Date : 02-12-2024

JEE ADVANCED - 4

SOLUTION

Paper - 1 & Paper - 2

Paper 1

Physics

1. *(b, c)* Electric field is perpendicular to length of wire at every point, so (b) is correct but (d) is not correct because electric field on the surface parallel to X-axis is not perpendicular to the surface.

Flux, $\phi = \frac{q_{\text{enclosed}}}{\varepsilon} = \frac{\lambda l}{\varepsilon} = \frac{\lambda \times 1}{\varepsilon} = \frac{\lambda}{\varepsilon}$

2. *(a,b)* Time constant of given L-R circuit is

$$\tau = \frac{L}{R_2} = \frac{0.4}{2} = 0.2 \text{ s}$$

Steady state current in inductor,

$$i_0 = \frac{E}{R_2} = \frac{12}{2} = 6 \text{ A}$$

So, current in inductor,

$$i = i_0(1 - e^{-t/\tau})$$
$$= 6(1 - e^{-t/0.2})$$
$$= 6(1 - e^{-5t}) \text{ A}$$

Potential drop across inductor,

$$V_2 = L\frac{dI}{dt} = (0.4)(30)\,e^{-5t} = 12e^{-5t} \text{ V}$$

3. *(a, b)* $T = 27°\text{C} = 300 \text{K}$

Let, $m =$ Mass of neon gas in the mixture, then

Mass of argon $= 28 - m$

Number of gram moles of neon $= \frac{m}{20} = n_1$

Number of gram moles of argon $= \frac{28 - m}{40} = n_2$

Total pressure of mixture, $p =$ Partial pressure of neon + Partial pressure of argon

$$\therefore \quad p = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V}$$
$$= (n_1 + n_2)\frac{RT}{V}$$

Substituting the given values, we have

$$1 \times 10^5 = \left(\frac{m}{20} + \frac{28 - m}{40}\right)\left(\frac{8.314 \times 300}{0.02}\right)$$

$\Rightarrow \quad m = 4.074 \text{ g}$

So, mass of neon $= 4.074$ g

Mass of argon $= 28 - m = 23.926$ g

4. *(b)* Energies of photons are, $E_\lambda = \frac{12375}{\lambda(\text{Å})}$ eV

$$\therefore \quad E_1 = \frac{12375}{4144} = 2.99 \text{ eV}$$

Similarly, $E_2 = 2.49$ eV

and $E_3 = 1.99$ eV

As only E_1 and E_2 are greater than work function, only these can emit electrons.

Now, intensity available for each wavelengths,

$$I' = \frac{I}{3} = \frac{3.6 \times 10^{-3}}{3}$$
$$= 1.2 \times 10^{-3} \text{ W/m}^2$$

Energy incident on given area ($1 \text{ cm}^2 = 10^{-4} \text{ m}^2$),

$$E = 1.2 \times 10^{-3} \times 10^{-4} = 1.2 \times 10^{-7} \text{ J}$$

Number of photons,

$$N = \frac{\text{Energy}}{\text{Energy of 1 photon}}$$

So, $N_1 = \frac{1.2 \times 10^{-7}}{2.99 \times 1.6 \times 10^{-19}}$

$= 2.5 \times 10^{11}$ per second

and $N_2 = \frac{1.2 \times 10^{-7}}{2.49 \times 1.6 \times 10^{-19}}$

$= 3 \times 10^{11}$ per second

So, number of photons = number of electrons

$= N_1 + N_2 = 2.5 \times 10^{11} + 3 \times 10^{11}$ per second

In 2 s, emitted electrons $= 2 \times 5.5 \times 10^{11}$

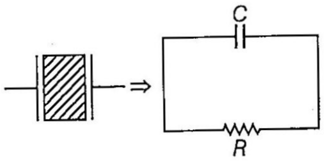
$= 11 \times 10^{12}$ electrons

Problem Solving Tactics

First calculate the energy of each beam using formula $\frac{12375}{\lambda(\text{Å})}$ eV. After that, calculate the energy of incident beam using, $E = IA$, where A is area. Finally, find the number of incident photons using the following formula.

$$N = \frac{\text{Energy}}{\text{Energy of 1 photon}}$$

5. *(c)* We have following equivalent circuit,



Resistance of circuit,

$$R = \frac{d}{\sigma A} \qquad \left[\because R = \rho \cdot \frac{l}{A} = \frac{l}{\sigma A}\right]$$

and $$C = \frac{K \varepsilon_0 A}{d}$$

Here, d is distance between plates.

Time constant,

$$\tau = CR = \frac{K \varepsilon_0}{\sigma}$$

$$= \frac{5 \times 8.85 \times 10^{-12}}{7.4 \times 10^{-12}} = 5.98 \text{ s}$$

Charge at any instant decreases as $Q = q_0 e^{-t/\tau}$

Here, $q_0 = 8.85 \times 10^{-6}$ C

So, current due to charge leakage,

$$I = \frac{dQ}{dt} = -\frac{q_0}{\tau} \cdot e^{-t/\tau}$$

So, current at $t = 12$ s (taking only magnitude),

$$I = \frac{8.85 \times 10^{-6}}{5.98} \times e^{-\frac{12}{5.98}}$$

$$= 0.198 \times 10^{-6} \text{ A}$$

$$= 0.198 \mu\text{A}$$

6. ***(d)*** Applying law of conservation of energy,

Total PE at point D = Total KE at point D

$$\Rightarrow \quad 2mgR - mgR \sin 45° = \frac{1}{2} mu^2$$

$$\Rightarrow \quad 2mgR - mg\frac{R}{\sqrt{2}} = \frac{1}{2} mu^2$$

$$\Rightarrow \quad 4mgR - \sqrt{2}\, mgR = mu^2$$

$$\Rightarrow \quad 4gR - \sqrt{2}\, gR = u^2$$

$$\Rightarrow \quad u = \sqrt{4gR - \sqrt{2}\, gR}$$

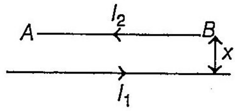
Maximum height attained,

$$H_{\max} = \frac{R}{\sqrt{2}} + \frac{u^2 \sin^2 45}{2g} = \frac{R}{\sqrt{2}} + \frac{4gR - \sqrt{2}\, gR}{2g \times 2}$$

$$= \frac{R}{\sqrt{2}} + R - \frac{R}{2\sqrt{2}}$$

$$= \left(1 + \frac{1}{2\sqrt{2}}\right) R$$

(b) Restoring force is acquired due to repulsion of wires.



Force for balancing in equilibrium position, $F_g = F_m$

F_g = Force due to gravity

F_m = Force due to magnetic effect of electric curre

$$F_g = F_m$$

$$mg = \frac{\mu_0 I_1 I_2}{2\pi x} \qquad \ldots \text{(i)}$$

Equation of SHM is obtained by differentiating F_m w.r.t. x, we have,

$$\frac{dF_m}{dx} = -\frac{\mu_0 I_1 I_2}{2\pi x^2}$$

$$dF_m = -\frac{\mu_0 I_1 I_2}{2\pi x^2} dx = -\left(\frac{mg}{x}\right) dx$$

[From Eq. (i)]

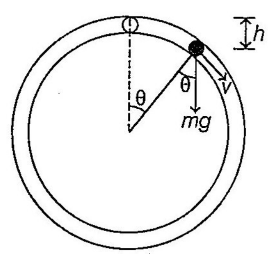
Hence, acceleration of the wire, $\frac{dF_m}{m} = -\left(\frac{g}{x}\right) dx$

$$a = -\frac{g}{x} dx$$

$$\Rightarrow \quad \omega^2 = \frac{g}{x}$$

So, $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{0.1}{9.8}} = 0.63 \text{ s}$$

8. ***(0)*** $h = \left(R + \frac{d}{2}\right)(1 - \cos\theta)$



Velocity of ball at angle θ is

$$v^2 = 2gh$$

$$= 2\left(R + \frac{d}{2}\right)(1 - \cos\theta) g \qquad \ldots \text{(i)}$$

Let N be the normal reaction (away from centre) at angle θ, then

$$mg \cos\theta - N = \frac{mv^2}{\left(R + \frac{d}{2}\right)}$$

Substituting value of v^2 from Eq. (i), we get

$$mg \cos\theta - N = 2\, mg\, (1 - \cos\theta)$$

$$\therefore \quad N = mg\, (3\cos\theta - 2)$$

The ball will lose contact with the inner sphere, when $N = 0$

or $$3\cos\theta - 2 = 0$$

or $$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

After this, it makes contact with outer sphere and normal reaction starts acting towards the centre. Thus, for

$$\theta \leq \cos^{-1}\left(\frac{2}{3}\right)$$

$$N_B = 0 \text{ and } N_A = mg\,(3\cos\theta - 2)$$

and for $\theta \geq \cos^{-1}\left(\frac{2}{3}\right)$

$$N_A = 0 \text{ and } N_B = mg\,(2 - 3\cos\theta)$$

The corresponding graphs are as follows

9. *(40)* In 10 s, number of nuclei has been reduced to half (25% to 12.5%).

Therefore, its half-life, $t_{1/2} = 10\,\text{s}$

From initial amount of 100% to reduction till 6.25%, it takes four half-lives.

$$100\% \xrightarrow{t_{1/2}} 50\% \xrightarrow{t_{1/2}} 25\% \xrightarrow{t_{1/2}} 12.5\% \xrightarrow{t_{1/2}} 6.25\%$$

$$\therefore \quad t_1 = 4\,t_{1/2} = 4\,(10) = 40\,\text{s}$$

$$t_1 = 40\,\text{s}$$

10. *(1)* Constant velocity means net acceleration of the system is zero or net pulling force on the system is zero. While calculating the pulling force, tension forces are not taken into consideration. Therefore,

$$M_1 g = M_2\, g \sin 37^\circ + \mu\, M_2\, g \cos 37^\circ + \mu\, M_3\, g$$

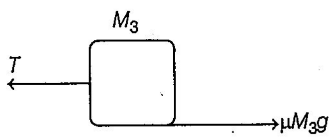
$$\text{or } M_1 = M_2 \sin 37^\circ + \mu M_2 \cos 37^\circ + \mu\, M_3$$

Substituting the values, we get

$$M_1 = (4)\left(\frac{3}{5}\right) + (0.25)\,(4)\left(\frac{4}{5}\right) + (0.25)\,(4) = 4.2\,\text{kg}$$

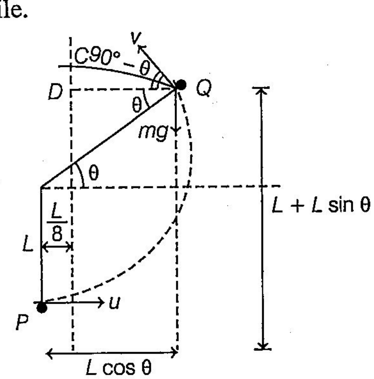
Since, M_3 is moving with uniform velocity,

$$T = \mu\, M_3\, g = (0.25)\,(4)\,(9.8) = 9.8\,\text{N}$$



$$\therefore \quad \frac{T}{g} = \frac{9.8}{9.8} = 1\,\text{kg}$$

11. *(3)* Let the string slacks at point Q as shown in figure. From P to Q, path is circular and beyond Q, path is parabolic. At point C, velocity of particle becomes horizontal, therefore QD = half the range of the projectile.



Now, we have following equations :

$$T_Q = 0. \text{ Therefore, } mg \sin\theta = \frac{mv^2}{L} \qquad \ldots\text{(i)}$$

$$v^2 = u^2 - 2gh = u^2 - 2gL\,(1 + \sin\theta) \qquad \ldots\text{(ii)}$$

$$QD = \frac{1}{2}\,\text{(Range)}$$

$$\Rightarrow \left(L\cos\theta - \frac{L}{8}\right) = \frac{v^2 \sin 2\,(90^\circ - \theta)}{2g}$$

$$= \frac{v^2 \sin 2\theta}{2g} \qquad \ldots\text{(iii)}$$

Eq. (iii) can be written as $\left(\cos\theta - \frac{1}{8}\right) = \left(\frac{v^2}{gL}\right)\sin\theta\cos\theta$

Substituting value of $\left(\frac{v^2}{gL}\right) = \sin\theta$ [from Eq. (i)]

From Eq. (i), we get

$$\left(\cos\theta - \frac{1}{8}\right) = \sin^2\theta \cdot \cos\theta$$

$$= (1 - \cos^2\theta)\cos\theta$$

or $\cos\theta - 1/8 = \cos\theta - \cos^3\theta$

$\therefore$ $\cos^3\theta = 1/8$

or $\cos\theta = 1/2$

or $\theta = 60^\circ$

From Eq. (i),

$$v^2 = gL\sin\theta = gL\sin 60^\circ$$

or $v^2 = \frac{\sqrt{3}}{2}\,gL$

$$\Rightarrow \quad 2\sqrt{3}v^2 = 3gL \quad \Rightarrow \quad \frac{2\sqrt{3}v^2}{gL} = 3$$

12. *(4)* Let, N_0 be the initial number of nuclei of ^{238}U.

After time t, $N_U = N_0\left(\frac{1}{2}\right)^n$

Here, n = number of half-lives $= \frac{t}{t_{1/2}} = \frac{1.5 \times 10^9}{4.5 \times 10^9} = \frac{1}{3}$

$$N_U = N_0\left(\frac{1}{2}\right)^{\frac{1}{3}}$$

and $N_{Pb} = N_0 - N_U = N_0\left[1 - \left(\frac{1}{2}\right)^{1/3}\right]$

$$\therefore \quad \frac{N_U}{N_{Pb}} = \frac{\left(\frac{1}{2}\right)^{1/3}}{1 - \left(\frac{1}{2}\right)^{1/3}} = 3.861$$

= Value of Z is closest to 4

13. *(9)* Angular momentum is conserved.

$$I_1\,\omega_1 = I_2\,\omega_2$$

$$\omega_2 = \frac{I_1}{I_2}\,\omega_1 = \left(\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2mr^2}\right)\times \omega_1$$

$$= \frac{\frac{1}{2}\times 50\times (0.4)^2 \times 10}{\frac{1}{2}\times 50\times (0.4)^2 + 2\times 6.25\times (0.2)^2}$$

$$\approx 9.00\ \text{rad s}^{-1}$$

14. *(c)* P. $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}+\frac{1}{r}\right) = \frac{1}{r} \Rightarrow f = r$

$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = \frac{2}{r} \Rightarrow f_{eq} = \frac{r}{2}$

Q $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}\right) \Rightarrow f = 2r$

$\Rightarrow \frac{1}{f} + \frac{1}{f} = \frac{2}{f} = \frac{1}{f_{eq}} \Rightarrow f_{eq} = r$

R. $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(-\frac{1}{r}\right) = -\frac{1}{2r} \Rightarrow f = -2r$

$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = -\frac{2}{2r} \Rightarrow f_{eq} = -r$

S. $\Rightarrow \frac{1}{f_{eq}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \Rightarrow f_{eq} = 2r$

Hence, P $\to$ 2, Q $\to$ 4, R $\to$ 3, S $\to$ 1

15. *(b)* $Z = \sqrt{(X_L - X_C)^2 + R^2}$

$\therefore Z = R$ when $\omega = \frac{1}{\sqrt{LC}}$ (i.e. $X_L = X_C$)

Thus, P $\to$ 4

$$I = \frac{E}{Z}$$

When $f \to f_r$ (f_r is resonance frequency), $Z \to R$ and I reaches maximum.

Thus, Q $\to$ 3

We know that $X_C = \frac{1}{\omega C}$ and $I = \frac{E}{X_C} \Rightarrow I \propto f$, hence if $f \to \infty, I \to \infty$

Thus, R $\to$ 2 and so I increases linearly.

As, $X_L = \omega L \Rightarrow \frac{1}{X_L} \propto \frac{1}{f}$

Thus, S $\to$ 1

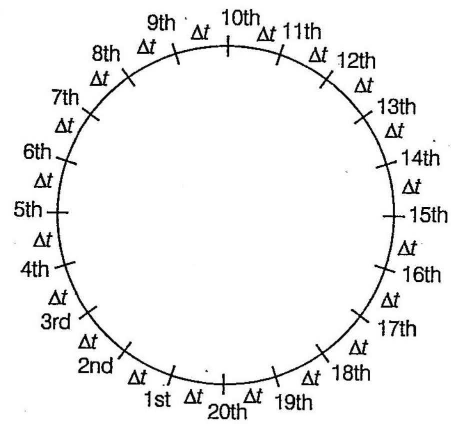
Hence, P $\to$ 4, Q $\to$ 3, R $\to$ 2, S $\to$ 1

16. *(b)*

P. $\because$ Time of flight $= T = \frac{2u}{g}$

$\therefore\ u = \frac{gT}{2} = \frac{10\times 10}{2} = 50\ \text{ms}^{-1}$

Q.



From figure, $20\Delta t = T = 10$ s

$\therefore\ \Delta t = 0.5$ s

Thus, the time interval in projection of balls is $\Delta t = 0.5$ s

For 15th ball,

$$t = (20-15)\Delta t$$
$$= 5\Delta t = 5\times 0.5 = 2.5\ \text{s}$$

$$\therefore\ h = ut + \frac{1}{2}at^2$$
$$= 50\times 2.5 - \frac{1}{2}\times 10\times 2.5^2$$
$$= 125 - 31.25 = 93.75\ \text{m}$$

R. For 10th ball,

$$t = (20-10)\Delta t = 10\Delta t$$
$$= 10\times 0.5 = 5\text{s}$$
$$\because\ v = u + at = 50 - 10\times 5 = 0$$

S. In time $\Delta t = 0.5$ s one ball is projected.

$\therefore$Number of balls projected per minute = 120.

Hence, P $\to$ 2, Q $\to$ 1, R $\to$ 4, S $\to$ 3.

17. *(c)* KE of 1 mole $= \frac{f}{2}RT$

$f = 3$ for monatomic molecules

$f = 5$ for diatomic molecules

$f = 6$ for triatomic non-linear molecules

$f = 7$ for triatomic linear molecules.

Hence, P $\to$ 5, Q $\to$ 3, R $\to$ 2, S $\to$ 1.

Chemistry

1. (*a, b, c*) $CuFeS_2$ (copper pyrite) is converted into copper by following steps:

***Step* I** Crushing (grinding) followed by concentration by froth-floatation process.

***Step* II** Roasting of ore in the presence of SiO_2 which removes iron as slag ($FeSiO_3$).

$$2CuFeS_2 + O_2 \longrightarrow Cu_2S + 2FeS + SO_2\uparrow$$
$$2FeS + 3O_2 \longrightarrow 2SO_2\uparrow + 2FeO$$
$$FeO + SiO_2 \longrightarrow \underset{\text{(Slag)}}{FeSiO_3}$$

***Step* III** Self-reduction in Bessemer converter.

$$2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$$
$$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$$

Copper obtained is blister copper (98% pure).

***Step* IV** Refining of blister copper is done by electrolysis.

Impure copper $\rightarrow$ Anode

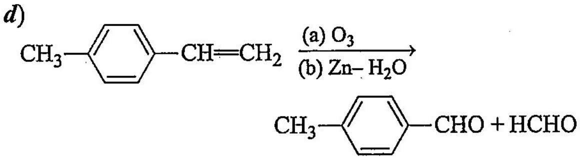
Pure copper $\rightarrow$ Cathode

At anode, $Cu \longrightarrow Cu^{2+} + 2e^-$

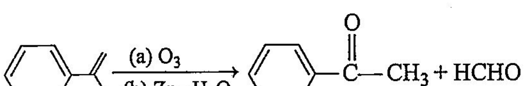
At cathode, $Cu^{2+} + 2e^- \longrightarrow Cu$

Carbon reduction method is not used. Hence, option (d) is incorrect.

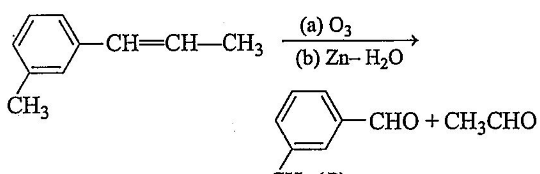
2. (*c, d*)



(*P*)
Gives Cannizzaro but not Haloform reaction



(*Q*)
Gives Haloform reaction but not Cannizzaro reaction



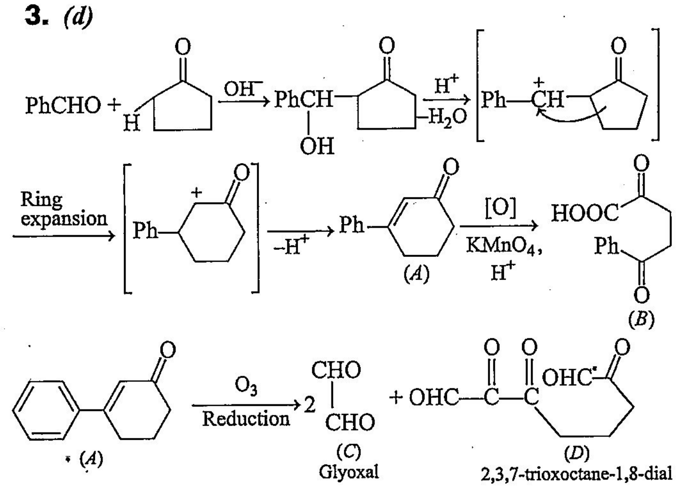
(*R*)
Gives Cannizzaro but not Haloform reaction

Haloform reactions are given by those compounds which have a methyl ketone group (— $COCH_3$) and Cannizzaro reactions are given by aldehydes lacking α-H atom. Hence, both *P* and *R* give Cannizzaro reaction and only *Q* gives Haloform reaction.

Problem Solving Tactics

- Identify the types of compounds *P*, *Q* and *R* based on their functional groups.
- Recall the characteristic reactions associated with each functional group.
- Compare the reactions of the compounds with the given options.

3. (*d*)



4. (*b*) The correct sequence is

***Step* I** $FeCr_2O_4 \xrightarrow[\Delta]{\text{NaOH, Air}} Na_2CrO_4$

***Step* II** $Na_2CrO_4 \xrightarrow{\text{C}, \Delta} Cr_2O_3$

***Step* III** $Cr_2O_3 \xrightarrow{\text{Al}, \Delta} Cr$

5. (*c*) The reaction can be considered as

$$Zn(s) + Ni^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Ni(s)$$

The cell involving the reaction would be

$$Zn(s)|Zn^{2+}(aq)||Ni^{2+}(aq)|Ni(s)$$

$$E^\circ_{cell} = -0.24 + 0.75 = 0.51\,V$$

$$\log K_{eq} = \frac{nFE^\circ}{2.303RT} = \frac{2 \times 96500 \times 0.51}{2.303 \times 8.314 \times 298} = 17.25$$

$\therefore$ $K_{eq} = 1.78 \times 10^{17}$

	$Zn(s) + Ni^{2+}(aq) \rightleftharpoons$	$Zn^{2+}(aq) + Ni(s)$
Initially	1	0
At equilibrium	$1-x$	x

$$K_{eq} = \frac{[Zn^{2+}]}{[Ni^{2+}]} = \frac{x}{1-x} = 1.78 \times 10^{17}$$

$\therefore$ $x \simeq 1.0\,M$

So, $[Ni^{2+}] = \dfrac{[Zn^{2+}]}{1.78 \times 10^{17}}$

$$= \frac{1}{1.78 \times 10^{17}}$$

$$= 5.62 \times 10^{-18}\ \text{M}$$

Problem Solving Tactics	To solve this question, first use the Nernst equation to calculate the equilibrium constant of reaction. Then, solve for the equilibrium concentration of Ni^{2+} ions.

6. (*b*) Use $K_p = K_C\ (RT)^{\Delta n}$ to solve this problem.

$$Se_6(g) \rightleftharpoons 3Se_2(g)$$

	$Se_6(g)$	$3Se_2(g)$
Initially	1	0
At equilibrium	$1-x$	$3x$

Total pressure at equilibrium

$$= 1 - x + 3x = 1 + 2x$$

Pressure of $Se_2\ (p_{Se_2}) = \frac{3x}{1+2x} \times p$

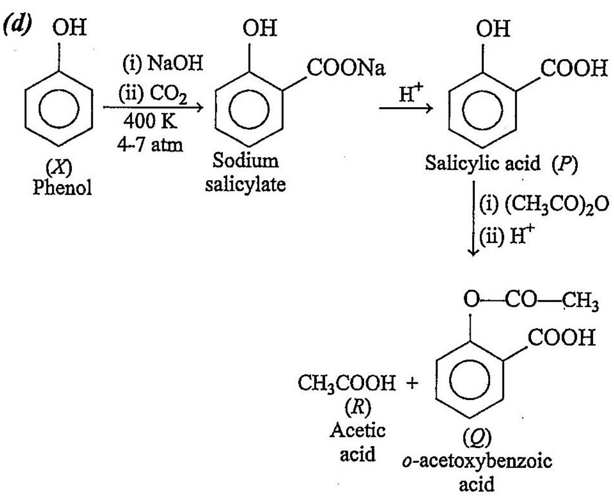
Pressure of $Se_6\ (p_{Se_6}) = \frac{1-x}{1+2x} \times p$

$$K_p = \frac{p^3_{Se_2}}{p_{Se_6}} = \frac{\left[\frac{3x}{1+2x} \times p\right]^3}{\frac{1-x}{1+2x} \times p} = 0.1687$$

Now, $K_p = K_C \cdot (RT)^{\Delta n}$

$$K_C = \frac{K_p}{(RT)^{\Delta n}} = \frac{0.1687}{(0.0821 \times 973)^2} = 0.264 \times 10^{-4}$$

7. (*d*)



8. (*288*) H_2SO_4 and NO_2 are formed when rhombic sulphur (S_8) is oxidised by conc. HNO_3.

$$S_8 + 48HNO_3 \longrightarrow 8H_2SO_4 + 48NO_2 + 16H_2O$$

1 mole of rhombic sulphur produces = 16 moles of H_2O

Molar mass of water = 18 g mol^{-1}

∴Mass of water = 16 × 18 = 288 g

9. (*13*) $f(r) = 4\pi r^2$, $\psi_{r^2} = Kr^2\, e^{-\frac{2Zr}{na_0}}$

For 1*s*-orbital, $n = 1$ $f(r) = Kr^2 e^{-\frac{2Zr}{na_0}}$

For maximum, $\frac{df(r)}{dr} = 0$

$$\Rightarrow \quad e^{-\frac{2Zr}{a_0}} \times 2r + r^2 \times e^{-\frac{2Zr}{a_0}} \times \left(\frac{-2Z}{a_0}\right) = 0$$

$$2 - \frac{2rZ}{a_0} = 0$$

or $$r = \frac{a_0}{Z} = \frac{52.9}{4} = 13.22 \times 10^{-14}\ \text{m}$$

$$\Rightarrow \quad r \approx 13 \times 10^{-14}\ \text{m}$$

10. (*11*) At 15 min, let α be the degree of dissociation of N_2O_5.

$$N_2O_5 \longrightarrow 2NO_2 + \frac{1}{2}O_2$$

	N_2O_5	$2NO_2$	$\frac{1}{2}O_2$
Initially	1	0	0
At 15 min	1 − α	2α	$\frac{\alpha}{2}$

Now, $\frac{\text{rate of effusion of } NO_2}{\text{rate of effusion of } O_2} = \frac{2\alpha}{\frac{\alpha}{2}}\sqrt{\frac{32}{46}}$

$$= \left(\frac{\text{mole fraction of } NO_2}{\text{mole fraction of } O_2}\right)_{\text{outside}}$$

$$\Rightarrow \quad 4 \times \sqrt{\frac{32}{46}} = \frac{0.66}{x_{O_2}}$$

$$x_{O_2} = \frac{0.66\sqrt{46}}{4 \times \sqrt{32}} \approx 0.2$$

⇒ Mole fraction of N_2O_5 in mixture collected outside after 15 min = 0.14

$$\therefore \quad \frac{r_{N_2O_5}}{r_{NO_2}} = \frac{1-\alpha}{2\alpha}\sqrt{\frac{46}{108}} = \frac{0.14}{0.66}$$

$$\Rightarrow \quad \frac{1-\alpha}{2\alpha} = 0.33 \Rightarrow \alpha = 0.60$$

Now, applying first order kinetics,

$$k \times 15 = \ln \frac{1}{1-0.6}$$

$$\Rightarrow \quad k = \frac{1}{15}\ln\frac{5}{2}$$

$$\Rightarrow \quad k = 0.061$$

$$\therefore \quad t_{1/2} = \frac{\ln 2}{0.061}$$

$$= 11.35\ \text{min} \approx 11\ \text{min}$$

Problem Solving Tactics	To solve this problem, firstly use Graham's law of diffusion to calculate the degree of dissociation. Then, apply first order reaction to obtain the rate constant (R). Finally substitute this value of k in half-life equation, i.e. $\left(\frac{0.693}{k} = t_{1/2}\right)$.

11. (*143*) Mass percent of $C = \frac{12}{44} \times 0.138 \times \frac{100}{0.15} = 25$

$$H = \frac{1}{9} \times 0.054 \times \frac{100}{0.15} = 4.0$$

$$N = \frac{14}{17} \times 0.238 \times \frac{100}{2} = 9.8$$

$$Cl = \frac{35.5}{143.5} \times 0.251 \times \frac{100}{0.125} = 49.67$$

$$O = 11.53$$

Empirical formula determination

Element	C	H	N	Cl	O
m%	25	4.0	9.8	49.67	11.53
n = (*m*% / *M*)	2.08	4.0	0.7	1.4	0.7
Simple ratio	3	6	1	2	1

Empirical formula = $C_3H_6NCl_2O$

Empirical formula mass

$= 12 \times 3 + 6 \times 1 + 14 + 2 \times 35.5 + 16 = 143$

12. (*9*) Given, monoatomic gas is being expanded adiabatically,

$p_1 = 1.0\,kPa = 1000\,Pa, V_1 = 1\,dm^3$

$= 0.001\,m^3, n = 0.4, p_2 = 0.1\,kPa = 100\,Pa$

For monoatomic gas, $\gamma = \frac{5}{3}$ and $C_V = \frac{3}{2}R$

$$\therefore T_1 = \frac{p_1V_1}{nR} = \frac{1000 \times 0.001}{0.4 \times 8.314} = 0.300\,K$$

$$\therefore \quad V_2 = 3.98\,dm^3$$

Also, $T_2 = 0.300\left(\frac{1}{3.98}\right)^{\left(\frac{5}{3}-1\right)}$ $\left[\because T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right]$

$$\therefore \quad T_2 = 0.300(0.398) = 0.12\,K$$

Now, $W = -nC_V \times R(T_2 - T_1)$

$$\therefore \quad W = -0.4 \times \frac{3}{2} \times 8.314(0.12 - 0.300)$$

$$\simeq 0.90\,J = 9 \times 10^{-4}\,kJ$$

13. (*7*) Since, density of H_2O (at 23°C) is $1\,g\,cm^{-3}$ (concentration of H_2O)

$$= 1000\,g\,L^{-1} = \frac{1000}{18}\,mol\,L^{-1} = \frac{10^6}{18}\,mol\,m^{-3}$$

Now, $\Lambda^\circ_{(H_2O)} = \lambda^\circ_{(H^+)} + \lambda^\circ_{(OH^-)}$

$= (3.4982 \times 10^{-2} + 1.98 \times 10^{-2})\,S\,m^2\,mol^{-1}$

$= 5.4782 \times 10^{-2}\,S\,m^2\,mol^{-1}$

$$\Lambda_C = \frac{\kappa}{C} = \frac{5.7 \times 10^{-6}\,S\,m^{-1}}{\frac{10^6}{18}\,mol\,m^{-3}}$$

$= 1.026 \times 10^{-10}\,S\,m^2\,mol^{-1}$

$$\alpha = \frac{\Lambda_C}{\Lambda^\circ} = \frac{1.026 \times 10^{-10}\,S\,m^2\,mol^{-1}}{5.4782 \times 10^{-2}\,S\,m^2\,mol^{-1}}$$

$= 1.873 \times 10^{-9}$

$$\therefore [H^+] = [OH^-] = C\alpha = \frac{1000}{18} \times 1.873 \times 10^{-9}$$

$= 1.040 \times 10^{-7}\,M$

$\therefore \quad pH = -\log [H^+]$

$= -\log [1.040 \times 10^{-7}]$

$= 6.98 \approx 7$

14. (*c*) The correct match is P-(4), Q-(1), R-(2), S-(3).

P. Silver is extracted by amalgamation process.

$$Ag + Hg \longrightarrow \underset{\text{Amalgam}}{Ag(Hg)} \xrightarrow{\text{Distillation}} Ag(s) + Hg(g)\uparrow$$

Q. Calcium is extracted by electrolysis of fused $CaCl_2$.

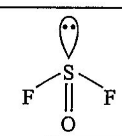
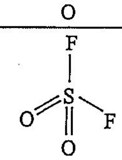
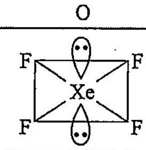
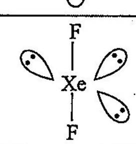
R. Zinc is extracted by carbon reduction method.

$$ZnO + C \longrightarrow Zn + CO$$

S. The first step in extraction of copper from $CuFeS_2$ is roasting in O_2.

$$2CuFeS_2 + O_2 \longrightarrow Cu_2S + 2FeS + SO_2$$

15. (*b*) The correct match is P-(4), Q-(1), R-(3), S-(2).

Chemical formulae	Structure	Hybridisation	Shape
OSF_2		sp^3	Trigonal pyramidal
O_2SF_2		sp^3	Tetrahedral
XeF_4		sp^3d^2	Square planar
XeF_2		sp^3d	Linear

16. (*b*) The correct match is P-(3), Q-(2), R-(1), S-(4).

P. $V_n = \frac{-1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r}\right) \Rightarrow K_n = \frac{1}{8\pi\varepsilon_0}\left(\frac{Ze^2}{r}\right)$

$\frac{V_n}{K_n} = -2$

Q. $E_n^x = \frac{-Ze^2}{8\pi\varepsilon_0 r}$

$E_n^x \propto r^{-1} \quad \therefore \quad x = -1$

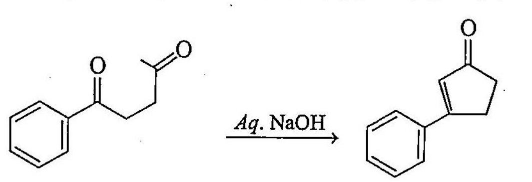
R. Angular momentum $= \sqrt{l(l+1)}\,\frac{h}{2\pi} = 0$ for 1s as $l = 0$

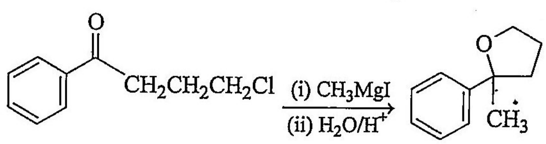
S. $r_n = \frac{a_0 n^2}{Z}$

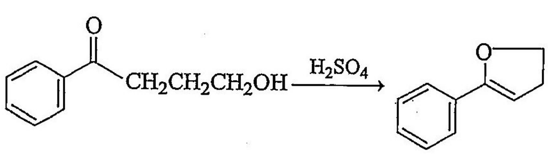
$\therefore \quad \frac{1}{r_n} \propto Z^y$

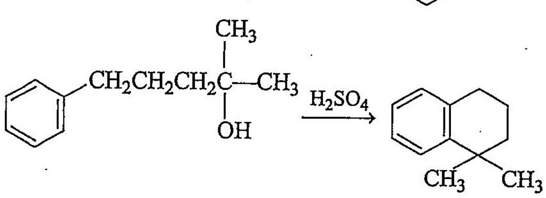
So, $\quad y = 1$

17. (*d*) The correct match is P-(5), Q-(3), R-(1), S-(2).

P.


Q.


R.


S.


Mathematics

1. (*a,d*) We have,

$z_1 = 5 + 12i$ and $|z_2| = 4$

(a) $|z_1 + iz_2| \le |z_1| + |iz_2| = 13 + 4 = 17$

(b) $|z_1 + (1+i)z_2| \ge ||z_1| - |1+i||z_2|| = |13 - 4\sqrt{2}| = 13 - 4\sqrt{2}$

Now, $\left|z_2 + \frac{4}{z_2}\right| \le |z_2| + \frac{4}{|z_2|} = 4 + 1 = 5$

$\Rightarrow \quad \left|z_2 + \frac{4}{z_2}\right| \ge \left||z_2| - \frac{4}{|z_2|}\right| = |4 - 1| = 3$

(c) Minimum value of $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{5}$

(d) Maximum value of $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3}$

2. (*a,b*) We have,

$\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) + \cos^{-1}k = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) = \frac{\pi}{2} - \cos^{-1}k = \sin^{-1}k,$

where $k \in [-1, 1]$

$x^2 - 6x + 17/2 = k$

$\Rightarrow \quad (x-3)^2 - \frac{1}{2} = k \Rightarrow (x-3)^2 = k + \frac{1}{2}$

(a) The largest value of k is 1 for two distinct solutions.

(b) For two distinct real roots, $k + \frac{1}{2} > 0$

$\Rightarrow \quad k > \frac{-1}{2} \Rightarrow k \in \left(-\frac{1}{2}, 1\right)$

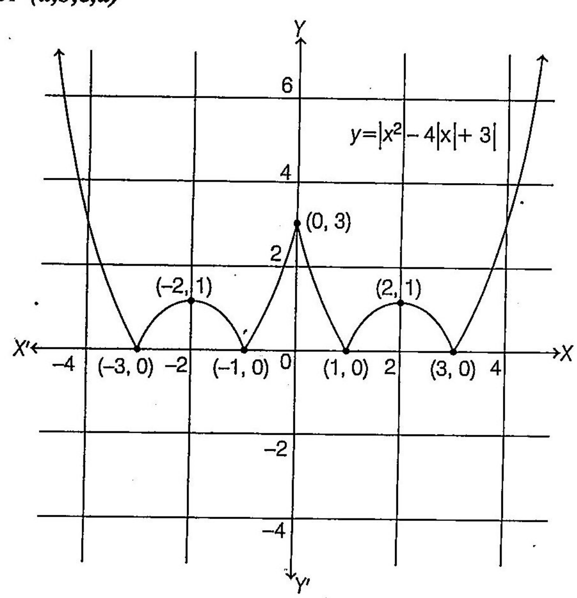
(c) For real roots, $k + \frac{1}{2} \ge 0 \Rightarrow k \ge \frac{-1}{2}$

But $k \in [-1, 1] \;\therefore\; k \in \left[-\frac{1}{2}, 1\right]$

(d) The equation has unique solution, if

$k + \frac{1}{2} = 0 \Rightarrow k = -\frac{1}{2}$

3. (*a,b,c,d*)



(a) Graph of

$y=|x^2-4|x|+3|$ is shown above.

Clearly, the above equation $|x^2-4|x|+3|=0$

has four solutions ± 1 and ± 3.

So, (a) is correct option.

Sum of roots $=+1-1+3-3=0$

So, (b) is also correct.

(c) Also, $y=f(x)$ and $y=a$ intersect at 8 distinct point, if $0<a<1$.

Thus, $f(x)=a$ has 8 real roots for $0<a<1$

(d) Option (d) is also the correct option as $y\geq 0$.

Therefore, all the options are correct.

4. *(c)* Consider the given series

$$\sum_{r=0}^{100}(-1)^r({}^{99}C_r+{}^{99}C_{r-1})\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\ldots\text{upto 10 terms}\right]$$

$$=\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\ldots\text{upto 10 terms}\right]$$

$$[\because {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r]$$

$$=\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r\left(\frac{1}{2}\right)^r+\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r\left(\frac{3}{4}\right)^r+\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r\left(\frac{7}{8}\right)^r+\ldots\text{upto 10 terms}$$

$$=\sum_{r=0}^{100}{}^{100}C_r\left(\frac{-1}{2}\right)^r+\sum_{r=0}^{100}{}^{100}C_r\left(\frac{-3}{4}\right)^r+\sum_{r=0}^{100}{}^{100}C_r\left(\frac{-7}{8}\right)^r+\ldots\text{upto 10 terms}$$

$$=\left(1-\frac{1}{2}\right)^{100}+\left(1-\frac{3}{4}\right)^{100}+\ldots\text{upto 10 terms}$$

$$\left[\because \sum_{r=0}^{100}{}^nC_r(-x)^r=(1-x)^n\right]$$

$$=\left(\frac{1}{2}\right)^{100}+\left(\frac{1}{4}\right)^{100}+\ldots\text{upto 10 terms}$$

$$=\left(\frac{1}{2}\right)^{100}\left[\frac{1-\left(\left(\frac{1}{2}\right)^{100}\right)^{10}}{1-\frac{1}{2^{100}}}\right]$$

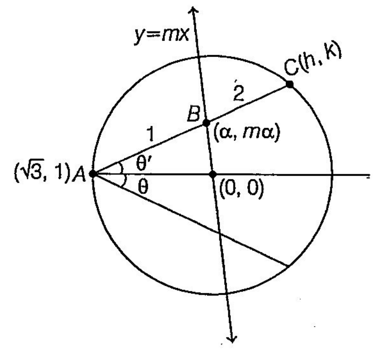
$$=\frac{1}{2^{100}}\left[\frac{1-\frac{1}{2^{1000}}}{1-\frac{1}{2^{100}}}\right]$$

$$=\frac{1}{2^{1000}}\left[\frac{2^{1000}-1}{2^{100}-1}\right]$$

5. *(a)* The line can be written as $y=mx$ and curve as $x^2+y^2=4$

Let $C(h,k)$ be a point on the circle and $A(\sqrt{3},1)$ be given point, then

$$\frac{h+2\sqrt{3}}{3}=\alpha \quad \text{[using section formula]}$$



$\Rightarrow\ h=3\alpha-2\sqrt{3}$ and $\dfrac{k+2}{3}=m\alpha \Rightarrow k=3m\alpha-2$

$\because\ (h,k)$ lies on the circle.

$$\therefore\ (3\alpha-2\sqrt{3})^2+(3m\alpha-2)^2=4$$

$$\Rightarrow (9\alpha^2+12-12\sqrt{3}\alpha)+(9m^2\alpha^2+4-12m\alpha)=4$$

$$\Rightarrow 9(1+m^2)\alpha^2-12\alpha(\sqrt{3}+m)+12=0$$

$$\Rightarrow 3(1+m^2)\alpha^2-4\alpha(\sqrt{3}+m)+4=0$$

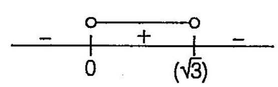
Discriminant >0

$$\Rightarrow 16(\sqrt{3}+m)^2-4\times 3(1+m^2)\times 4>0$$

$$\Rightarrow (3+m^2+2\sqrt{3}m)-3-3m^2>0$$

$$\Rightarrow 2\sqrt{3}m-2m^2>0$$

$$\Rightarrow m(\sqrt{3}-m)>0$$



$$0<m<\sqrt{3}$$

i.e. $m\in(0,\sqrt{3})$

Problem Solving Tactics Use section formula to find the coordinate of point B on chord AC and frame the equation of circle as quadratic equation, then use condition Discriminant > 0.

6. *(a)* Given, functional equation is

$$2f(x-1)-f\left(\frac{1-x}{x}\right)=x \quad \text{...(i)}$$

On replacing x by $\frac{1}{x}$, we get

$$2f\left(\frac{1}{x}-1\right)-f\left(\frac{1-\frac{1}{x}}{\frac{1}{x}}\right)=\frac{1}{x}$$

$$\Rightarrow \quad 2f\left(\frac{1-x}{x}\right)-f(x-1)=\frac{1}{x} \quad \text{...(ii)}$$

On multiplying by 2 in Eq. (i) and then adding Eqs. (i) and (ii), we get

$$3f(x-1)=2x+\frac{1}{x}.$$

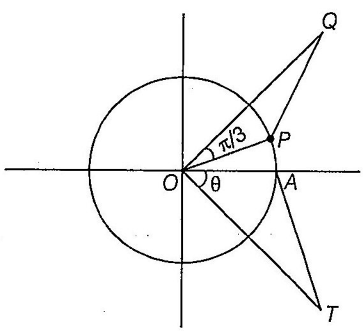
Now, replacing x by $x+1$, we get [to generate $f(x)$]

$$3f[(x+1)-1]=2(x+1)+\frac{1}{(x+1)}$$

$$\therefore \quad f(x)=\frac{1}{3}\left(2(x+1)+\frac{1}{x+1}\right)$$

$$=\frac{2(1+x)^2+1}{3(1+x)}$$

7. *(b)*



Let θ be the angle between **OA** and **OT**.

$$|\mathbf{PQ}|=\sqrt{r^2+1-2r\cos\frac{\pi}{3}}$$

$$=\sqrt{r^2+1-2r\cdot\frac{1}{2}}=\sqrt{r^2-r+1}$$

$$|\mathbf{AT}|=\sqrt{r^2+1-2r\cos\theta}$$

$$\therefore \quad |\mathbf{PQ}|=2|\mathbf{AT}|$$

$$\therefore \quad |\mathbf{PQ}|^2=4|\mathbf{AT}|^2$$

$$\Rightarrow \quad r^2-r+1=4(r^2+1-2r\cos\theta)$$

$$\Rightarrow \quad 8r\cos\theta=3r^2+r+3$$

$$\Rightarrow \quad \cos\theta=\frac{3r^2+r+3}{8r}$$

$$\because \quad -1\le\cos\theta\le 1$$

$$\therefore \quad -1\le\frac{3r^2+r+3}{8r}\le 1$$

$$\Rightarrow \quad 3r^2+9r+3\ge 0 \text{ and } 3r^2-7r+3\le 0$$

$$\Rightarrow \quad r^2+3r+1\ge 0 \text{ and } 3r^2-7r+3\le 0$$

$$r\in\left[\frac{7-\sqrt{13}}{6},\frac{7+\sqrt{13}}{6}\right] \quad [\because r>0]$$

which is always true.

8. *(20)* $\because 1+2+3+4+5+6+7+8+9=45$, a number consisting all these digits will be divisible by 9. Thus, the number will be divisible by 36, if and only if it is divisible by 4. The number formed by its last two digits must be divisible by 4. The possible values of the last pair to the following:

12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76, 84, 92, 96.

i.e. There are 16 ways of choosing last two digits.

The remaining digits can be arranged in ${}^7P_7=7!$ ways,

Therefore, number of favourable ways $=16\times 7!$

and number of total ways $=9!$

$\therefore$Required probability, $p=\frac{16\times 7!}{9!}=\frac{16}{9\times 8}=\frac{2}{9}$

$$\therefore \quad 90p=20$$

9. *(7)* Given, $AB=BA^2 \Rightarrow B=A^{-1}BA^2$ and $B^3=I$

$\Rightarrow (A^{-1}BAA)(A^{-1}BAA)(A^{-1}BAA)=I \quad [\because B=A^{-1}BA^2]$

$\Rightarrow \quad (A^{-1}BA)(BA)(BAA)=I \quad [\because A^{-1}A=I]$

$\Rightarrow \quad A^{-1}B(AB)(AB)AA=I$

$\Rightarrow A^{-1}B(BA^2)(BA^2)AA=I \quad [\because AB=BA^2]$

$\Rightarrow \quad A^{-1}BBA(AB)A^4=I$

$\Rightarrow \quad A^{-1}BBA(BA^2)A^4=I \quad [\because AB=BA^2]$

$\Rightarrow \quad A^{-1}BB(AB)A^6=I$

$\Rightarrow \quad A^{-1}BB(BA^2)A^6=I \quad [\because AB=BA^2]$

$\Rightarrow \quad A^{-1}B^3A^8=I$

$\Rightarrow \quad (A^{-1}I)A^8=I \quad [\because B^3=I]$

$\Rightarrow \quad A^{-1}A^8=I$

$\Rightarrow \quad A^7=I=A^k \quad [\because A^k=I]$

$\Rightarrow \quad A^k=A^7$

$\therefore$The least value of k is 7.

10. *(121)* Consider two sets 1, 3, 5, 7, 9 ...(i)

and 0, 2, 4, 6, 8 ...(ii)

Required number of ways = [any two from set (i) + any two from set (ii) (excluding zero)] 3!+ [0 along with any one from set (ii) (except 0)] 4 + all three digits alike

$$= ({}^5C_2 + {}^4C_2)3! + {}^4C_1 \cdot 4 + 9$$
$$= (10+6)\times 6 + 4\times 4 + 9$$
$$= 96 + 16 + 9 = 121$$

11. *(2)* Given, $\dfrac{z^2 - z + 1}{z^2 + z + 1} = |z|$

$$\Rightarrow \frac{z^2+1}{-z} = \frac{|z|+1}{|z|-1} \qquad [\because |z| \neq 1]$$

$$\Rightarrow z + \frac{1}{z} = -\left(\frac{|z|+1}{|z|-1}\right)$$

$\Rightarrow z + \dfrac{1}{z}$ must be purely real.

$$\Rightarrow \text{Im}\left(re^{i\theta} + \frac{1}{r}e^{-i\theta}\right) = 0$$

$$\Rightarrow r\sin\theta - \frac{1}{r}\sin\theta = 0$$

$$\Rightarrow \quad \theta = 0 \text{ or } \theta = \pi \qquad [\because r = |z| \neq 1]$$

$$\therefore \quad \frac{x^2 - x + 1}{x^2 + x + 1} = |x| \qquad \text{...(i)}$$

For $x > 0$, other than 1, Eq. (i) has one solution.
For $x < 0$, other than -1, Eq. (i) has one solution.
$\therefore$Required number of complex numbers is 2.

12. *(10)* We have,

$$\mathbf{a} = \sqrt{3}\hat{\mathbf{i}} - \hat{\mathbf{j}},\ \mathbf{b} = \frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}$$

$$\mathbf{x} = \mathbf{a} + (q^2 - 3)\mathbf{b}$$

$$\Rightarrow \mathbf{x} = (\sqrt{3}\hat{\mathbf{i}} - \hat{\mathbf{j}}) + (q^2-3)\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

$$\Rightarrow \mathbf{x} = \left(\sqrt{3} + \frac{q^2-3}{2}\right)\hat{\mathbf{i}} - \left(1 - \frac{\sqrt{3}}{2}(q^2-3)\right)\hat{\mathbf{j}}$$

and $\mathbf{y} = -p\mathbf{a} + q\mathbf{b}$

$$\Rightarrow \mathbf{y} = -p(\sqrt{3}\hat{\mathbf{i}} - \hat{\mathbf{j}}) + q\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

$$\Rightarrow \mathbf{y} = \left(-\sqrt{3}p + \frac{q}{2}\right)\hat{\mathbf{i}} + \left(p + \frac{\sqrt{3}q}{2}\right)\hat{\mathbf{j}}$$

$\therefore \mathbf{x}\cdot\mathbf{y} = 0$

$$\Rightarrow \left(\sqrt{3} + \frac{q^2-3}{2}\right)\left(-\sqrt{3}p + \frac{q}{2}\right) - \left(1 - \frac{\sqrt{3}}{2}(q^2-3)\right)\left(p + \frac{\sqrt{3}q}{2}\right) = 0$$

$$\Rightarrow p = \frac{q(q^2-3)}{4}$$

$$\therefore p = f(q) \Rightarrow \frac{dp}{dq} = \frac{1}{4}(3q^2 - 3)$$

$f(q)$ is decreasing when

$$3q^2 - 3 < 0 \Rightarrow q \in (-1, 1)$$
$$a = -1 \text{ and } b = 1$$
$$\therefore \quad 5|b-a| = 5|1-(-1)| = 5(2) = 10$$

13. *(5)* Given, $y_n = \dfrac{{}^{3n}C_n}{{}^{2n}C_n}$

$$\Rightarrow y_n = \frac{3n!}{n!(2n)!} \times \frac{n!n!}{(2n)!}$$
$$= \frac{3n(3n-1)\ldots(2n+1)}{2n(2n-1)\ldots(n+1)}$$
$$= \frac{(2n+1)(2n+2)(2n+3)\ldots(2n+n)}{(n+1)(n+2)(n+3)\ldots(n+n)}$$
$$= \left(\frac{2n+1}{n+1}\right)\left(\frac{2n+2}{n+2}\right)\left(\frac{2n+3}{n+3}\right)\ldots\left(\frac{2n+n}{n+n}\right)$$
$$= \left(1 + \frac{n}{n+1}\right)\left(1 + \frac{n}{n+2}\right)\ldots\left(1 + \frac{n}{n+n}\right)$$

Now, let $P = \lim\limits_{n\to\infty}(y_n)^{1/n}$

$$= \lim_{n\to\infty}\left[\left(1 + \frac{n}{n+1}\right)\left(1 + \frac{n}{n+2}\right)\ldots\left(1 + \frac{n}{n+n}\right)\right]^{1/n}$$

On taking log on both sides, we get

$$\log P = \lim_{n\to\infty}\frac{1}{n}\left[\log\left(1 + \frac{n}{n+1}\right) + \log\left(1 + \frac{n}{n+2}\right) + \ldots + \log\left(1 + \frac{n}{n+n}\right)\right]$$

$$\Rightarrow \log P = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1 + \frac{1}{1 + r/n}\right)$$

$$\Rightarrow \log P = \int_0^1 \log\left(1 + \frac{1}{1+x}\right)dx = \int_0^1 \log\left(\frac{x+2}{x+1}\right)dx$$

$$\Rightarrow \log P = \int_0^1 \log(x+2)dx - \int_0^1 \log(x+1)dx$$

$$\Rightarrow \log P = [(x+2)\log(x+2) - (x+2)]_0^1 - [(x+1)\log(x+1) - (x+1)]_0^1$$

$$\Rightarrow \log P = [3\log 3 - 3] - [2\log 2 - 2] - [(2\log 2 - 2) - (0 - 1)]$$

$$\Rightarrow \log P = \log 27 - 3 - \log 4 + 2 - \log 4 + 2 - 1$$

$$\Rightarrow \log P = \log 27 - \log 16 = \log\frac{27}{16}$$

$$\therefore \quad P = \frac{27}{16}$$

Here, $a = 27$ and $b = 16$

$$\Rightarrow \quad 2b - a = 32 - 27 = 5$$

14. *(d)* We have,

$$f(m) = \sum_{i=0}^{m}\binom{30}{30-i}\binom{20}{m-i}$$

$$=\sum_{i=0}^{m}\binom{30}{i}\binom{20}{m-i} \qquad [\because {}^nC_r={}^nC_{n-r}]$$

$$={}^{20}C_m+{}^{30}C_1\cdot{}^{20}C_{m-1}+\ldots+{}^{30}C_m={}^{50}C_m$$

$$[\because {}^pC_r+{}^pC_{r-1}\,{}^qC_1+{}^pC_{r-2}\,{}^qC_2+\ldots+{}^qC_r={}^{p+q}C_r]$$

P. Clearly, $f(m)$ is maximum when $m=25$

$\therefore$Maximum value of $f(m)$ is ${}^{50}C_{25}$.

Q. Clearly,

$$\sum_{m=0}^{50} f(m)=\sum_{m=0}^{50}{}^{50}C_m={}^{50}C_0+{}^{50}C_1+\ldots+{}^{50}C_{50}=2^{50}$$

R. Clearly, $\sum_{m=0}^{50}[f(m)]^2=\sum_{m=0}^{50}({}^{50}C_m)^2=({}^{50}C_0)^2$

$$+({}^{50}C_1)^2+\ldots+({}^{50}C_{50})^2={}^{100}C_{50}$$

$$[\because ({}^nC_0)^2+({}^nC_1)^2+\ldots+({}^nC_n)^2={}^{2n}C_n]$$

S. Consider,

$$3f(0)-8f(1)+13f(2)-18f(3)+\ldots+253f(50)$$

$$=\sum_{m=0}^{50}(-1)^m(3+5m)f(m)$$

$$=\sum_{m=0}^{50}(-1)^m(3+5m)\,{}^{50}C_m$$

$$=3\left(\sum_{m=0}^{50}(-1)^m\,{}^{50}C_m\right)+5\left(\sum_{m=0}^{50}(-1)^m\cdot m\,{}^{50}C_m\right)$$

$$=3\left(\sum_{m=0}^{50}(-1)^m\,{}^{50}C_m\right)+5\left(\sum_{m=1}^{50}(-1)^m\cdot 50\cdot{}^{49}C_{m-1}\right)$$

$$=3\left(\sum_{m=0}^{50}(-1)^m\,{}^{50}C_m\right)-250\left(\sum_{m=1}^{50}(-1)^{m-1}\,{}^{49}C_{m-1}\right)$$

$$=3(1-1)^{50}-250(1-1)^{49}=0$$

15. *(c)* We have,

$$\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\frac{\pi}{2}$$

P. If $a=1$ and $b=0$, then

$$\sin^{-1}x+\cos^{-1}y+\cos^{-1}0=\frac{\pi}{2}$$

$$\sin^{-1}x+\cos^{-1}y+\frac{\pi}{2}=\frac{\pi}{2} \qquad \left[\because \cos^{-1}0=\frac{\pi}{2}\right]$$

$\Rightarrow$ $\sin^{-1}x+\cos^{-1}y=0$

$\Rightarrow$ $\sin^{-1}x=-\cos^{-1}y$

$\Rightarrow$ $x^2+y^2=1$

Q. If $a=1$ and $b=1$, then

$$\sin^{-1}x+\cos^{-1}y+\cos^{-1}xy=\frac{\pi}{2}$$

$\Rightarrow$ $\cos^{-1}x-\cos^{-1}y=\cos^{-1}xy$

$\Rightarrow$ $\cos^{-1}(xy+\sqrt{1-x^2}\sqrt{1-y^2})=\cos^{-1}xy$

$\Rightarrow$ $xy+\sqrt{1-x^2}\sqrt{1-y^2}=xy$

$\Rightarrow$ $(1-x^2)(1-y^2)=0$

$\Rightarrow$ $(x^2-1)(y^2-1)=0$

R. If $a=1$ and $b=2$, then

$$\sin^{-1}x+\cos^{-1}y+\cos^{-1}2xy=\frac{\pi}{2}$$

$\Rightarrow$ $\cos^{-1}x-\cos^{-1}y=\cos^{-1}2xy$

$\Rightarrow$ $xy+\sqrt{1-x^2}\sqrt{1-y^2}=2xy$

$\Rightarrow$ $\sqrt{1-x^2}\sqrt{1-y^2}=xy$

$\Rightarrow$ $(1-x^2)(1-y^2)=x^2y^2$

$\Rightarrow$ $x^2+y^2=1$

S. If $a=2$ and $b=2$, then

$$\sin^{-1}2x+\cos^{-1}y+\cos^{-1}2xy=\frac{\pi}{2}$$

$\Rightarrow$ $\cos^{-1}2x-\cos^{-1}y=\cos^{-1}2xy$

$\Rightarrow$ $2xy+\sqrt{(1-4x^2)}\sqrt{1-y^2}=2xy$

$\Rightarrow$ $(4x^2-1)(y^2-1)=0$

16. *(c)* We have,

$$2m^3-3m^2-3m+2=0$$

$\Rightarrow$ $2m^3-3m(m+1)+2=0$

$\Rightarrow$ $2(m^3+1)-3m(m+1)=0$

$\Rightarrow$ $(m+1)[2(m^2+1-m)-3m]=0$

$\Rightarrow$ $(m+1)[2m^2+2-2m-3m]=0$

$\Rightarrow$ $(m+1)[2m^2-5m+2]=0$

$\Rightarrow$ $(m+1)(2m-1)(m-2)=0$

$\Rightarrow$ $m=-1, m=\frac{1}{2}$ and $m=2$

Also, we have equation of line which can be written as, $m_i^2x-m_iy=-1$

$$\Rightarrow \quad \frac{x}{-\frac{1}{m_i^2}}+\frac{y}{\frac{1}{m_i}}=1$$

P. Clearly, algebraic sum of the intercepts made by the lines on X-axis

$$=-\sum\frac{1}{m_i^2}=-\left(1+4+\frac{1}{4}\right)=-\frac{21}{4}$$

Q. Clearly, algebraic sum of the intercepts made by the lines on Y-axis $=\sum\frac{1}{m_i}=-1+2+\frac{1}{2}$

$$=1+\frac{1}{2}=\frac{3}{2}$$

R. Let $d_i\,(i=1,2,3)$ denotes the perpendicular distance of the line from the origin, respectively.

Then, $d_i = \dfrac{1}{\sqrt{m_i^4 + m_i^2}}$

Required sum $= \dfrac{1}{\sqrt{1+1}} + \dfrac{1}{\sqrt{\dfrac{1}{16}+\dfrac{1}{4}}} + \dfrac{1}{\sqrt{16+4}}$

$= \dfrac{1}{\sqrt{2}} + \dfrac{4}{\sqrt{5}} + \dfrac{1}{2\sqrt{5}} = \dfrac{1}{10}(5\sqrt{2} + 8\sqrt{5} + \sqrt{5})$

$= \dfrac{1}{10}(5\sqrt{2} + 9\sqrt{5})$

S. Let l_i denotes the length of line intercepted between the coordinate axes. Then,

$$l_i = \sqrt{\left(\frac{1}{m_i^2}\right)^2 + \left(\frac{1}{m_i}\right)^2}$$

Required sum $= \sqrt{1+1} + \sqrt{16+4} + \sqrt{\dfrac{1}{16}+\dfrac{1}{4}}$

$= \sqrt{2} + 2\sqrt{5} + \dfrac{\sqrt{5}}{4} = \dfrac{(4\sqrt{2}+9\sqrt{5})}{4}$

17. *(c)* We have, $f(x+y) = e^y f(x) + e^x f(y)$

On differentiating w.r.t. x, taking y as constant, we get

$$f'(x+y) = e^y f'(x) + e^x f(y)$$

On putting $x=0$ and $y=x$, we get

$$f'(0+x) = e^x f'(0) + e^0 f(x)$$

$\Rightarrow \quad f'(x) = e^x \cdot (2) + f(x) \qquad [\because f'(0)=2]$

$\Rightarrow \quad f'(x) - f(x) = 2e^x$

Let $\quad f(x) = y$,

then $\quad f'(x) = \dfrac{dy}{dx}$

$$\frac{dy}{dx} - y = 2e^x$$

$$\text{IF} = e^{\int -1\,dx} = e^{-x}$$

Solution of differential equaiton is

$$y \cdot e^{-x} = \int 2e^x \cdot e^{-x}\,dx$$

$\Rightarrow \quad ye^{-x} = 2x + C$

$\Rightarrow \quad y = 2xe^x + Ce^x$

$\therefore \quad f(x) = 2xe^x + Ce^x$

$f(x) = 2xe^x \qquad [\because f(0)=0 \Rightarrow C=0]$

P. Domain of $f(x)$ is $(-\infty, \infty)$.

Q. Range of $f(x)$ is $\left[-\dfrac{2}{e}, \infty\right)$.

R. $\int_0^1 f(x)dx = \int_0^1 2xe^x\,dx = [2e^x(x-1)]_0^1 = 2$

S. $\lim_{x\to 0} f(x) = \lim_{x\to 0} 2xe^x = 0$

Paper 2

Physics

1. *(b)* Potential energy of particle is

$$U = mV_x = 0.01 \times 0.08x^2 = 8\times 10^{-4}x^2$$

Also, $U = \dfrac{1}{2}kx^2$

$\Rightarrow \quad k = \dfrac{2\times U}{x^2} = 16\times 10^{-4}\,\text{Nm}^{-1}$

Angular frequency of particle is

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{16\times 10^{-4}}{0.01}} = 0.4 \text{ rad s}^{-1}$$

Total energy of oscillator is

$$E = \frac{1}{2}ma^2\omega_0^2 = 8\times 10^{-4}$$

$\Rightarrow \quad a^2 = \dfrac{2\times 8\times 10^{-4}}{m\omega_0^2} = \dfrac{2\times 8\times 10^{-4}}{0.01\times (0.4)^2} = 1$

$\Rightarrow \quad a = 1\,\text{m}$

So, $x = a\sin(\omega_0 t + \phi) = \sin(0.4t + \phi)$

or $\quad x = a\cos(\omega_0 t + \phi) = \cos(0.4t + \phi)$

2. *(a)* Force on loop,

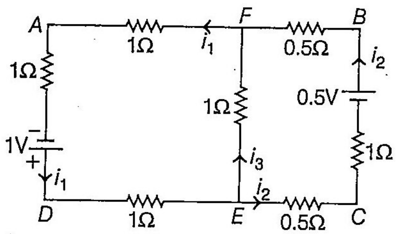
$$= F_{KLM} + F_{MNK}$$
$$= B_0 I(2R)\hat{\mathbf{i}} + B_0 I(2R)\hat{\mathbf{i}} = 4B_0 IR\hat{\mathbf{i}}$$

3. *(c)* Induced emf in loops $AFED$ and $FBCE$ are

$$e_1 = \frac{d\phi}{dt} = A\times\frac{dB}{dt} = 1\times 1\times 1 = 1\,\text{V}$$

and $\quad e_2 = A\times\dfrac{dB}{dt} = 1\times 0.5\times 1 = 0.5\,\text{V}$

As magnetic field is increasing, we have following circuit,



So, we have,

$$i_1 = i_2 + i_3$$

And loop rule in loop $ECBF$ gives.

$$-0.5 \times i_2 - 1 \times i_2 + 0.5 - i_2 \times 0.5 + 1 \times i_3 = 0$$

Also from loop $ADEF$;

$$-1 \times i_1 + 1 - 1 \times i_1 - 1 \times i_3 - 1 \times i_1 = 0$$

Solving, we get

$$i_1 = \frac{7}{22} \text{A}$$

$$i_2 = \frac{3}{11} \text{A}$$

$$i_3 = \frac{1}{22} \text{A}$$

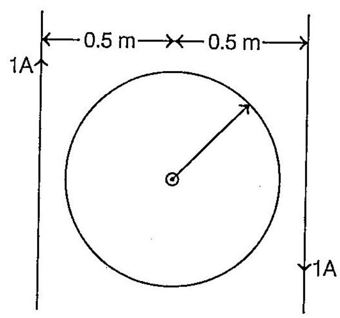
Problem Solving Tactics

First find the value induced emf using,

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt} BA = A \frac{dB}{dt}$$

After calculating the emf e_1 and e_2 using above formula, applying Kirchhoff's law to find the value of current in various branches.

4. *(b)*.



Torque (τ) experienced by a current loop in a uniform magnetic field (B), $\tau = |\mathbf{m} \times \mathbf{B}| = mB \sin\theta$

where, m = Magnetic dipole moment = IA

I = Current flowing through the coil = 1 A

A = Area of the coil.

Magnetic field at the loop is the net magnetic field, $B_{net} = B_1 + B_2$, where B_1 and B_2 are the field produced by wires 1 and 2. Hence $B_{net} = 2 \times \frac{\mu_0 I}{2\pi d}$

Now, $I = 1$ A, $d = 0.5$ m

$$\therefore \quad B_{net} = \frac{2\mu_0}{\pi} \text{T}$$

Now, dipole moment of the coil

$$m = NIA = I\pi r^2$$

$$= \pi \times (1 \times 10^{-2})^2$$

$$= \pi \times 10^{-4} \text{ A-m}^2$$

Now, torque using $\tau = mB \sin\theta$

$$= \pi \times 10^{-4} \times \frac{2\mu_0 I}{2\pi d} \times \frac{1}{2} \qquad \left(\because \sin 30° = \frac{1}{2}\right)$$

$$= \mu_0 \times 10^{-4} \text{ N-m}$$

5. *(a,b,c,d)* As A is suspended vertically from B, hence horizontal motion will not effect the vertical motion and hence A will have same horizontal velocity as that of B.

At height h, the length of the string will remain same and

$$y + \sqrt{x^2 + h^2} = L = \text{constant}$$

Motion of B and A are related to each other, so it is easy to say that A will have horizontal velocity and that will be equal to velocity of B.

According to the string constraint, total length of string must be constant.

$$y + \sqrt{x^2 + h^2} = L$$

On differentiating w.r.t. t, we get

$$\frac{dy}{dt} + \frac{x}{\sqrt{x^2 + h^2}} \frac{dx}{dt} = 0 \qquad \left[\frac{-dy}{dt} = v_A\right]$$

$$v_A = \frac{x}{\sqrt{x^2 + h^2}} v_B \qquad \left[\frac{dx}{dt} = v_B\right]$$

From figure, we can say that

$$\tan\theta = \frac{x}{h} \Rightarrow \sec^2\theta \left[\frac{d\theta}{dt}\right] = \frac{1}{h} \frac{dx}{dt}$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{v_B}{h \sec^2\theta} = \frac{v_B \cos^2\theta}{h}$$

Resultant velocity of A, $v = \sqrt{v_A^2 + v_B^2}$.

6. *(a,b,c,d)* $\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}}$

$$C_{P(He)} = \frac{5R}{2}, C_{P(H_2)} = \frac{7R}{2}$$

$$\therefore \quad C_{P\,mix} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}$$

$$= \frac{2 \times \frac{5R}{2} + 2 \times \frac{7R}{2}}{2 + 2} = \frac{12R}{4} = 3R$$

$$\therefore \quad C_{V_{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$= \frac{2 \times \frac{3R}{2} + 2 \times \frac{5R}{2}}{2+2} = 2R$$

$$\Rightarrow \quad \gamma_{mix} = \frac{3R}{2R} = \frac{3}{2}$$

For sudden compression,

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

By ideal gas equation, this can be written as

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

$$300 \times (3)^{3/2 - 1} = T \times (3/2)^{3/2 - 1}$$

$$T = 300\sqrt{2}\ \text{K}$$

At higher temperature, C_p and C_V varies with temperature.

So, γ also changes with temperature.

For adiabatic chamber $\Delta Q = 0$, so $\Delta U + \Delta W = 0$

So, work done and internal energy are inter convertable.

7. *(b,c)* For no slipping, there will not be any relative motion.

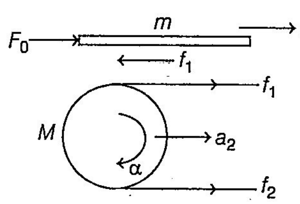
So, $\quad v = \omega R$

Also, $\quad \tau_{net} = I\alpha$

where, τ_{net} is net torque,

I is moment of inertia and α is angular acceleration.

The forces on different surfaces are shown below.



Here, in above diagram, all the forces are shown.

where,

f_1 = friction between plank and cylinder,

f_2 = friction between cylinder and ground,

a_1 = acceleration of plank and

a_2 = acceleration of CM of cylinder.

As, there is no slipping anywhere

$$a_1 = 2a_2 \quad \text{...(i)}$$

$$a_1 = \frac{F_0 - f_1}{m} \quad \text{...(ii)}$$

$$a_2 = \frac{f_1 + f_2}{M} \quad \text{...(iii)}$$

$$\therefore \quad \tau_{net} = I\alpha \Rightarrow \alpha = \frac{\tau_{net}}{I}$$

$$\Rightarrow \quad \alpha = \frac{(f_1 - f_2)R}{I} = \frac{(f_1 - f_2)R}{\frac{1}{2}MR^2}$$

$$= \frac{2(f_1 - f_2)}{MR} \quad \text{...(iv)}$$

$$a_2 = R\alpha = \frac{2(f_1 - f_2)}{M} \quad \text{...(v)}$$

Solving Eqs. (i) and (v), we get

$$\Rightarrow \quad a_1 = \frac{8F_0}{3M + 8m}$$

$$\Rightarrow \quad a_2 = \frac{4F_0}{3M + 8m}$$

$$\Rightarrow \quad f_1 = \frac{3MF_0}{3M + 8m}$$

$$\Rightarrow \quad f_2 = \frac{MF_0}{3M + 8m}$$

8. *(4)* Resistance, $r = \frac{\rho l}{a}$

Volume, $x = a \times l$

So, $$r = \frac{\rho l^2}{al} = \frac{\rho l^2}{x} \Rightarrow l = \left(\frac{r}{\rho}\right)^{1/2}$$

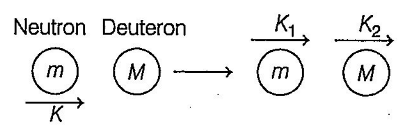
Also, $$a = \text{area} = \frac{\pi d^2}{4} \Rightarrow r = \frac{\rho l \times a}{a \times a} = \frac{\rho x}{a^2}$$

$$\Rightarrow \quad r = \frac{\rho x}{\left(\frac{\pi}{4} d^2\right)^2} \Rightarrow d^4 = \frac{16\rho x}{\pi^2 r}$$

$$\Rightarrow \quad d = \left(\frac{16\rho x}{\pi^2 r}\right)^{1/4}$$

$$\therefore \quad k = 4$$

9. *(8)*



Momentum conservation gives

$$\sqrt{2mK} = \sqrt{2mK_1} + \sqrt{2MK_2} \quad \text{...(i)}$$

Also, as collision is elastic,

$$K = K_1 + K_2 \quad \text{...(ii)}$$

Eqs. (i) and (ii) gives,

$$\sqrt{2mK} = \sqrt{2m(K - K_2)} + \sqrt{2MK_2}$$

$$\Rightarrow (\sqrt{2mK} - \sqrt{2MK_2})^2 = (\sqrt{2m(K - K_2)})^2$$

$$\Rightarrow \quad K_2(m + M) = 2\sqrt{mMKK_2}$$

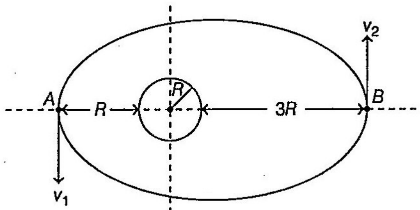
$$\Rightarrow \quad K_2 = \frac{4mM}{(M+m)^2} \cdot K$$

So, fraction of energy lost, $\eta = \frac{K_2}{K} = \frac{4mM}{(m+M)^2}$

So, $\eta = \frac{4 \times 1 \times 2}{(1+2)^2} = \frac{8}{9}$

$9\eta = 8$

10. (8)



Using, angular momentum conservation at A and B, we get

$$mv_1 (2R) = mv_2 (4R)$$

$$\Rightarrow \quad v_1 = 2v_2 \quad \text{...(i)}$$

Using conservation of energy, we get

$$\frac{1}{2}mv_1^2 - \frac{GMm}{2R} = \frac{1}{2}mv_2^2 - \frac{GMm}{4R} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$v_2 = \sqrt{\frac{GM}{6R}}$$

and $$v_1 = \sqrt{\frac{2GM}{3R}}$$

Now, if r = radius of curvature at B, then

$$\frac{mv_2^2}{r} = \frac{GMm}{(4R)^2}$$

$$\Rightarrow \quad r = \frac{16v_2^2R^2}{GM}$$

$$= \frac{8R}{3} \quad \text{(by substituting for } v_2\text{)}$$

$\therefore \quad N = 8$

11. *(2)* This problem is based on Moseley's observation can be mathematically expressed as

$$\sqrt{\nu} = a (Z - b)$$

where ν is frequency, Z is atomic number and a, b are constants. This relation is known as Moseley's law.

According to Moseley's law, K_α X-rays emission

$$\sqrt{\nu} = a (Z - 1)$$

$$\Rightarrow \quad (Z-1)^2 \propto \nu$$

or $$(Z-1)^2 \propto \frac{1}{\lambda}$$

$$\Rightarrow \quad \frac{(Z_{Mo} - 1)^2}{(Z_{Cu} - 1)^2} = \frac{\lambda_{Cu}}{\lambda_{Mo}}$$

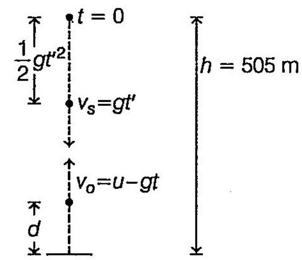
$$\Rightarrow \quad \lambda_{Cu} = \lambda_{Mo} \frac{(Z_{Mo} - 1)^2}{(Z_{Cu} - 1)^2}$$

$$= 0.71 \times \left(\frac{41}{28}\right)^2$$

$$= 1.52 \text{ Å}$$

$$\lambda = 2 \text{ Å}$$

12, *(1500)* The sound wave received at $t = 5$ s must have been emitted earlier. Let t' be the time at which the source emits a sound wave which is detected at $t = 5$s. So, the time for which this sound travels $t - t'$ i.e. $5 - t'$. Distance travelled by waves is $v(5 - t')$. Let the distance moved by the detector be d, then



$$d = ut - \frac{1}{2}g t^2 = 5u - \frac{25}{2}g$$

and $$h - \frac{1}{2}gt'^2 - d = v(5 - t')$$

or $$505 - \frac{1}{2}gt'^2 - 5u + \frac{25}{2}g = 300(5 - t')$$

$$\Rightarrow \quad 505 - 5t'^2 - 5(50) + \frac{25}{2}(10) = 1500 - 300t'$$

$$5t'^2 - 300t' + 1500 - 505 + 250 - 125 = 0$$

$$5t'^2 - 300t' + 1120 = 0$$

$$t'^2 - 60t' + 224 = 0 \Rightarrow t' = 4 \text{ s}$$

$\therefore$ Apparent frequency $= \left(\frac{v + v_0}{v - v_s}\right) f_0$

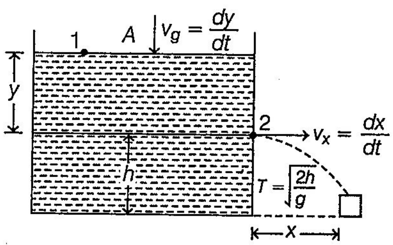
$$v_0 = u - gt = 50 - 10(5) = 0$$

$$v_s = gt' = 10(4) = 40 \text{ ms}^{-1}$$

$$\therefore \quad f' = \left(\frac{300 + 0}{300 - 40}\right) 1300$$

$$= 1500 \text{Hz}$$

13. *(100)* Here, velocity of efflux at point 2 is given by $v = \sqrt{2gy}$ and as condition is given in question that block should move in such a way that stream always hits the block.



So, velocity of block, $v_b = \frac{dx}{dt}$. Also, the velocity at point 1 will be rate of decrease in level of water, i.e. $\frac{dy}{dt}$.

Besides these concept, we have to use equation of continuity.

Velocity of efflux, $v = \sqrt{2gy}$

Range, $x = \sqrt{2gy} \times \sqrt{\frac{2h}{g}}$

The velocity of the block must be $\left(\frac{dx}{dt}\right)$

$\therefore \quad v_b = \frac{dx}{dt} = \sqrt{\frac{2h}{g}} \times \sqrt{2g} \times \frac{1}{2\sqrt{y}}\frac{dy}{dt}$

$v_b = \frac{\sqrt{h}}{\sqrt{y}} \cdot \frac{dy}{dt}$ ...(i)

Using equation of continuity, $\frac{Ady}{dt} = a\sqrt{2gy}$ ...(ii)

By Eqs. (i) and (ii), we get

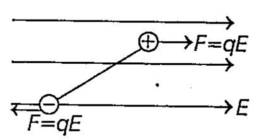
$$v_b = \sqrt{\frac{h}{y}} \times \frac{a}{A}\sqrt{2gy}$$

$$v_b = \sqrt{2gh} \times \frac{a}{A} = 20 \times \frac{1}{20} = 1\,\text{ms}^{-1}$$

$$= 100\,\text{cm s}^{-1}$$

14. *(2)* Torque on the arrangement is

$$\tau = |\mathbf{p} \times \mathbf{E}| = qLE\sin\theta$$



If I = moment of intertia of the arrangement, then

$$I = 2 \times m \times \left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$$

Also torque, $\tau = I\alpha \; - qLE\sin\theta = \frac{mL^2}{2}\alpha$

$\Rightarrow \quad \alpha = -\frac{2qE}{mL}\sin\theta$

For small angles, $\sin\theta \approx \theta$

$\Rightarrow \quad \alpha = \frac{-2qE}{mL}\theta$

So, $|\alpha|$ = angular acceleration $= \frac{2qE}{mL}\theta$

$\therefore \quad n = 2$

15. *(2)* Since, $\alpha = -\frac{2qE}{mL}\theta \Rightarrow \frac{d^2\theta}{dt^2} + \frac{2qE}{mL} \cdot \theta = 0$

So, time period is $T = 2\pi\sqrt{\frac{mL}{2qE}}$

So, time required by rod to get aligned with the field is

$$\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$$

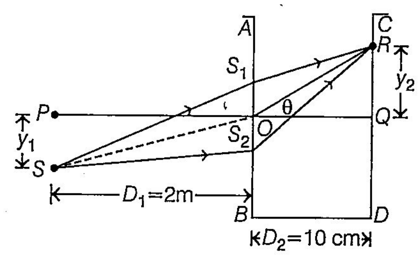
$\therefore \quad p = 2$

16. *(2)* Given, $y_1 = 40\,\text{cm}$, $D_1 = 2\text{m} = 200\,\text{cm}$, $D_2 = 10\,\text{cm}$

$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5}$$

$$\alpha = \tan^{-1}(1/5)$$

$$\sin\alpha = \frac{1}{\sqrt{26}} = \frac{1}{5} = \tan\alpha$$



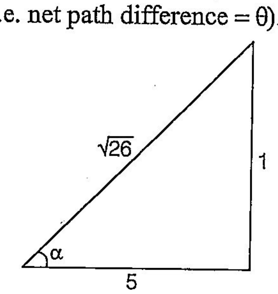
Path difference between SS_1 and SS_2 is

$$\Delta x_1 = SS_1 - SS_2$$

or $\quad \Delta x_1 = d\sin\alpha = (0.8\,\text{mm})\left(\frac{1}{5}\right)$

or $\quad \Delta x_1 = 0.16\,\text{mm}$ ...(i)

Now,let a point R on the screen, central bright fringe observed (i.e. net path difference = θ).



Path difference between S_2R and S_1R would be

$$\Delta x_2 = S_2R - S_1R$$

or $\Delta x_2 = d\sin\theta$

Central bright fringe will be observed when net path difference is zero.

or $\Delta x_2 - \Delta x_1 = 0$

$\Delta x_2 = \Delta x_1$

or $d\sin\theta = 0.16$

or $(0.8)\sin\theta = 0.16$

or $\sin\theta = \dfrac{0.16}{0.8} = \dfrac{1}{5}$

$\tan\theta = \dfrac{1}{\sqrt{24}} \approx \sin\theta = \dfrac{1}{5}$

Hence, $\tan\theta = \dfrac{y_2}{D_2} = \dfrac{1}{5}$

$\therefore$ $y_2 = \dfrac{D_2}{5} = \dfrac{10}{5} = 2\,\text{cm}$

17. *(1)* The central bright fringe will be observed at point Q. If the path difference created by the liquid slab of thickness $t = 10\,\text{cm}$ or 100 mm is equal to Δx_1, so that the net path difference at Q becomes zero.

So, $(\mu - 1)t = \Delta x_1$

or $(\mu - 1)(100) = 0.16$

or $\mu - 1 = 0.0016$

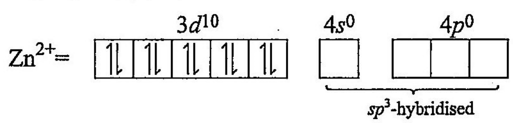
or $\mu = 1.0016 \simeq 1$

Chemistry

1. *(a)* Diamagnetic molecules are

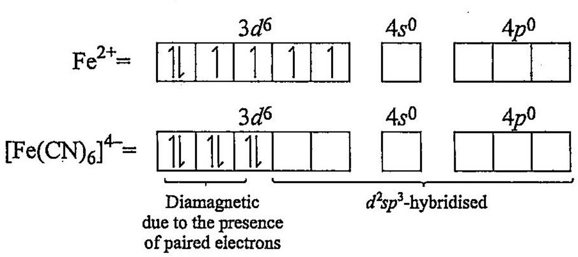
$[Zn(OH)_4]^{2-}$, $K_4[Fe(CN)_6]$, $[PdBr_4]^{2-}$, $[Ni(CO)_4]$.

- In $[Zn(OH)_4]^{2-}$, $Zn^{2+} = 3d^{10}\,4s^0$

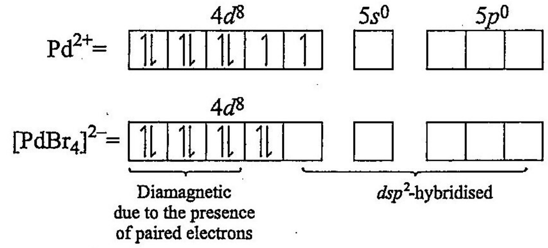


$[Zn(OH)_4]^{2-}$ is diamagnetic due to the paired electrons.

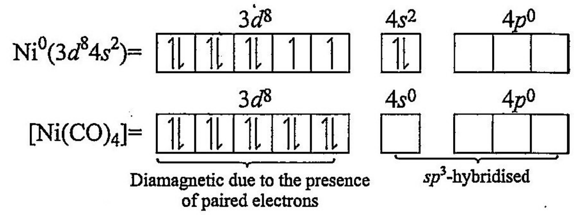
- In $K_4[Fe(CN)_6]$, Fe is in + 2 oxidation state.



- In $[PdBr_4]^{2-}$, Pd is in + 2 oxidation state with electronic configuration $4d^8\,5s^0$.



- In $[Ni(CO)_4]$, Ni is in zero oxidation state.



2. (*b*) By Soddy-Fajans' group displacement law, emission of one α-particle displaces the daughter element two groups left and emission of one β-particle displaces the daughter element one group right.

$$\underset{\text{III B}}{{}^{235}_{92}U} \xrightarrow{-\alpha} \underset{G_1 = \text{III B}}{{}^{231}_{90}A} \xrightarrow{-\alpha} \underset{G_2 = \text{II A}}{{}^{227}_{88}B}$$

$$\xrightarrow{-\beta} \underset{G_3 = \text{III B}}{{}^{227}_{89}C} \xrightarrow{-\beta} \underset{G_4 = \text{III B}}{{}^{227}_{90}B}$$

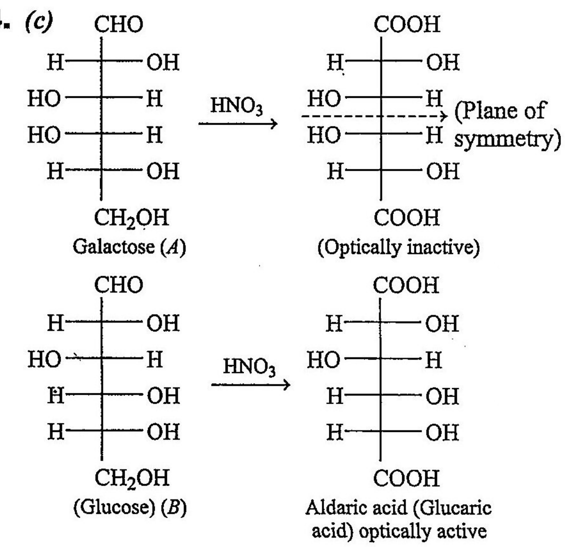
Actinide series

Elements (90-103) are in III B [actinide series including Ac(89)].

Element Ra (88) is of II A.

3. (*d*) Rhombohedral system have $a = b = c$ sides and $\alpha = \beta = \gamma \neq 90°$ angles.

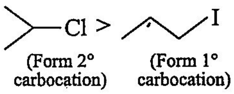
4. (*c*)



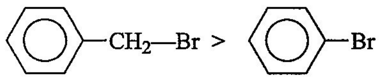
5. *(a,c,d)*

(a) For S_N2, 1° RX > 2° RX

(b) For S_N1, rate is found to be difficult with

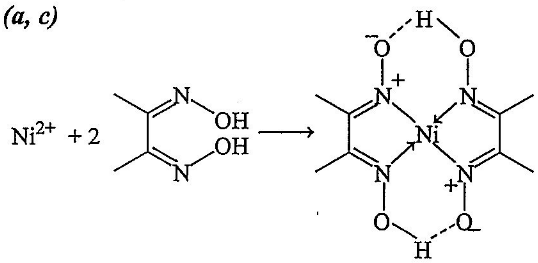


(c) For S_N2, rate is found to be difficult with aryl halides hence,



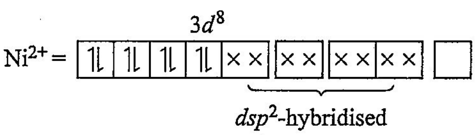
(d) Rate of elimination reaction (E_2 or E_1) is always faster with 3° carbon bearing the leaving group.

6. *(a, c)*

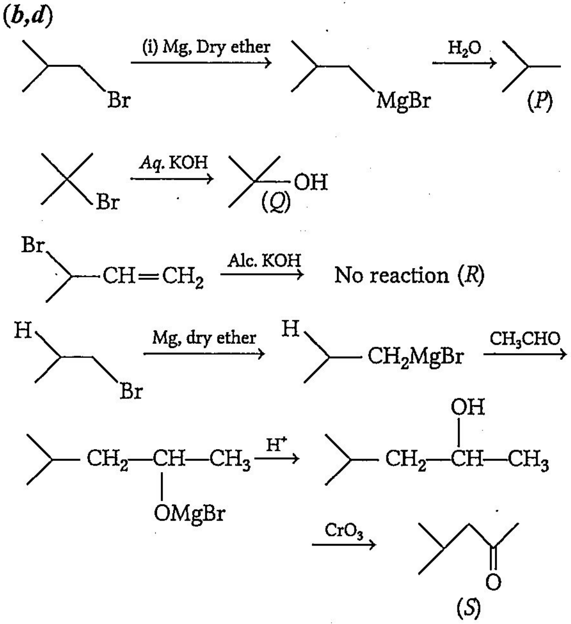


Ni(dmg)$_2$

In Ni(dmg)$_2$ complex, nickel is in +2 oxidation state electronic configuration of $Ni^{2+} = 3d^8 4s^0$.



7. *(b,d)*



8. *(5)* The molecular weight of ketone = 100

and molecular formula = $C_6H_{12}O$ $(6 \times 12 + 12 \times 1 + 16 = 100)$

The possible structure will be

(i) $CH_3—CH_2—CH_2—CH_2—C(=O)—CH_3 \xrightarrow{NaBH_4}$

$CH_3—CH_2—CH_2—CH_2—C^*(OH)(CH_3)—CH_3$

(ii) $CH_3—CH_2—CH_2—C(=O)—CH_2CH_3 \xrightarrow{NaBH_4}$

$CH_3—CH_2—CH_2—{}^*C(OH)(H)—CH_2—CH_3$

(iii) $CH_3—CH(CH_3)—CH_2—C(=O)—CH_3 \xrightarrow{NaBH_4}$

$CH_3—CH(CH_3)—CH_2—{}^*C(H)(OH)—CH_3$

(iv) $CH_3—CH_2—\overset{*}{C}H(CH_3)—C(=O)—CH_3 \xrightarrow{NaBH_4}$

$CH_3—CH_2—\overset{*}{C}H(CH_3)—{}_*C(H)(OH)—CH_3$

(v) $CH_3—CH(CH_3)—C(=O)—CH_2—CH_3 \xrightarrow{NaBH_4}$

$CH_3—CH(CH_3)—{}_*C(OH)(H)—CH_2—CH_3$

(vi) $CH_3—C(CH_3)_2—C(=O)—CH_3 \xrightarrow{NaBH_4}$

$CH_3—C(CH_3)_2—C_*(OH)(H)—CH_3$

Out of these, only (iv) have chiral carbon and will not give racemic product on reduction. Hence, the total number of ketones that give a racemic product ON reaction with $NaBH_4$ is 5.

Problem Solving Tactics The ketone already has a chiral centre and undergoes reduction. It will form another chiral centre in the compound. These chiral centres are adjacent to each other, compounds form the diastereomers. These not form racemic mixture while enantiomers give.

9. (2) $[Ni(CN)_4]^{2-}$, $[Pt(Cl)_4]^{2-}$

Hybridisation $= dsp^2$

Shape of molecule = Square planar.

- PF_5 , PCl_5

  Hybridisation $= sp^3d$

  Shape of molecule = Trigonal bipyramidal.
- BrF_5

  Hybridisation $= sp^3d^2$

  Shape of molecule = Square pyramidal.
- SF_6, $[CrF_6]^{3-}$, $[Co(NH_3)_6]^{3+}$

  Hybridisation $= sp^3d^2, d^2sp^3$

  Shape of molecules = Octahedral
- CH_4 , NH_4^+, $[Ni(CO)_4]$

  Hybridisation $= sp^3$

  Shape of molecules = Tetrahedral.

10. (8) At constant volume and temperature for a gas $p \propto w$.

Given, for N_2, $p_1 = 3$ atm, $p_2 = \frac{1}{2}$ atm, $w_1 = 14$ kg, $t = 30$ mins.

$$\because \quad \frac{p_1}{p_2} = \frac{w_1}{w_2} \Rightarrow \frac{3}{\frac{1}{2}} = \frac{14}{w_2}$$

$$\Rightarrow \quad w_2 = \frac{14}{6} \text{ kg } N_2$$

$\therefore$ Mass of N_2 diffused

$$= 14 - \frac{14}{6} = \frac{70}{6} \text{ kg}$$

Similarly, for H_2

$p_1 = 3$ atm, $p_2 = \frac{1}{2}$ atm $w_1 = 1$ kg

$$\therefore \quad w_2 = \frac{1}{6} \text{ kg}$$

$\therefore$ Mass of H_2 diffused $= 1 - \frac{1}{6} = \frac{5}{6}$ kg

According to Graham's law of diffusion,

$$\Rightarrow \frac{r_{N_2}}{r_{H_2}} = \sqrt{\frac{(M_{H_2})}{(M_{N_2})}} \text{ or } \frac{w_{H_2}}{w_{N_2}} \times \frac{t_{N_2}}{t_{H_2}} = \sqrt{\frac{2}{28}}$$

$$\Rightarrow \quad \frac{\frac{5}{6}}{\frac{70}{6}} \times \frac{30}{t_{H_2}} = \sqrt{\frac{1}{14}} \Rightarrow t \simeq 8 \text{ mins}$$

11. (8) Photon will be in Brackett series

$[\because 0.31 \le E \le 0.85 \text{ (in eV)}]$ for Brackett

$$0.85\left(1-\frac{1}{4}\right) = 13.6\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$$

$$0.85\left(1-\frac{1}{4}\right) = \frac{13.6}{16}\left(1-\left(\frac{4}{n}\right)^2\right)$$

$$\Rightarrow \quad \frac{4}{n} = \frac{1}{2} \Rightarrow n = 8$$

Hence, $n = 8$

Problem Solving Tactics The solve this problem, you should learn the energy range of photon for different series like, Brackett pfund in order to know the value of n (ground state energy level).

12. (1) By Freundlich adsorption isotherm, for adsorption of solution on a solid surface.

$$\frac{x}{m} = K(C)^{\frac{1}{n}}$$

$$\Rightarrow \quad \frac{x}{m} = 0.50(C)^{\frac{1}{3}}$$

Let, solute absorbed $= x$ g

Then, equilibrium concentration $= (2-x)$g

$$\therefore \quad \frac{x}{2} = 0.50(2-x)^{1/3}$$

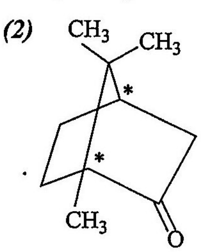
$$\Rightarrow \quad x = 1(2-x)^{\frac{1}{3}}$$

$$\Rightarrow \quad x^3 = (2-x)$$

$$\therefore \quad x^3 + x - 2 = 0$$

Thus, $x = 1$ g

13. (2)



$\Rightarrow$ * Represents chiral centres.

The total number of stereoisomers $= 2^n = 2^2 = 4$

where, n = no. of chiral centres.

However, the given molecule cannot show geometrical isomerism due to lot of steric repulsion and strain. So, only mirror images can exist. Therefore, 2 stereoisomers are possible.

14. *(166)* $\Delta G^\circ = -RT \ln K$ ...(i)

Also, $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ ...(ii)

Comparing equations (i) and (ii),

$$-RT \ln K = \Delta H^\circ - T\Delta S^\circ$$

Given, $K = \frac{p_c}{p^\circ}$ and $p^\circ = 1\,\text{bar}$

So, $-RT \ln\left(\frac{p_c}{1}\right) = \Delta H^\circ - T\Delta S^\circ$

$$\ln\left(\frac{p_C}{1}\right) = -\frac{\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R} \quad \text{...(iii)}$$

In graph, slope $= \frac{-\Delta H^\circ}{R}$

$$\text{Slope} = 10^4\left(\frac{Y_2 - Y_1}{X_2 - X_1}\right) = 10^4\left(\frac{-7+3}{12-10}\right)$$

$$= 10^4\left(\frac{-4}{2}\right) = -2\times 10^4$$

$\therefore$ $\frac{-\Delta H^\circ}{R} = -2\times 10^4$

$\Rightarrow 2\times 10^4 \times 8.3 = \Delta H^\circ$

$\Rightarrow \Delta H^\circ = 16.6\times 10^4\ \text{J mol}^{-1} \approx 166\,\text{kJ mol}^{-1}$

15. *(141)* From graph, $\frac{10^4}{T} = 10$

$\therefore$ $T = 10^3\ \text{K}$

Also, $\ln\left(\frac{p_c}{1}\right) = -3$

Substituting the value in Eq. (iii),

$$\ln\left(\frac{p_c}{1}\right) = \frac{-\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$$

$$-3 = \frac{-2\times 10^4}{10^3} + \frac{\Delta S^\circ}{R}$$

$\Rightarrow \frac{\Delta S^\circ}{R} = -3 + 20 = 17$

$\Rightarrow \Delta S^\circ = 17\times 8.3 = 141.1\,\text{JK}^{-1}\text{mol}^{-1} \approx 141\,\text{JK}^{-1}\ \text{mol}^{-1}$.

16. *(100.10)* The complete reaction is as follow

$$AgNO_3(aq) \longrightarrow Ag^+(aq) + NO_3^-(aq)$$

Given, molality, $m = 0.1$ m

van't Hoff factor, $i = 2$

$$\Delta T_b = iK_b m$$

where, ΔT_b = Change in boiling point of water.

K_b = Ebullioscopic constant $= 0.5\,\text{K kg mol}^{-1}$

$\therefore$ $\Delta T_b = 2\times 0.5\times 0.1 = 0.1$

Now, $\Delta T_b = T_b - T_b^\circ$

$T_b = \Delta T_b + T_b^\circ$

$T_b = 0.1 + 100 = 100.1°\text{C}$

$\therefore$Boiling point of solution (*A*), x°C $= 100.10$°C.

17. *(25)* According to statement given in paragraph, the reactions can be written as

$$\underset{0.05\,m}{AgNO_3(aq)} \longrightarrow \underset{0.05\,m}{Ag^+(aq)} + \underset{0.05\,m}{NO_3^-(aq)}$$

$$\underset{0.05\,m}{BaCl_2(aq)} \longrightarrow \underset{0.05\,m}{Ba^{2+}(aq)} + \underset{0.1\,m}{2Cl^-(aq)}$$

Ag^+ and Cl^- combine to form AgCl precipitate.

$$Ag^+(aq) + Cl^-(aq) \longrightarrow AgCl(s)$$

At $t = 0$ 0.05 m 0.1 m

At $t = \infty$ 0 0.05 m

Total concentration of all ions in solution

$[Cl^-] + [NO_3^-] + [Ba^{2+}] = 0.05 + 0.05 + 0.05 = 0.15\,M$

$\therefore$ $\Delta T_b = 0.5\times 0.15 = 0.075$°C

$\therefore$ $\Delta T_b = T_b - T_b^\circ$

$0.075 = T_b - 100$

$T_b = 100.075$°C

$\therefore$ Boiling point of solution $B = 100.075$° C

and boiling point of solution $A = 100.1$° C

Now, difference in boiling point of solution

$|y| = 100.1 - 100.075$

$= 0.025$° C

$= 25\times 10^{-3}$°C

Mathematics

1. *(d)* $\therefore y = c_1e^{2x} + c_2e^x + c_3e^{-x}$

$\Rightarrow e^x y = c_1e^{3x} + c_2e^{2x} + c_3$

On differentiating both sides w.r.t. *x*, we get

$$e^x\left(\frac{dy}{dx}\right) + y\cdot e^x = 3c_1e^{3x} + 2c_2e^{2x} + 0$$

$$\Rightarrow \quad e^{-x}\left(\frac{dy}{dx} + y\right) = 3c_1e^x + 2c_2$$

Again, on differentiating both sides w.r.t. *x*, we get

$$e^{-x}\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) + \left(\frac{dy}{dx} + y\right)(-e^{-x}) = 3c_1e^x$$

$$\Rightarrow \quad e^{-2x}\left(\frac{d^2y}{dx^2} - y\right) = 3c_1$$

Finally, on differentiating both sides w.r.t *x*, we get

$$e^{-2x}\left(\frac{d^3y}{dx^3} - \frac{dy}{dx}\right) + \left(\frac{d^2y}{dx^2} - y\right)(-2e^{-2x}) = 0$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

On comparing with $\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, we get

$\therefore$ $a = -2$, $b = -1$ and $c = 2$

$$\frac{a^2+c^2}{abc}=\frac{(-2)^2+(2)^2}{(-2)\times(-1)\times(2)}=\frac{8}{4}=2$$

Problem Solving Tactics	From the given condition $y=c_1e^{2x}+c_2e^{x}+c_3e^{-x}$ in similar format to the given diffenential equation and by comparing both equations, find the value of a, b and c.

2. *(c)* We know the prime digits are 2, 3, 5, 7. If we fix 2 at the first place, then rest of $2n-1$ places can be filled in 4^{2n-1} ways. Sum of 2 consecutive digits is prime when consecutive digits are (2, 3) or (2, 5), thus 2 will be fixed at all alternate places.

i.e.

2		2		2

For filling n places by 2, we have only 1 way and for filling rest of n places, by 3 or 5.

Number of ways $=2^n$

$\therefore$ Number of favourable ways $=2^n$

Required probability $=\frac{2^n}{4^{2n-1}}=\frac{2^n\times 4}{2^{4n}}=\frac{4}{2^{3n}}=4\times 2^{-3n}$

Now, on comparing with $a\times b^{cn}$,

we get $a=4, b=2$ and $c=-3$

Therefore, $a^2+b^2-c^2=(4)^2+(2)^2-(-3)^2$

$=16+4-9=11$

3. *(c)* Let $I=\int_0^1\frac{4x^3[1+(x^4)^{2016}]}{(1+x^4)^{2018}}dx$

On putting $x^4=t\Rightarrow 4x^3dx=dt$

$$I=\int_0^1\frac{1+t^{2016}}{(1+t)^{2018}}dt$$

$$\Rightarrow I=\int_0^1\frac{dt}{(1+t)^{2018}}+\int_0^1\frac{t^{2016}}{(1+t)^{2018}}dt$$

$$\Rightarrow I=\int_0^1\frac{dt}{(1+t)^{2018}}+\int_0^1\frac{1}{t^2\left(1+\frac{1}{t}\right)^{2018}}dt$$

$$\Rightarrow I=\left[\frac{(1+t)^{-2017}}{-2017}\right]_0^1+\left[\frac{\left(1+\frac{1}{t}\right)^{-2017}}{2017}\right]_0^1$$

$$\Rightarrow I=\frac{-1}{2017}\left[\frac{1}{2^{2017}}-1\right]+\frac{1}{2017}\left[\frac{1}{2^{2017}}-0\right]$$

$$\Rightarrow I=\frac{1}{2017}\left[-\frac{1}{2^{2017}}+1+\frac{1}{2^{2017}}\right]=\frac{1}{2017}$$

Here, $m=1$ and $n=2017$

$$\frac{m+n}{504.5}=\frac{2018}{504.5}=4$$

4. *(a)* We have, $r=\max|x_i-x_j|\,\forall\, 1\le i,j\le n$

and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$

Now, $(x_i-\bar{x})^2=\left(x_i-\frac{x_1+x_2+\ldots+x_n}{n}\right)^2$

$$=\frac{1}{n^2}[(x_i-x_1)+(x_i-x_2)+\ldots+(x_i-x_{i-1})+(x_i-x_{i+1})+\ldots+(x_i-x_n)]^2\le r^2$$

$$\Rightarrow\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2\le\frac{nr^2}{n-1}$$

$$\Rightarrow S^2\le\frac{nr^2}{(n-1)}$$

$$\Rightarrow S\le r\sqrt{\frac{n}{n-1}}$$

5. *(a,b,c,d)* We have, $P(X=x)=k\,(x+1)\left(\frac{1}{5}\right)^x$

Since, $\sum_{x=0}^{\infty}P(X=x)=1$

$$\Rightarrow k\left[1+2\left(\frac{1}{5}\right)+3\left(\frac{1}{5}\right)^2+\ldots\infty\right]=1$$

$$\Rightarrow k\left\{\left(1-\frac{1}{5}\right)^{-2}\right\}=1$$

$$\Rightarrow k\left(\frac{25}{16}\right)=1$$

$$\Rightarrow k=\frac{16}{25}$$

So, $P(X=x)=\frac{16}{25}(x+1)\left(\frac{1}{5}\right)^x$

Option (a), $P(X=0)=\frac{16}{25}(0+1)\left(\frac{1}{5}\right)^0=\frac{16}{25}$

Option (b), $P(X\le 1)=P(X=0)+P(X=1)$

$=\frac{16}{25}+\frac{2}{5}\times\frac{16}{25}$

$=\frac{16}{25}\left(1+\frac{2}{5}\right)$

$=\frac{16}{25}\times\frac{7}{5}=\frac{112}{125}$

Option (c), $P(X\ge 1)=1-P(X=0)$

$=1-\frac{16}{25}=\frac{9}{25}$

Option (d), $E(X)=\sum_{x=0}^{\infty}xP(X=x)$

$$\Rightarrow \quad E(X)=\sum_{x=0}^{\infty} x(x+1)\left(\frac{1}{5}\right)^x$$

$$\Rightarrow \quad E(X)=1\times 2\left(\frac{1}{5}\right)+2\times 3\left(\frac{1}{5}\right)^2+3\times 4\left(\frac{1}{5}\right)^3+\ldots.\infty \quad \ldots(i)$$

$$\therefore \quad \frac{1}{5}E(X)=1\times 2\left(\frac{1}{5}\right)^2+2\times 3\left(\frac{1}{5}\right)^3+\ldots\infty \quad \ldots(ii)$$

On substracting Eq. (ii) from Eq. (i), we get

$$\frac{4}{5}E(X)=\frac{2}{5}+\frac{2}{5}\left\{\frac{2}{5}+\frac{3}{5^2}+\frac{4}{5^3}+\ldots\infty\right\}$$

$$=\frac{2}{5}\left\{1+\frac{2}{5}+\frac{3}{5^2}+\ldots\infty\right\}$$

$$=\frac{2}{5}\left(1-\frac{1}{5}\right)^{-2}=\frac{2}{5}\times\frac{25}{16}=\frac{5}{8}$$

So, $E(X)=\frac{5}{4}\times\frac{5}{8}=\frac{25}{32}$

6. ***(a,b,c)*** Consider the given differential equation

$$\tan^2 x\frac{dy}{dx}=\sec x(1-y)-\frac{dy}{dx}$$

$$\Rightarrow \quad (1+\tan^2 x)\frac{dy}{dx}=\sec x(1-y)$$

$$\Rightarrow \quad \sec^2 x\frac{dy}{dx}+y\sec x=\sec x$$

$$\Rightarrow \quad \frac{dy}{dx}+y\cos x=\cos x$$

$$\text{IF}=e^{\int\cos x dx}=e^{\sin x}$$

$\therefore$ Solution is $ye^{\sin x}=\int e^{\sin x}\cos x\,dx=e^{\sin x}+C$

Now, for $x=0$ and $y=1$, we have

$$1=1+C\Rightarrow C=0$$

$$\Rightarrow \quad ye^{\sin x}=e^{\sin x} \quad \Rightarrow y=1$$

$$\therefore \quad f(x)=1$$

which is continuous, periodic, differential and even function.

7. ***(a,b,c,d)***

$$\because \quad 2x-3y+4z+2=0 \quad \ldots(i)$$

$$2x-3y+4z+6=0 \quad \ldots(ii)$$

$$4x-6y+8z+3=0$$

$$\Rightarrow \quad 2x-3y+4z+\frac{3}{2}=0 \quad \ldots(iii)$$

and $\quad 2x-3y+4z-6=0 \quad \ldots(iv)$

Since, the planes are all parallel planes.

$$\therefore \quad P_1=\frac{|2-6|}{\sqrt{2^2+3^2+4^2}}=\frac{4}{\sqrt{29}}$$

Similarly, $P_2=\dfrac{\left|2-\frac{3}{2}\right|}{\sqrt{2^2+3^2+4^2}}=\dfrac{\left(\frac{1}{2}\right)}{\sqrt{29}}$

$$=\frac{1}{2\sqrt{29}}$$

$$P_3=\frac{|2-(-6)|}{\sqrt{2^2+3^2+4^2}}=\frac{8}{\sqrt{29}}$$

$$\therefore \quad P_1=\frac{4}{\sqrt{29}},P_2=\frac{1}{2\sqrt{29}} \text{ and } P_3=\frac{8}{\sqrt{29}}$$

So, $P_1=8P_2$ and $P_3=16P_2$ and $P_1+2P_2+3P_3=\sqrt{29}$

and $P_1+8P_2=P_3$

$$\Rightarrow \quad P_1+8P_2-P_3=0$$

8. ***(10)*** It is given that α and β are the roots of equation

$$\frac{1-8(\log_{10}x)^2}{\log_{10}x-2[\log_{10}x]^2}=1$$

Let $\quad \log_{10}x=y$

So, $\quad \dfrac{1-8y^2}{y-2(y)^2}=1$

$$1-8y^2=y-2y^2$$

$$\Rightarrow \quad -8y^2+2y^2-y+1=0$$

$$\Rightarrow \quad -6y^2-y+1=0$$

$$\Rightarrow \quad 6y^2+y-1=0$$

$$\Rightarrow \quad 6y^2+3y-2y-1=0$$

$$\Rightarrow \quad 3y-1=0,2y+1=0$$

$$y=\frac{1}{3} \text{ and } y=\frac{-1}{2}$$

Therefore, $\log_{10}x=-\frac{1}{2}$ and $\log_{10}x=\frac{1}{3}$

$$\Rightarrow \quad x=10^{-\frac{1}{2}} \text{ or } x=10^{1/3}$$

$$\Rightarrow \quad x=\frac{1}{10^{1/2}} \text{ or } x=10^{1/3}$$

So, $\quad \alpha=\frac{1}{10^{1/2}}$ and $\beta=10^{1/3}$ $\quad [\because \alpha<\beta]$

Now, $\alpha^2\beta^3+1=\left(\frac{1}{10^{1/2}}\right)^2(10^{1/3})^3+1$

$$=\frac{1}{10}\times 10+1\Rightarrow 1+1=2$$

$$\therefore \quad (\alpha^2\beta^3+1)^2=2^2=4$$

Again, $\alpha^4=10^{-2}$

$$\Rightarrow \quad \alpha^4=\frac{1}{100} \text{ or } \frac{1}{\alpha^4}=100$$

Now, $\frac{1}{10}\left[\frac{(\alpha^2\beta^3+1)^2}{\alpha^4}-30000\alpha^4\right]$

$$=\frac{1}{10}\left[(2)^2 100-30000\times\frac{1}{100}\right]$$

$$=\frac{1}{10}[400-300]=\frac{100}{10}=10$$

9. *(5)* Let $\alpha=\cos^{-1}\sqrt{p}$,

$\beta=\cos^{-1}\sqrt{1-p}$

and $\gamma=\cos^{-1}\sqrt{1-q}$

Then, $\cos\alpha=\sqrt{p}$

$\cos\beta=\sqrt{1-p}$

and $\cos\gamma=\sqrt{1-q}$

$\Rightarrow \sin\alpha=\sqrt{1-p}$, $\sin\beta=\sqrt{p}$ and $\sin\gamma=\sqrt{q}$

We can write $\alpha+\beta+\gamma=\frac{3\pi}{4}$

$\Rightarrow \quad \alpha+\beta=\frac{3\pi}{4}-\gamma$

$\Rightarrow \quad \cos(\alpha+\beta)=\cos\left(\frac{3\pi}{4}-\gamma\right)$

$\Rightarrow \quad \cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\cos\frac{3\pi}{4}\cos\gamma+\sin\frac{3\pi}{4}\sin\gamma$$

$\Rightarrow \sqrt{p}\sqrt{1-p}-\sqrt{1-p}\sqrt{p}=-\frac{1}{\sqrt{2}}\cos\gamma+\frac{1}{\sqrt{2}}\sin\gamma$

$\Rightarrow \quad 0=-\cos\gamma+\sin\gamma$

$\Rightarrow \quad 0=-\sqrt{1-q}+\sqrt{q}$

$\Rightarrow \quad \sqrt{q}=\sqrt{1-q}$

$\Rightarrow \quad q=1-q$

$\Rightarrow \quad q=\frac{1}{2}$

$\Rightarrow \quad 10q=5$

10. *(25)* We have,

$$f\left(\frac{\pi}{6}+x\right)+f\left(\frac{\pi}{3}-x\right)=\frac{\pi}{2}$$

x is replaced by $\frac{\pi}{3}-x$, we get

$$f\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)+f\left(\frac{\pi}{3}-\frac{\pi}{3}+x\right)=\frac{\pi}{2}$$

$\Rightarrow \quad f\left(\frac{\pi}{2}-x\right)+f(x)=\frac{\pi}{2}$

$\Rightarrow \quad f\left(\frac{\pi}{2}-x\right)=\frac{\pi}{2}-f(x) \quad \ldots(i)$

Now, let $I=\int_0^{\pi/2}[\cos f(x)]^2\,dx$

$$I=\int_0^{\pi/2}\left[\cos f\left(\frac{\pi}{2}-x\right)\right]^2 dx$$

$$\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$\Rightarrow \quad I=\int_0^{\pi/2}\left[\cos\left(\frac{\pi}{2}-f(x)\right)\right]^2 dx \quad$ [using Eq. (i)]

$\Rightarrow \quad I=\int_0^{\pi/2}[\sin f(x)]^2\,dx$

$\Rightarrow \quad 2I=\int_0^{\pi/2}[\cos^2 f(x)+\sin^2 f(x)]\,dx$

$\Rightarrow \quad 2I=\int_0^{\pi/2}dx \Rightarrow 2I=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$

$\therefore \quad k\pi=\frac{\pi}{4}$

$\Rightarrow 100k=25$

11. *(1)* Given, $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1$

$[\mathbf{b}\times\mathbf{c}\ \ \mathbf{c}\times\mathbf{a}\ \ \mathbf{a}\times\mathbf{b}]=(\mathbf{b}\times\mathbf{c})\cdot[(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})]$

$$=(\mathbf{b}\times\mathbf{c})\cdot[((\mathbf{c}\times\mathbf{a})\cdot\mathbf{b})\,\mathbf{a}-((\mathbf{c}\times\mathbf{a})\cdot\mathbf{a})\,\mathbf{b}]$$

$$=(\mathbf{b}\times\mathbf{c})\cdot[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\mathbf{a}$$

$$=[\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]^2=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$$

$[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=[\mathbf{b}\times\mathbf{c}\ \ \mathbf{c}\times\mathbf{a}\ \ \mathbf{a}\times\mathbf{b}]$

$\Rightarrow \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$

$\Rightarrow \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=1$

Projection of $\mathbf{b}+\mathbf{c}$ on $\mathbf{a}\times\mathbf{b}$ is

$$=\frac{(\mathbf{b}+\mathbf{c})\cdot(\mathbf{a}\times\mathbf{b})}{|\mathbf{a}\times\mathbf{b}|}=\frac{[\mathbf{a}\ \mathbf{b}\,\mathbf{c}]}{[\mathbf{a}\times\mathbf{b}]}=\frac{1}{1}=1$$

12. *(150)* We have,

$$f(x)=\begin{vmatrix}\cos(x+\alpha) & \sin(x+\alpha) & \sin(\beta-\gamma)\\ \cos(x+\beta) & \sin(x+\beta) & \sin(\gamma-\alpha)\\ \cos(x+\gamma) & \sin(x+\gamma) & \sin(\alpha-\gamma)\end{vmatrix}$$

$$f'(x)=\begin{vmatrix}-\sin(x+\alpha) & \sin(x+\alpha) & \sin(\beta-\gamma)\\ -\sin(x+\beta) & \sin(x+\beta) & \sin(\gamma-\alpha)\\ -\sin(x+\gamma) & \sin(x+\gamma) & \sin(\alpha-\gamma)\end{vmatrix}$$

$$+\begin{vmatrix}\cos(x+\alpha) & \cos(x+\alpha) & \sin(\beta-\gamma)\\ \cos(x+\beta) & \cos(x+\beta) & \sin(\gamma-\alpha)\\ \cos(x+\gamma) & \cos(x+\gamma) & \sin(\alpha-\gamma)\end{vmatrix}$$

$$+\begin{vmatrix}\cos(x+\alpha) & \sin(x+\alpha) & 0\\ \cos(x+\beta) & \sin(x+\beta) & 0\\ \cos(x+\gamma) & \sin(x+\gamma) & 0\end{vmatrix}$$

$\Rightarrow \quad f'(x)=0+0+0=0$

$\Rightarrow \quad f$ is a constant function.

$\therefore f(x) = 6 \; \forall x \in R$ $\qquad [\because f(2) = 6]$

$\therefore \sum_{x=1}^{25} f(r) = \sum_{x=1}^{25} f(x) = \sum_{x=1}^{25} 6 = 150$

13. *(4)* We have, $y = \ln\left(\frac{2+\sqrt{4-x^2}}{2-\sqrt{4-x^2}}\right) - \sqrt{4-x^2}$

On putting $x = 2\sin\theta$,

$\therefore \; y = \ln\left(\frac{2+2\cos\theta}{2-2\cos\theta}\right) - 2\cos\theta$

$= \ln\left(\cot^2\frac{\theta}{2}\right) - 2\cos\theta$

$\Rightarrow \; y = 2\ln\cot(\theta/2) - 2\cos\theta$

$\Rightarrow \; \frac{dy}{d\theta} = \frac{-\text{cosec}^2(\theta/2)}{\cot(\theta/2)} + 2\sin\theta$

$= \frac{-2\cos^2\theta}{\sin\theta}$ and $\frac{dx}{d\theta} = 2\cos\theta$

$\therefore \; \frac{dy}{dx} = -\cot\theta$

Equation of tangent at P is

$(y - 2\ln\cot(\theta/2) + 2\cos\theta)$

$= -\cot\theta(x - 2\sin\theta)$

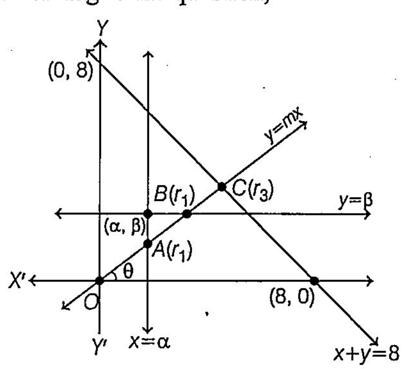
$\therefore \quad T = (0, 2\ln\cot(\theta/2))$

and $\quad P = (2\sin\theta, 2\ln\cot(\theta/2) - 2\cos\theta)$

So, $PT^2 = (-2\sin\theta)^2 + (2\cos\theta)^2$

$= 4(\sin^2\theta + \cos^2\theta) = 4 \times 1 = 4$

14. *(12)* According to the question,



$f(\alpha, \beta) \le 0$

$\left|\frac{\beta}{\alpha} - \frac{3}{2}\right| + (3\alpha - 2\beta)^6 + \sqrt{e\alpha + 2\beta - 2e - 6} \le 0$

$\Rightarrow \; \left|\frac{\beta}{\alpha} - \frac{3}{2}\right| + (3\alpha - 2\beta)^6 + \sqrt{e\alpha + 2\beta - 2e - 6} = 0$

$\Rightarrow \; \frac{\beta}{\alpha} - \frac{3}{2} = 0, 3\alpha - 2\beta = 0$ and $e\alpha + 2\beta - 2e - 6 = 0$

$\Rightarrow \; \beta = \frac{3\alpha}{2}$ and $e\alpha + 2\beta - 2e - 6 = 0$

$\Rightarrow \; (\alpha - 2)e + 2(\beta - 3) = 0$

$\Rightarrow \; \alpha = 2$ and $\beta = 3$

Let equation of straight line be $\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$

Let $OA = r_1, OB = r_2$ and $OC = r_3$

Then, $A = (r_1\cos\theta, r_1\sin\theta)$

$B = (r_2\cos\theta, r_2\sin\theta)$

and $\quad C = (r_3\cos\theta, r_3\sin\theta)$

Point A lies on $x = 2$

$\Rightarrow \quad r_1\cos\theta = 2 \Rightarrow r_1 = \frac{2}{\cos\theta}$ $\qquad$ ...(i)

Point B lies on $y = 3$,

$r_2\sin\theta = 3$

$\Rightarrow \quad r_2 = \frac{3}{\sin\theta}$ $\qquad$ ...(ii)

and point C lies on $x + y = 8$

$\Rightarrow \; r_3\cos\theta + r_3\sin\theta = 8$

$\Rightarrow \quad r_3 = \frac{8}{\cos\theta + \sin\theta}$ $\qquad$ ...(iii)

$\because \quad r_1 r_2 r_3 = 48\sqrt{2}$

$\Rightarrow \; \left(\frac{2}{\cos\theta}\right)\left(\frac{3}{\sin\theta}\right)\left(\frac{8}{\cos\theta + \sin\theta}\right) = 48\sqrt{2}$

$\Rightarrow \; \sqrt{2}\cos\theta \cdot \sin\theta(\cos\theta + \sin\theta) = 1$

$\Rightarrow \; 2\sin\theta \cdot \cos\theta\left(\sin\left(\theta + \frac{\pi}{4}\right)\right) = 1$

$\Rightarrow \; \sin 2\theta \cdot \sin\left(\theta + \frac{\pi}{4}\right) = 1$

Both $\sin 2\theta$ and $\sin\left(\theta + \frac{\pi}{4}\right)$ should be equal to 1.

$\therefore \quad \theta = \frac{\pi}{4}$ $\qquad$ ...(iv)

From Eqs. (i), (ii) and (iii), we get

$r_1 = 2\sqrt{2}, r_2 = 3\sqrt{2}$

and $\quad r_3 = \frac{8}{\sqrt{2}} = 4\sqrt{2}$

Hence, $[OA + OB + OC] = [r_1 + r_2 + r_3]$

$= [9\sqrt{2}] = 12$

15. *(0.25)* Now, equation of OA

$y = mx$

$\Rightarrow \; y = \tan\theta \cdot x$ $\qquad [\because m = \tan\theta]$

$\Rightarrow \; y = \tan\frac{\pi}{4} \cdot x$ $\qquad \left[\text{from Eq. (iv), } \theta = \frac{\pi}{4}\right]$

i.e. $x - y = 0$

Therefore, distance of line OC from (α, α) is 0.

Thus, $k = 0$

Now, $\frac{k+2}{8} + \frac{0+2}{8} = 0.25$

16. *(0.45)* So, $p=\frac{1}{2}$ and $q=1-\frac{1}{2}=\frac{1}{2}$

For getting total sum less than 15 in 4 trials, we will throw coins to get

(a) $3+3+3+3<15$ (b) $5+3+3+3<15$

and so we need atleast three success.

Probability of this event

$$={}^4C_3\times\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1+{}^4C_4\left(\frac{1}{2}\right)^4$$

$$=4\times\frac{1}{8}\times\frac{1}{2}+1\times\frac{1}{16}=\frac{5}{16}$$

$$\therefore \text{Odds in favour}=\frac{P(E)}{P(\overline{E})}=\frac{\frac{5}{16}}{\left(1-\frac{5}{16}\right)}=\frac{\frac{5}{16}}{\frac{11}{16}}$$

$$=\frac{5}{16}\times\frac{16}{11}=\frac{5}{11}=0.45$$

17. *(5.9)* If the probability of bulb being non-defective be taken as the success.

Then, $p=\frac{9}{10}$ and $q=1-p=1-\frac{9}{10}$

$\Rightarrow \quad q=\frac{1}{10}$

$\therefore$ Probability of non-defective $={}^5C_5p^5q^0$

$$=1\times\left(\frac{9}{10}\right)^5=(0.9)^5$$

On comparing with a^b, we get

$a=0.9$ and $b=5$

So, $a+b=0.9+5=5.9$