrolysis of ammonium sulphate

(4) make the medium strongly acidic

Ans. **NTA (2)**

Sol. $Fe(OH)_2\downarrow$ will ppt out.

62. $CH_3CH_2–OH \xrightarrow[\text{(iii) NaOH, CaO, } \Delta]{\text{(i) Jone's Reagent}\ \text{(ii) } KMnO_4} P$

Consider the above reaction sequence and identify the major product P.
(1) Methoxymethane (2) Methanal (3) Methanoic acid (4) Methane

Ans. **NTA (4)**

Sol.

$$CH_3–CH_2–OH \xrightarrow{\text{Jones reagent}} CH_3–CH=O \xrightarrow{KMnO_4} CH_3–COOH \xrightarrow[\Delta]{NaOH,CaO} CH_4$$

63. The number of ions from the following that have the ability to liberate hydrogen from a dilute acid is _____ .

Ti^{2+}, Cr^{2+} and V^{2+}

(1) 3 (2) 2 (3) 1 (4) 0

Ans. **NTA (1)**

Sol. $Ti^{2+} \rightarrow Ti^{+4}$

$Cr^{+2} \rightarrow Cr^{+3}$

$V^{2+} \rightarrow V^{+3}$

They will oxidise to their most stable oxidation state

64. Given below are two statements :

Statement I : On passing $HCl_{(g)}$ through a saturated solution of $BaCl_2$, at room temperature white turbidity appears.

Statement II : When HCl gas is passed through a saturated solution of NaCl, sodium chloride is precipitated due to common ion effect.

In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Statement I is correct but Statement II is incorrect
(2) Both Statement I and Statement II are correct
(3) Statement I is incorrect but Statement II is correct
(4) Both Statement I and Statement II are incorrect

Ans. **NTA (1)**
RESO (2)

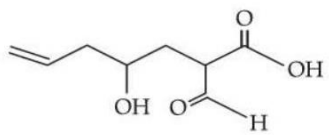
Sol. In both case IP > K_{sp} and ppt will occur.

65. Coagulation of egg, on heating is because of :
(1) Denaturation of protein occurs
(2) The secondary structure of protein remains unchanged
(3) Breaking of the peptide linkage in the primary structure of protein occurs.
(4) Biological property of protein remains unchanged

Ans. **NTA (1)**

Sol. Denaturation of protein is change of secondary & tertiary structure of protein nat result coagulation of albumin of egg.

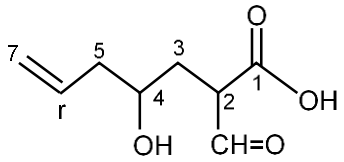
66. The correct nomenclature for the following compound is :



(1) 2–carboxy–4–hydroxyhept–7–enal
(2) 2–carboxy–4–hydroxyhept–6–enal
(3) 2–formyl–4–hydroxyhept–7–enoic acid
(4) 2–formyl–4–hydroxyhept–6–enoic acid

Ans. **NTA (4)**

Sol.



2–formyl–4–hydroxyhept–6–enoic acid

67. The correct statements from the following are :
(A) The decreasing order of atomic radii of group 13 elements is Ti > In > Ga > Al > B.
(B) Down the group 13 electronegativity decreases from top to bottom.
(C) Al dissolves in dil. HCl and liberals H_2 but conc. HNO_3 renders Al passive by forming a protective oxide layer on the surface.
(D) All elements of group 13 exhibits highly stable +1 oxidation state.
(E) Hybridisation of Al in $[Al(H_2O)_6]^{3+}$ ion is sp^3d^2.
(1) (C) and (E) only (2) (A) and (C) only
(3) (A), (C) and (E) only (4) (A), (B) (C) and (E) only

Ans. **NTA (1)**

Sol. (A) atomic radii Al > Ga incorrect.
(B) Incorrect EN Al < Ga.
(C) Correct.
(D) Only for $Tl^{+} > Tl^{+3}$.
(E) Correct.

68. The quantity of silver deposited when one coulomb charge is passed $AgNO_3$ solution :
(1) Chemical equivalent of silver (2) 1 g of silver
(3) 0.1 g atom of silver (4) 1 electrochemical equivalent of silver

Ans. **NTA (4)**

Sol. $Ag^{+}(aq) + e^{-} \longrightarrow Ag(s)$

$$\frac{W}{E} = \frac{Q}{F}$$

$$W = Q\frac{E}{F}$$

Or $W = 1\ Z$

69. The number of moles of methane required to produce 11 g CO_2 (g) after complete combustion is :
(Given molar mass of methane is g mol^{-1} : 16)
(1) 0.5 (2) 0.75 (3) 0.35 (4) 0.25

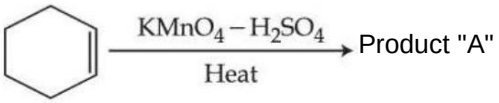
Ans. **NTA (4)**

Sol. $CH_4 + 2O_2 \xrightarrow{\Delta} CO_2 + 2H_2O$

$\frac{1}{4}$ mol $\frac{11}{44}$ mol

$\frac{1}{4}$ mol

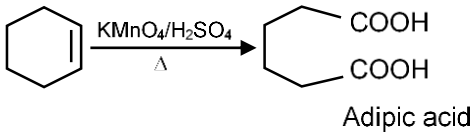
70. Consider the given chemical reaction :



Product "A" is :
(1) oxalic acid (2) acetic acid (3) adipic acid (4) picric acid

Ans. **NTA (3)**

Sol.



Adipic acid

71. Combustion of 1 mole of benzene is expressed at

$C_6H_6(l) + \frac{15}{2} O_2(g) \rightarrow 6\,CO_2(g) + 3\,H_2O(l)$

The standard enthalpy of combustion of 2 mol of benzene is – 'x' kJ.

x = _____.

Given

1. standard Enthalpy of formation of 1 mol of $C_6H_6(l)$, for the reaction
   $6C\ (graphite) + 3H_2(g) \rightarrow C_6H_6(l)$ is 48.5 kJ mol^{-1}.
2. Standard Enthalpy of formation of 1 mol of CO_2 (g), for the reaction
   $C(graphite) + O_2(g) \rightarrow CO_2(g)$ is –393.5 kJ mol^{-1}.
3. Standard and Enthalpy of formation of 1 mol of H_2O (l), for the reaction
   $H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O(l)$ is –286 kJ mol^{-1}.

Ans. **NTA (6535)**

Sol. * = – 1 + 6 × 2 + 3 × 3
= – 48.5 + 6 × (–393.5) + 3 × (–286)
= – 48.5 + 2361 – 858
= – 3267.5
= ≈ 6535 kJ

72. Consider the following single step reaction in gas phase at constant temperature.

$2A_{(g)} + 2B_{(g)} \rightarrow C_{(g)}$

The initial rate of the reaction is recorded as r_1 when the reaction starts with 1.5 atm pressure of A and 0.7 atm pressure of B. After some time, the rate r_2 is recorded when the pressure of C becomes 0.5 atm. The ratio $r_1 : r_2$ is _____ × 10^{-1}. (Nearest integer)

Ans. **NTA (315)**

Sol. **r =** $kP_A^2P_B$

$r_1 = k\,(1.5)^2\,0.7$ ... (1)

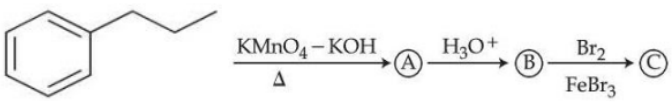
2A(g) +	B(g) →	C(g)
1.5 atm	0.7 atm	0
1.5 – 2 × 0.5	0.7 – 0.5	0.5 atm
= 0.5 atm	= 0.2 atm	

$r_2 = k\,(0.5)^2\,0.2$ ... (2)

(1) / (2)

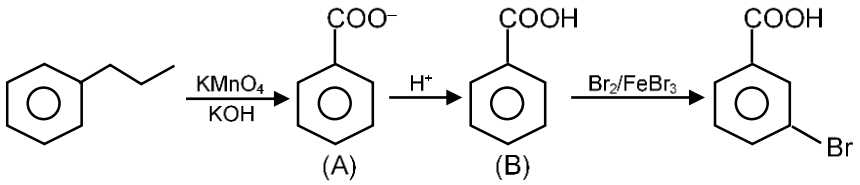
$\frac{r_1}{r_2} = \frac{(1.5)^2 \times 0.7}{(0.5)^2 \times 0.2} = 31.5 = 315 \times 10^{-1}$

73. The product (C) in the following sequence of reactions has _____ π bonds.



Ans. **NTA (4)**

Sol.



74. In the Claisen-Schmidt reaction to prepare 351 g of dibenzalacetone using 87 g of acetone, the amount of benzaldehyde required is _____ g. (Nearest integer)

Ans. **NTA (318)**

Sol. $2Ph-CH=O + CH_3-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{OH^-} Ph-CH=CH-\underset{\underset{O}{\|}}{C}-CH=CH-Ph$

3.0 mol 87gm, 1.5 mol 351gm, 1.5 mol

wt of Ph–CH=O = 3 × 106 = 318 gm

75. The fusion of chromite ore with sodium carbonate in the presence of air leads to the formation of products A and B along with the evolution of CO_2. The sum of spin-only magnetic moment values of A and B is _____ B.M. (Nearest integer)

[Given atomic number : C = 6, Na : 11, O : 8, Fe : 26, Cr : 24]

Ans. **NTA (6)**

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \xrightarrow{\Delta} \underset{(A)}{8Na_2CrO_4} + \underset{(B)}{2Fe_2O_3} + 8CO_2$

For Cr^{+6} (n = 0)

For Fe^{+3} $3d^5$ (n = 5)

$= \sqrt{n(n+2)}$ BM

$= \sqrt{35}$ BM $\approx$ 6 BM