JEE MAIN - 12 **FULL SYLLABUS** **Date : 24-02-2025**

SOLUTION & ANSWER KEY

KEY SHEET

PHYSICS

1)	1	2)	2	3)	1	4)	4	5)	3
6)	4	7)	3	8)	1	9)	1	10)	4
11)	4	12)	4	13)	2	14)	1	15)	2
16)	1	17)	2	18)	3	19)	2	20)	4
21)	5	22)	6	23)	161	24)	625	25)	5

CHEMISTRY

26)	4	27)	3	28)	3	29)	3	30)	3
31)	3	32)	3	33)	3	34)	1	35)	3
36)	4	37)	3	38)	3	39)	1	40)	2
41)	1	42)	1	43)	1	44)	1	45)	3
46)	7	47)	5	48)	1	49)	8	50)	123

MATHEMATICS

51)	3	52)	2	53)	1	54)	1	55)	2
56)	4	57)	1	58)	2	59)	1	60)	1
61)	4	62)	2	63)	1	64)	1	65)	3
66)	1	67)	1	68)	3	69)	4	70)	1
71)	4	72)	3	73)	3	74)	38	75)	167

SOLUTIONS

PHYSICS

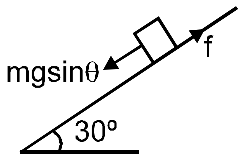
1. Sol. Conceptual

2. Sol. $P_i = P_f$

or $0 = 20(0.7 - v) - 50v$

or $v = 0.2 \text{ m/s}$

3. Sol.



Since, $\mu > \tan\theta$; the block will not slide.

Therefore, $f = mg\sin\theta = 2 \times 9.8 \times \frac{1}{2} = 9.8 \text{ N}$

4. Sol. Statement I:

$$v_{max} = \sqrt{\mu Rg} = \sqrt{(0.2) \times 2 \times 9.8}$$

$$v_{max} = 1.97 \text{ m/s}$$

$$7 \text{ km/h} = 1.944 \text{ m/s}$$

Speed is lower than v_{max}, hence it can take safe turn.

Statement II

$$v_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

$$= \sqrt{2 \times 9.8\left[\frac{1 + 0.2}{1 - 0.2}\right]} = 5.42 \text{ m/s}$$

$$18.5 \text{ km/h} = 5.14 \text{ m/s}$$

Speed is lower than v_{max}, hence it can take safe turn.

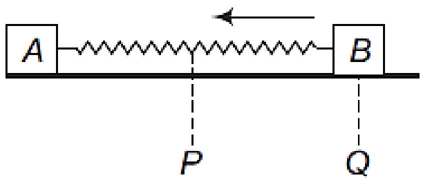
5. Sol. Velocity of the ball on striking $= \sqrt{2gh}$

After that ball goes to height less than (h) due to inelastic collision $= \sqrt{2g}(h - d)$

$$\therefore \sqrt{2g(h-d)} = \sqrt{2gh}e \Rightarrow h - d = e^2 h \Rightarrow \frac{h}{d} = \frac{1}{1 - e^2}$$

6. Sol. $P \rightarrow$ compressed state of spring

$Q \rightarrow$ natural length of spring



From P to Q

$$a_B = \frac{Kx}{m_B}$$

$$a_{CM} = \frac{m_B a_B}{m_A + m_B} = \frac{(m_B)(Kx)}{m_A + m_B}$$

From P to Q, compression x decreases. Therefore, a_{CM} decreases. After Q, A leaves contact with wall, spring comes in its natural length. Net force on system becomes zero. Therefore, a_{CM} becomes zero.

From P to Q, velocity of B, therefore velocity of COM will increase. After that a_{CM} becomes zero. Therefore, v_{CM} becomes constant.

7. Sol. Using conservation of angular momentum

$$\left(\mathrm{Mr}^2\right)\omega = \left(\mathrm{Mr}^2 + 2\mathrm{mr}^2\right)\omega'$$

$$\omega' = \frac{M\omega}{M + 2m}$$

8. Sol. Linear mass density $\left(\frac{m}{l}\right) = 0.2\mathrm{gm}^{-1}$

$= 0.2 \times 10^{-3}\,\mathrm{Kgm}^{-1}$

$B = 1\ \mathrm{T};\ g = 10\ \mathrm{ms}^{-2}$

$$\mathrm{F} = \mathrm{BI}\ell$$

$$\mathrm{F} = \mathrm{mg}$$

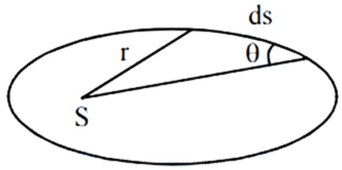
$$\therefore \mathrm{mg} = \mathrm{BI}\ell$$

$$I = \frac{mg}{B\ell} = \frac{0.2 \times 10^{-3} \times 10}{1} = 2 \times 10^{-3}$$

$$\mathrm{I} = 2\ \mathrm{mA}$$

9. Sol. Higher density means higher mass for same shape and size. Higher mass means higher moment of inertia for same shape and size.

10. Sol.



$$dA = \frac{1}{2} r\, d\ell = \frac{1}{2} r\, ds \sin\theta$$

$$\frac{dA}{dt} = \frac{1}{2} r \sin\theta \frac{ds}{dt} = \frac{Vr \sin\theta}{2}$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{mVr\sin\theta}{m} = \frac{L}{2\,m}$$

11. Sol. $K = \frac{AY}{\ell},\ K' = \frac{4AY}{\ell/2} = 8K$

$$\frac{U}{2} = \frac{\frac{1}{2} \times 8\ K \times \Delta\ell^2}{\frac{1}{2} \times K \times \Delta\ell^2} \Rightarrow\ U = 16\ J$$

12. Sol. R is the radius of bigger drop is the radius of *n* water drops. Water drops are combined to make bigger drop. So, Volume of *n* drops = volume of bigger drop

$$\Rightarrow n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \Rightarrow R = rn^{1/3} \Rightarrow n = \left(\frac{R}{r}\right)^3$$

$$\Delta U = T\left(Change\,in\,surface\,area\right) \Rightarrow \Delta U = T\left(n4\pi r^2 - 4\pi R^2\right)$$

$$\Rightarrow \Delta U = 4\pi T\left[\left(\frac{R}{r}\right)^3 r^2 - R^2\right] \Rightarrow \frac{4\pi T\left(\frac{R^3}{r} - R^2\right)}{J}$$

$$\Rightarrow \frac{\Delta U}{V} = \frac{4\pi T\left(\frac{R^3}{r} - R^2\right)}{J \times \frac{4}{3}\pi R^3} = \frac{3\ T}{J}\frac{1}{r} - \frac{1}{R}\Big]$$

$$x = 12\sin\omega t - 16\sin^3\omega t$$

13. $= 4\left[\sin 3\omega t\right]\left[by\,u\sin g \sin 3\theta = 3\sin\theta - 4\sin^3\theta\right]$

$$\therefore \left[\max imum\,accelaration\ A_{max} = (3\omega)^2 \times 4 = 36\chi\omega^2\right]$$

14. Sol. As $\frac{5\lambda}{2} = 20 \Rightarrow\ \lambda = 8$ cm

$$K = \frac{2\pi}{\lambda} = \frac{314}{4}$$

$$\omega = KV = \frac{2\pi}{8\times10^{-2}}\times 350 = 27475$$

$$\therefore y = 0.05\ sin\left(\frac{314}{4}x - 27475t\right)$$

15. Sol. $H = \sigma eAT^4 \Rightarrow H \propto A \propto r^2$

$$C = \frac{\sigma eA}{ms}\left(4Ts^3\Delta T\right) \Rightarrow C \propto \frac{A}{m} \propto \frac{r^2}{r^3} \propto \frac{1}{r}$$

16. Sol. Stages 1 and 2 are at same temperature also stages 4 and 5 are at same temperature.
As, V_P is more at higher temperature and same at all stages at equal temperature.

$\therefore\ V_{P3} > V_{P1} = V_{P2} > V_{P4} = V_{P5}$

(As $T_3 > T_2 = T_1 > T_4 = T_5$)

Hence (A).

17. Sol.

As $\sigma_1 = \sigma_2$ (Given) $\therefore \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}$, or $\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2}$ [Let r_1 and r_2 be two different radii]

Then the ratio of electric field intensities near the surface of spherical conductor,

$$\frac{E_1}{E_2} = \frac{q_1}{4\pi\varepsilon_0 r_1^2}\times\frac{4\pi\varepsilon_0 r_2^2}{q_2} = \frac{q_1}{q_2}\times\frac{r_2^2}{r_1^2} = 1 \text{ or } E_1 = E_2$$

18. Sol. The magnetic intensity H is $H = nI = 1000\times2.0 = 2\times10^3$ A / m.
The magnetic field B is given by

$$B = \mu_r\mu_0 H = 400\times4\pi\times10^{-7}\left(\text{ N / A}^2\right)\times2\times10^3(\text{ A / m}) = 1.0\text{ T}$$

Magnetisation is given by $M = \left(B - \mu_0 H\right)/\mu_0 \approx 8\times10^5$ A / m

19. Sol. $I = I_1 \sin\omega t + I_2 \cos\omega t$

$$\therefore I_0 = \sqrt{I_1^2 + I_2^2}$$

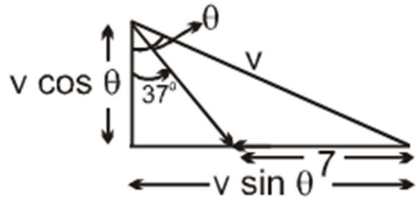
$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

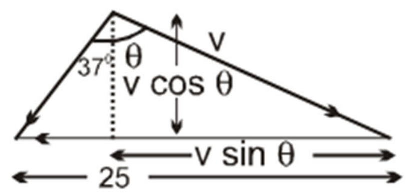
20. Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$.

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{\lambda_e}{\lambda_P} = \sqrt{\frac{m_P}{m_e}} = \sqrt{1831.4} = 42.79$$

21. Sol. Let $v =$ actual velocity of rain and $\theta =$ its angle with vertical: -





In fig. (A), $v\sin\theta = 7 + v\cos\theta\tan 37^\circ = 7 + \frac{3}{4}v\cos\theta$

$\Rightarrow 4v\sin\theta - 3v\cos\theta = 28$ (1)

In fig. (B), $25 = v\sin\theta + v\cos\theta\tan 37^\circ = v\sin\theta + \frac{3}{4}v\cos\theta$

$\Rightarrow 4v\sin\theta + 3v\cos\theta = 100$ (2)

Solving (1) and (2); we get $v = 20\ \text{m/s}$ and $\theta = 53^\circ$

22. Sol. Let's say the compression in the spring by: y . So, by work energy theorem we have

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$$

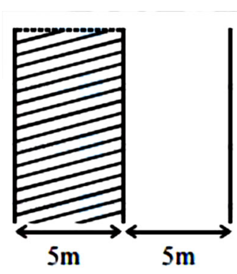
$$\Rightarrow y = \sqrt{\frac{m}{k}} \cdot v$$

$$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$$

$$\Rightarrow y = 2\ \text{m}$$

$$\Rightarrow \text{ final length of spring} = 8 - 2 = 6\ \text{m}$$

23. Sol.



$\text{A} = 100\ \text{m}^2$

Using $C = \frac{k \in_0 A}{d}$

$$C_1 = \frac{10\epsilon_0(100)}{5} = 200\epsilon_0$$

$$C_2 = \frac{\epsilon_0(100)}{5} = 20\epsilon_0$$

C_1 & C_2 are in series so $C_{eqv.} = \frac{C_1C_2}{C_1+C_2} = \frac{4000\epsilon_0}{220} = 160.9 \times 10^{-12} \simeq 161 pF$

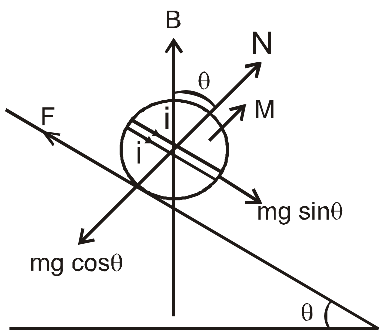
24. Sol. $i = 10 \text{ A}, \ A = 5 \text{ mm}^2 = 5 \times 10^{-6} \text{ m}^2$

and $v_d = 2 \times 10^{-3} \text{ m/s}$

We know, $i = neAv_d$

$\therefore 10 = n \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3} \Rightarrow n = 0.625 \times 10^{28} = 625 \times 10^{25}$

25. Sol.



$M = L \times 2R \times i$

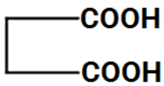
$MB \sin\theta = mg \sin\theta\, R$

$$i = \frac{\text{mg}}{2\text{BNI}} = 2.5 \text{ A}$$

CHEMISTRY

26. Sol. Relatively lowering will be minimum for $\left[Co\left(NH_3\right)_3Cl_3\right]$ as i = 1 and hence it will have maximum vapour pressure.

27. Sol. In the presence of Zn, only Br_2 can be removed, not as HBr In the presence of $\left(CH_3\right)_2S$



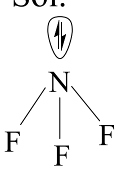
can not be formed (reductive ozonolysis)

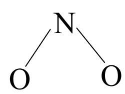
28. Sol. $PV^{\gamma} = \text{constant}$

$$\frac{dP}{dV} = -\gamma\frac{P}{V} \Rightarrow \frac{dP}{P} = -\gamma\frac{dV}{V}$$

29. Sol. Number of radial node in $3p$ orbital $= 3-1-1 = 1$ is its radial probability curve will meet the x- axis only at one point and this is the case in curve (A) and (C) is either for $3p$ or $2s$ orbital. Also, as the shell number increases, the distance of maximum probability increases from the nucleus. Hence (C) is correct.

30.. Sol.





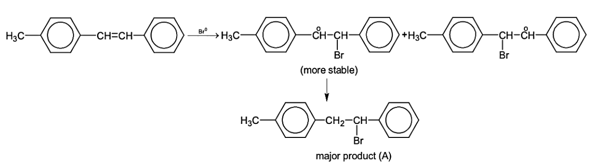
sp^2

pyramidal

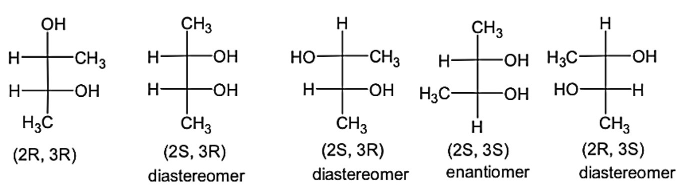
sp^3

31. Sol. Due to steric hindrance in II the lone pair e^- are not involved in resonance in, I electron withdrawing effect of NO_2 is high.

32. Sol. It is a case of free radical addition to $C=C$ bond.



33. Sol.



34. Sol. $NH_4CN(s) \rightleftharpoons \underset{P}{NH_3(g)} + \underset{P}{HCN(g)}$

At eq.

$\therefore$ Total pressure $= 2P = 0.298$ atm $p = 0.149$

Also $K_P = P^1_{NH_3} \times P^1_{HCN} = 0.149 \times 0.149 = 0.022\ atm^2$

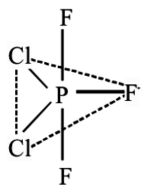
Now $P_{NH_3} = 0.25$ atm

$NH_4CN(s) \rightleftharpoons NH_3(g) + HCN(g)$

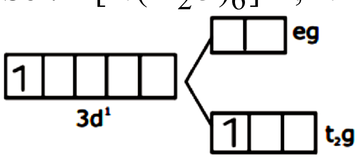
Initial	0.25	0
At eq	$0.25 + P^1$	P^1

$K_P = P^1\left(0.25 + P^1\right)$

$P^1 = 0.0694$ atm

35. Sol. 

36. Sol. $[Ti(H_2O)_6]^{3+}$, Ti^{3+}, $3d^1$ in octahedral field of ligand



$CFSE = -0.4\Delta_0$ $\quad CFSE = \dfrac{-0.4 \times 20300}{83.7} = 97$ kJ mol

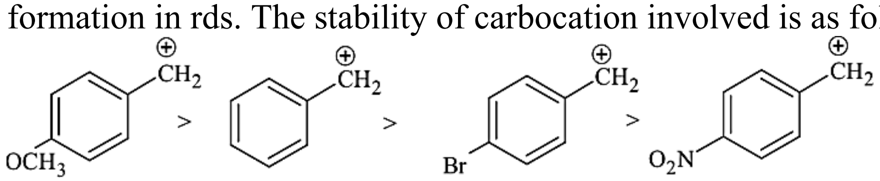
37. Sol. Acid base reaction takes place from stronger to weaker side

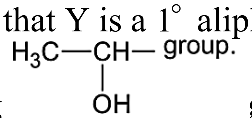
38. Sol. Theoretical

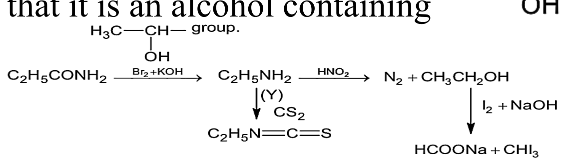
39. Sol. Solution will make buffer before equivalence point since before equivalence point both HA and NaA are present

40. Sol. Factual

41. Sol. Substitution of $-OH$ by Br atom takes place by carbocationic intermediate formation in rds. The stability of carbocation involved is as follows:



42. Sol. Reaction of X with $Br_2 + KOH$ suggests that X is an amide. Evolution of N_2 and formation of alcohol suggests that Y is a 1° aliphatic amine. Iodoform test of Z suggests that it is an alcohol containing  group.

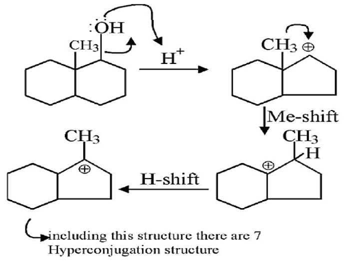


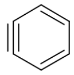
43. Entropy increases due to formation of more number of moles of NH_3 gas

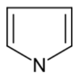
44. + Inductive Effect increases acidic strength decreases.

45. Sol. Electronic transition is $5 \rightarrow 2$ $\quad z = 3$

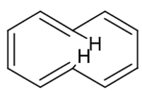
$$\Delta L = \frac{\Delta nh}{2\pi} = \frac{3h}{2\pi}$$

46. Sol. 

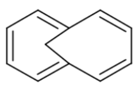
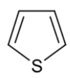
47. Sol.  $8\pi e^-$. satisfying $(4n+2)\pi e^-$ rule $\Rightarrow$ only $6\pi e^-$ are in one plane and in cyclic conjugate. Therefore, aromatic.

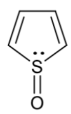


$6\pi e^-$ satisfying $(4n+2)\pi e^-$ rule $\Rightarrow$ aromatic



Due to repulsion between internal hydrogen atom ⇒ non aromatic

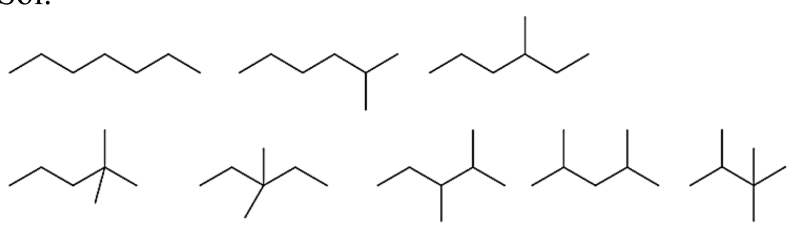
 



Non aromatic because cyclic delocalisation of πe^- is abscent.

48. Sol. EDTA is hexa-dentate ligand

49. Sol.



50. Sol. $\rightarrow v = \left(\frac{2\pi Ke^2}{h}\right) \times \frac{Z}{n}$, so, atomic no. $(Z) \propto$ slope of the graph.

So, statement (1) is True.

$\rightarrow \lambda = \frac{h}{\sqrt{2qVm}}$, slope of the graph $\propto \frac{1}{\sqrt{m}}$

So, statement (2) is True.

$\rightarrow$ From, $PV = nRT$ $V = \left(\frac{nR}{P}\right)T$, so, slope of graph $\propto \frac{1}{P}$

So, statement (3) is also correct. $\rightarrow P = (nRT)\frac{1}{V}$, so, slope $\propto T$

So, statement (4) is incorrect.

MATHEMATICS

51. Sol. Point on $L_1(\lambda+3, 3\lambda-1, -\lambda+6)$

Point on $L_2(7\mu-5, -6\mu+2, 4\mu+3)$

$\Rightarrow \lambda+3 = 7\mu-5$

$3\lambda-1 = -6\mu+2$ .... (1)

Point on $(2,-4,7)$ .... (2)

$\Rightarrow \lambda=-1, \mu=1$

Reflection is $(2,-4,-7)$

52. Sol. Let $x=\frac{1-u}{1+u} \therefore I=\int_0^1 \frac{\ln 2-\ln(1+u)}{1+u^2}dx \Rightarrow 2I=\ln 2\left(\tan^{-1}u\right)_0^1 \Rightarrow I=\frac{\pi \ln 2}{8}$

53. Sol. We have, $A^2-5A=-7I$

$\Rightarrow AAA^{-1}-5AA^{-1}=-7IA^{-1} \Rightarrow AI-5I=-7A^{-1}$

$\Rightarrow A^{-1}=\frac{1}{7}(5I-A)$

Also, $A^3-2A^2-3A+I=A(5A-7I)-2A^2-3A+I$

$=5A^2-7A-2A^2-3A+I=3A^2-10A+I$

$=3(5A-7I)-10A+I=5A-20I=5(A-4I)$

54. Sol. $(xy+1)\left(3x^2ydx-x^3dy\right)=\left(x^6+y^2\right)(xdy+ydx)$

$\Rightarrow (xy+1)d\left(\frac{x^3}{y}\right)=\left(\left(\frac{x^3}{y}\right)^2+1\right)d(xy)$

55. Sol. Analyse by graph

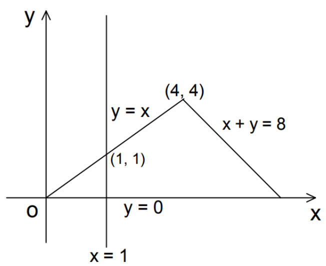
56. Sol. $\lim\limits_{x\to 0}\frac{x\tan^2 2x}{\tan 4x\sin^2 x}=\lim\limits_{x\to 0}\frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2}=1$

57. Sol. Let the sum of n terms exceed 1000.

$\therefore \frac{1\left(1-3^n\right)}{1-3}>1000 \Rightarrow 3^n>2001$

$\therefore n=7,8,9\ldots\ldots$

58. Sol.



59. Sol. $u \times v = (a-b)\times(a+b) = 2a \times b$

$\therefore |u \times v| = 2|\vec{a} \times b| = 2\sqrt{\left[a^2b^2\sin^2\theta\right]} = 2\sqrt{\left(a^2b^2 - a^2b^2\cos^2\theta\right)} = 2\sqrt{\left[16-(a-b)^2\right]}$

60. Sol. $\because$ sec x is even function and $\log\left(x+\sqrt{1+x^2}\right)$ is odd function, therefore given function is even function.

61. Sol. $A^3 = A^2 \cdot A = (2I - A)A = 2A - A^2 = 2A - (2I - A) = 3A - 2I$

$A^4 = A^3 \cdot A = (3A - 2I)A = 3A^2 - 2A = 3(2I - A) - 2A = 6I - 5A$

$A^5 = (6I - 5A)A = 6A - 5A^2 = 6A - 5(2I - A) = 11A - 10I$

$\Rightarrow P = 11$ and $K = -10$

62. Sol. $\cot\theta - \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos2\theta}{\sin\theta\cos\theta} \Rightarrow \dfrac{\cos2\theta}{\sin\theta\cos\theta} - \dfrac{\sin2\theta}{\cos^2\theta} = \dfrac{\cos3\theta}{\cos^2\theta\sin\theta}$

Similarly, $\dfrac{\cos6\theta}{\cos^5\theta\sin\theta} - \dfrac{\sin6\theta}{\cos^6\theta} = \dfrac{\cos7\theta}{\cos^6\theta\sin\theta}$

63. Sol. Using A.M.-G.M. inequality

$\dfrac{e^x + 2e^{-x}}{2} \geq \sqrt{e^x \cdot 2e^{-x}}$.

Thus $e^x + 2e^{-x} \geq 2\sqrt{2}$. Then $\dfrac{1}{e^x + 2e^{-x}} \leq \dfrac{1}{2\sqrt{2}}$

As $\dfrac{1}{e^x + 2e^{-x}}$ is always positive, we have $0 < \dfrac{1}{e^x + 2e^{-x}} \leq \dfrac{1}{2\sqrt{2}}$

Observe that $f(0) = 1/3$. Thus such that $f(c) = 1/3$.

Using extreme-value theorem, we can say that as f is continuous, f will attain a value $1/3$ at some point. Here we are able to identify the point as well.

64. Sol. (a): For R_1 : Let $a = 2+\sqrt{2}, b = 2-\sqrt{2}, c = 2^{5/4}$

$\therefore\ aR_1b \Rightarrow a^2 + b^2 = (2+\sqrt{2})^2 + (2-\sqrt{2})^2 = 12 \in Q$

$bR_1c \Rightarrow b^2 + c^2 = (2-\sqrt{2})^2 + \left(2^{5/4}\right)^2 = 6 \in Q$

$aR_1c \Rightarrow a^2 + c^2 = (2+\sqrt{2})^2 + \left(2^{5/4}\right)^2 = 6 + 8\sqrt{2} \notin Q$

$\therefore\ R_1$ is not transitive

For R_2 Let $a = 2^{3/2}, b = 3^{3/4}, c = 1$

Then, $aR_2b \Rightarrow a^2 + b^2 = \left(2^{3/2}\right)^2 + \left(3^{3/4}\right)^2 = 8 + 3\sqrt{3} \notin Q$

$bR_2c \Rightarrow b^2 + c^2 = \left(3^{3/4}\right)^2 + (1)^2 = 3\sqrt{3} + 1 \notin Q$

$aR_2c \Rightarrow a^2 + c^2 = \left(2^{3/2}\right)^2 + (1)^2 = 8 + 1 = 9 \in Q$

$\therefore$ R_2 is not transitive.

65. Sol. : $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

Adding, $2I = \int_{\pi/6}^{\pi/3} \left(\frac{1}{1+\sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} \right) dx$

$= \int_{\pi/6}^{\pi/3} 1 \cdot dx = \left(\frac{\pi}{3} - \frac{\pi}{6} \right) = \frac{\pi}{6} \therefore I = \frac{\pi}{12}$

Again Statement-II is true.

66. Sol. Let a tangent to the parabola be

$$y = mx + \frac{\sqrt{5}}{m} \ (m \neq 0)$$

As it is a tangent to the circle $x^2 + y^2 = 5/2$, we have

$$\left(\frac{\sqrt{5}}{m} \right) = \frac{\sqrt{5}}{\sqrt{2}} \sqrt{1+m^2} \Rightarrow \left(1+m^2\right)m^2 = 2$$

which gives $m^4 + m^2 - 2 = 0 \Rightarrow \left(m^2 - 1\right)\left(m^2 + 2\right) = 0$

As $m \in R, m^2 = 1 \therefore m = \pm 1$

Also $m = \pm 1$ does satisfy $m^4 - 3m^2 + 2 = 0$

Hence common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$.

67. Sol. Any point on the parabola $y^2 = 4ax$ is $\left(a^2, 2at\right)$

Equation of the chord of the ellipse $\frac{x^2}{2a^2} + \frac{y^2}{a^2} = 1$

Whose mid-point is (at $, 2at)$ is $T = S_1$

$\Rightarrow x\frac{at^2}{2a^2} + y\frac{2at}{a^2} = \frac{a^2t^4}{2a^2} + \frac{4a^2t^2}{a^2} \Rightarrow tx + 4y + at^3 + 8at$

As it passes through $\left(20a, -\frac{a^2}{2}\right) \Rightarrow 20at + 4\left(-\frac{a^2}{2}\right) = at^3 + 8at$

$\Rightarrow at^3 - 12at + 2a^2 = 0 \Rightarrow t^3 - 12t + 2a = 0 (a \neq 0)$

Now three should be 3 distinct values of t for three distinct chords

Let $f(t) = t^3 - 12t + 2a; f'(t) = 3t^2 - 12 = 0 \Rightarrow t = \pm 2$

So $f(2) f(-2) < 0 \Rightarrow (a+8)(a-8) < 0$

$\therefore 14$ values of a are possible $(a \neq 0)$

68. Sol. Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Which gives $\alpha + \delta = 0$ and $\alpha^2 + \beta\gamma = 1$

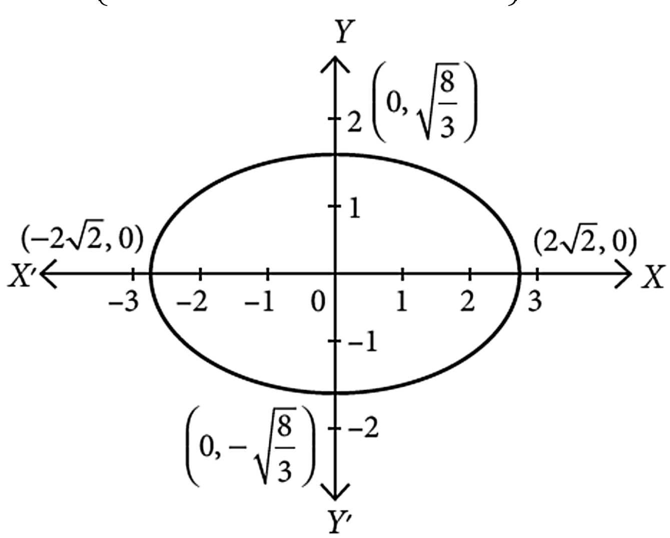
So we have $\text{Tr}(A) = 0$

$\det A = \alpha\delta - \beta\gamma = -\alpha^2 - \beta\gamma = -\left(\alpha^2 + \beta\gamma\right) = -1$

Thus statement-1 is true but statement-2 is false.

69. Sol. Given $R = \left\{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\right\}$

$\Rightarrow R = \left\{(x, y) : x, y \in Z, \dfrac{x^2}{8} + \dfrac{y^2}{8/3} \leq 1\right\}$



Now, domain of R^{-1} = Range of $R = \{-1, 0, 1\}$.

70. Sol. (a): We have, $A = \{m \in R :$ Both the roots of $x^2 - (m+1)x + m + 4 = 0$ are real $\}$

$\because$ Roots of $x^2 - (m+1)x + m + 4 = 0$ are real

$\therefore D \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0 \Rightarrow m^2 + 1 + 2m - 4m - 16 \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0 \Rightarrow m^2 - 5m + 3m - 15 \geq 0$

$\Rightarrow (m-5)(m+3) \geq 0 \Rightarrow m \in (-\infty,-3] \cup [5,\infty)$

$\Rightarrow A = (-\infty,-3] \cup [5,\infty)$ and $B = [-3,5)$

So, $A - B = (-\infty,-3) \cup [5,\infty)$

$A \cap B = \{-3\}$

$B - A = (-3,5)$

$A \cup B = (-\infty,-3] \cup [5,\infty) \cup [-3,5) = (-\infty,\infty) = R$

71. Sol.

Class	Mid values (x_i)	f_i	$f_i x_i$	$f_i x_i^2$
$10-20$	15	2	30	450
$20-30$	25	x	$25x$	$625x$
$30-40$	35	2	70	2450
		$N = \Sigma f_i$ $= 4 + x$	$\Sigma f_i x_i =$ $100 + 25x$	$\Sigma f_i x_i^2 =$ $2900 + 625x$

Here, $\overline{x} = \dfrac{\Sigma f_i x_i}{N} = \dfrac{100+25x}{4+x} = \dfrac{25(4+x)}{4+x} = 25$

Variance, $\left(\sigma^2\right) = \dfrac{\Sigma f_i x_i^2}{N} - (x)^2$

$= \dfrac{2900+625x}{4+x} - 625 = \dfrac{2900+625x-2500-625x}{4+x} = \dfrac{400}{4+x}$

72. Sol. Here, $p = \dfrac{1}{10}, q = \dfrac{9}{10}$ $\therefore$ $P($ not hitting in n trials $) = \left(\dfrac{9}{10}\right)^n \Rightarrow P($ hitting the target atleast once$) = 1 - \left(\dfrac{9}{10}\right)^n > \dfrac{1}{4} \Rightarrow \left(\dfrac{9}{10}\right)^n < \dfrac{3}{4}$, which is true for $n = 3$

So, least number of shots required is 3 .

73. Sol. Considering $[2x] = 0,1,2,3,4$ and 5 and corresponding $[y] = 5,4,3,2,1$, 0, we get six squares each of area $= 0.5$ sq. units

Hence, total area $= 6 \times 0.5 = 3$ sq. units

74. Sol. WLOG let E be be between C & D (as in solution

Assume $AD = 3$ m. We use power of a point to get that $AG = DE = \sqrt{2}m$ and

$AB = AG + GB = AG + BE = 10 + 2\sqrt{2}m$

Since now we have $AC = 10$, $BC = 20, AB = 10 + 2\sqrt{2}m$ in triangle ΔABC and cevian $AD = 3$ m. Now, we can apply Stewart's Theorem.

$$2000+180m^2=10(10+2\sqrt{2}m)^2+1000$$
$$1000+180m^2=1000+400\sqrt{2}m+80m^2$$
$$100m^2=400\sqrt{2}m$$

75. Sol. We know that α,β,γ are the roots of

$$x(x-200)(x+1/4)-1/4=x^3-\frac{799}{4}x^2-50x-\frac{1}{4}$$

By Vieta's formulas, we have:

$$\alpha+\beta+\gamma=\frac{799}{4}$$
$$\alpha\beta+\beta\gamma+\gamma\alpha=-50$$
$$\alpha\beta\gamma=\frac{1}{4}$$

Now, by tangent addition formulas, we have $\tan(\omega)=\dfrac{\alpha+\beta+\gamma-\alpha\beta\gamma}{1-\alpha\beta-\beta\gamma-\gamma\alpha}$. Substituting Vieta's formulas, we obtain

$$\tan(\omega)=\frac{\frac{799}{4}-\frac{1}{4}}{1-(-50)}=\frac{\frac{798}{4}}{51}=\frac{133}{34}.$$

Therefore, our answer is $133+34=167$ and we are done.