Shivam Classes
A Premier Institute JEE(Main & Adv.)
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SHIVAM CLASSES
- By IITians

**TEST**
**CLASS : XI**

**Test Date : 11-11-2024** **JEE (Advanced) - 3** **MAX. MARKS : 198**

## (SOLUTION) [ PHYSICS]

**1.(b)** $\vec{r} = \alpha t\,\hat{i} + \alpha t(1-\beta t)\hat{j}$

$$\mathbf{v} = \alpha\hat{i} + \alpha(1-2\beta t)\hat{j} \Rightarrow \mathbf{a} = -2\alpha\beta\hat{j}$$

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{a}}{va}$$

$$\Rightarrow \cos\left(\frac{\pi}{4}\right) = \frac{\alpha(1-2\beta t)(-2\alpha\beta)}{\sqrt{\alpha^2+\alpha^2(1-2\beta t)^2}\,(2\alpha\beta)}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = -\frac{\alpha(1-2\beta t)}{\alpha\sqrt{1+(1-2\beta t)^2}}$$

$$\Rightarrow 1+(1-2\beta t)^2 = 2(1-2\beta t)^2$$

$$\Rightarrow 1-2\beta t = \pm 1$$

$$\Rightarrow t = \frac{1}{\beta} \text{ and } t = 0$$

**2.(a)** $\mathbf{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

$$\text{Speed} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

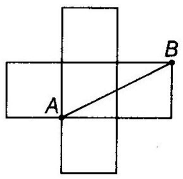
$$\Rightarrow \sqrt{125} = \sqrt{(10\sqrt{2}\cos 45°)^2 + (10\sqrt{2}\sin 45° - 10t)^2}$$

$$\Rightarrow 125 = 100 + (10-10t)^2 \Rightarrow (10-10t)^2 = 25$$

$$\Rightarrow |10-10t| = 5 \Rightarrow 10-10t = 5$$

$$\Rightarrow t = 0.5\text{ s} \Rightarrow 10-10t = -5 \Rightarrow t = 1.5\text{ s}$$

**3.(c)**




We can open the cube as shown in figure.

Shortest path length $= \sqrt{(2a)^2 + a^2} = \sqrt{5}a$

Time taken by ant $= \dfrac{\sqrt{5}a}{u}$

**4.(b)** Given, $Y = c^{\alpha}h^{\beta}G^{\gamma}$

Writing the dimensional formula of $Y$, $c$, $h$, and $G$, we get

$$[ML^{-1}T^{-2}] = [M^0L^1T^{-1}]^{\alpha}\,[ML^2T^{-1}]^{\beta}\,[M^{-1}L^3T^{-2}]^{\gamma}$$

$$\Rightarrow [ML^{-1}T^{-2}] = [M^{\beta-\gamma}\,L^{\alpha+2\beta+3\gamma}\,T^{-\alpha-\beta-2\gamma}]$$

$\therefore$ $\beta - \gamma = 1$ ...(i)

$\alpha + 2\beta + 3\gamma = -1$ ...(ii)

$-\alpha - \beta - 2\gamma = -2$

$\Rightarrow$ $\alpha + \beta + 2\gamma = 2$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

$$\alpha = 7, \beta = -1, \gamma = -2$$

**5.(abc)** Area under $a$-$t$ graph $= \Delta v = v_2 - v_1$

Area under $v$-$t$ graph $= \Delta x = x_2 - x_1$

**6.(ab)**

| | |
|---|---|
| $a = -bv,$ | $a = -bv$ |
| $\dfrac{dv}{dt} = -bv,$ | $\Rightarrow v\dfrac{dv}{dx} = -bv$ |
| $\Rightarrow \int_{v_0}^{v}\dfrac{dv}{v} = -b\int_0^t dt,$ | $\Rightarrow \int_{v_0}^{v} dv = -b\int_0^x dx$ |
| $\Rightarrow \ln\left(\dfrac{v}{v_0}\right) = -bt,$ | $\Rightarrow v - v_0 = -bx$ |
| $\Rightarrow v = v_0 e^{-bt}$ | $\Rightarrow v = v_0 - bx$ |
| $\Rightarrow 0 = v_0 e^{-bt}$ | $\Rightarrow 0 = v_0 - bx$ |
| $\Rightarrow t = \infty$ | $\Rightarrow x = \dfrac{v_0}{b}$ |

**7.(abcd)**

(a) Since, their relative acceleration is zero, their relative velocity will be constant till one hits the ground.

(b) Both will hit the ground with the same speed.

(c) Second stone is thrown when the first stone is at its highest point.

(d) Since, they hit the ground with same speed, in case of elastic collisions they will rebound to the same height.

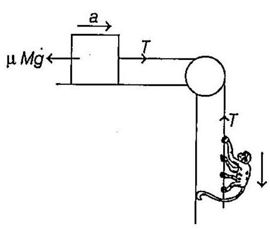
**8.(ad)**

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta V}{V} = 3\frac{\Delta r}{r}$$

$$\Rightarrow \frac{\Delta V}{V}\times 100 = 3\left(\frac{\Delta r}{r}\times 100\right) = 3\% \text{ and } A = 4\pi r^2$$

$$\frac{\Delta A}{A} = 2\frac{\Delta r}{r} \Rightarrow \frac{\Delta A}{A}\times 100 = 2\left(\frac{\Delta r}{r}\times 100\right) = 2\%$$

**9. (d)**
**10. (c)**




$$a_{MR} = b \Rightarrow a_M = b + a_R = b + a$$

For block,

$$\Sigma F_x = ma_x \Rightarrow T - \mu Mg = Ma \quad \text{...(i)}$$

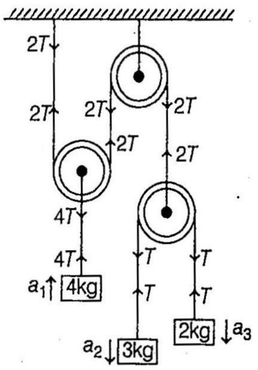
For monkey,

$$mg - T = m(b+a) \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

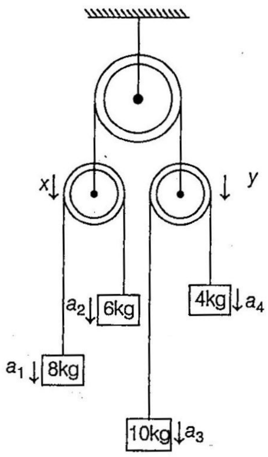
$$a = \frac{m(g-b) - \mu Mg}{M+m} \text{ and } T = \frac{Mm(\mu g + g - b)}{M+m}$$

**11. (c)**




$$4T-40=4a_1 \quad \text{...(i)}$$
$$30-T=3a_2 \quad \text{...(ii)}$$
$$20-T=2a_3 \quad \text{...(iii)}$$
$$4a_1=a_2+a_3 \quad \text{...(iv)}$$
$$\Rightarrow \quad 4T-40=\frac{30-T}{3}+\frac{20-T}{2}$$
$$\Rightarrow \quad 24T-240=60-2T+60-3T$$
$$\Rightarrow \quad 29T=360 \Rightarrow T=\frac{360}{29}\text{N}$$
$$4\left(\frac{360}{29}\right)-40=4a_1$$
$$\Rightarrow \quad a_1=\frac{70}{29}\text{m/s}^2\uparrow$$

**12.(b)**




$$80-T=8a_1 \quad \text{...(i)}$$
$$60-T=6a_2 \quad \text{...(ii)}$$
$$100-T=10a_3 \quad \text{...(iii)}$$
$$40-T=4a_4 \quad \text{...(iv)}$$
$$+a_1+a_2-x-x=0 \Rightarrow 2x=a_1+a_2 \quad \text{...(v)}$$
$$+x+y=0 \quad \text{...(vi)}$$
$$+a_3+a_4-y-y=0 \quad \text{...(vii)}$$

From Eqs. (v), (vi) and (vii), we get

$$a_1+a_2+a_3+a_4=0 \quad \text{...(viii)}$$
$$\frac{80-T}{8}+\frac{60-T}{6}+\frac{100-T}{10}+\frac{40-T}{4}=0$$
$$\Rightarrow \quad T=\frac{4800}{77}\text{N} \Rightarrow x=\frac{a_1+a_2}{2}=\frac{10}{11}\text{m/s}^2\downarrow$$

**13. A.**(A)-R;(B)-Q;(C)-P;D-S

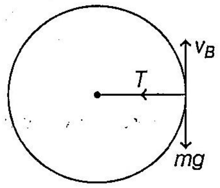
(A) Using work-energy theorem between $A$ and $B$,

$$W_{mg}+W_T=\Delta K$$
$$\Rightarrow \quad -mgR+0=\frac{1}{2}mv_B^2-\frac{1}{2}m(3\sqrt{gR})^2 \Rightarrow v_B=\sqrt{7gR}$$

(B) Using work-energy theorem between $A$ and $C$,

$$W_{mg}+W_T=\Delta K$$
$$\Rightarrow \quad -mg2R+0=\frac{1}{2}mv_C^2-\frac{1}{2}m(3\sqrt{gR})^2 \Rightarrow v_C=\sqrt{5gR}$$

(C) At $B$,




$$T=\frac{mv_B^2}{R} \Rightarrow T=7mg$$

(D) At $C$,

$$T+mg=\frac{mv_C^2}{R} \Rightarrow T=4mg$$

**14. B. (A)-R;(B)-P;(C)-S;(D)-T**

(A) Work done by gravity $=-mgh$

(B) Work done by friction $=-\int(\mu mg\cos\theta)\,dl$

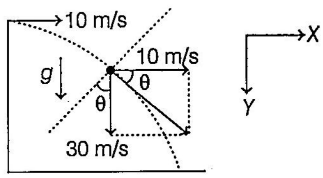
$$=-\mu mg\int dl\cos\theta$$
$$=-\mu mg\int_0^l dx=-\mu mgl$$

(C) Using work-energy theorem for block,

$$W_{mg}+W_N+W_{\text{friction}}+W_F=\Delta K$$
$$\Rightarrow \quad -mgh+0-\mu mgl+W_F=0-0 \Rightarrow W_F=mg(h+\mu l)$$

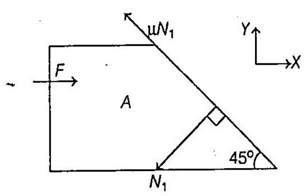
(D) $W_{\text{all forces}}=\Delta K=0-0=0$

**15.(100)**



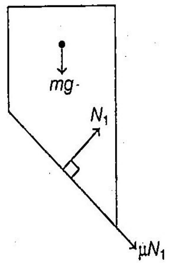

$$\mathbf{u}=10\hat{\mathbf{i}}, \quad \mathbf{v}=\mathbf{u}+\mathbf{a}t=10\hat{\mathbf{i}}+(10\hat{\mathbf{j}})(3)=10\hat{\mathbf{i}}+30\hat{\mathbf{j}}$$
$$\text{Speed}=\sqrt{10^2+30^2}=10\sqrt{10}\text{ m/s}$$
$$\tan\theta=\frac{30}{10}=3$$
$$\text{Centripetal acceleration}=\frac{v_\perp^2}{R} \Rightarrow g\cos\theta=\frac{(10\sqrt{10})^2}{R}$$
$$\Rightarrow \quad 10\frac{1}{\sqrt{10}}=\frac{1000}{R} \Rightarrow R=100\sqrt{10}$$

**16.(5)** For $A$,




$$\Sigma F_x=0 \Rightarrow F-N_1\sin 45°-\mu N_1\cos 45°=0$$
$$\Rightarrow \quad F=\frac{N_1}{\sqrt{2}}(1+\mu) \quad \text{...(i)}$$

For $B$,




$$\Sigma F_y = 0 \quad \Rightarrow N_1 \cos 45° = mg + \mu N_1 \sin 45°$$

$$N_1 = \frac{\sqrt{2}\, mg}{1-\mu} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$F = mg\frac{(1+\mu)}{(1-\mu)}$$

**17. (14)**

**18.(7)**

## (SOLUTION) [ CHEMISTRY ]

**19.** (b) $KE = -E_n$

$$= -\left(\frac{13.6Z^2}{n^2}\right)$$

For H-atom, $Z=1$ and $n=3$ (total waves) = 3 as shown

$$\therefore \quad KE = +\frac{13.6 \times (1)^2}{(3)^2} = 1.51 \text{eV}$$

**20.** (d) $p_{total} = p_{atm} + p_{gas} = 749 + 29.2 \times 10$

$$= 1041 \text{mmHg}$$

**21.(d)** $CH_4$ $sp^3$, tetrahedral.

**22.** (c) For Balmer series,

$$\frac{1}{\lambda} = \bar{R}_H Z^2 \left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

For $He^+$, $Z = 2$

$$\therefore \quad \frac{1}{\lambda} = 4\bar{R}_H \left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$= 4R_H\left[\frac{n^2-4}{4n^2}\right] = R_H\left[\frac{n^2-4}{n^2}\right]$$

$$\therefore \quad \lambda = \frac{n^2}{R_H\,(n^2-4)}$$

$$= \frac{1}{R_H}\frac{n^2}{(n+2)(n-2)}$$

$$\lambda = k\frac{n^2}{(n+2)\,(n-2)}$$

$$\therefore \quad k = \frac{1}{R_H}$$

**23.** (a, b, c)

$CH_3OH = 0.1$ mol $= 0.1 \times 32$ g $= 3.2$ g

$CH_3CH_2OH = 0.1$ mol $= 0.1 \times 46$ g $= 4.6$ g

$H_2O = 0.2$ mol $= 0.2 \times 18$ g $= 3.6$ g

Total weight = 11.4 g

If $CH_3OH$ is solute and others are solvent then 0.1 mol $CH_3OH$ is in 8.2 g of $CH_3OH$ and $H_2O$.

$$\therefore \text{Molarity of } CH_3OH = \frac{0.1 \text{ mol}}{0.0082 \text{ kg}}$$

$$= 12.195 \text{ molal}$$

Thus, (a) is correct.

$CH_3CH_2OH$ is in maximum amount.

Thus, its molality

$$= \frac{0.1 \text{ mol}}{0.0068 \text{ kg}} = 14.706 \text{ molal}$$

Thus, (b) is correct.

$$\text{Mass \% of } CH_3CH_2OH = \frac{4.6}{11.4} \times 100$$

$$= 40.35\%$$

Thus, (c) is correct.

$$\text{Mass \% of } CH_3OH = \frac{3.2}{100} \times 100 = 28.07\%$$

Thus, (d) is incorrect.

**24.(a,b,c)**

(D) Cs has lowest I.E. (out of all stable elements).

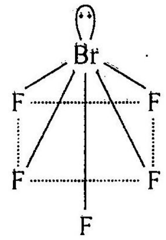
**25.(a,b,c,d)**

$XeO_4$, $XeO_3F_2$, $XeO_6^{4-}$, $XeO_3$ .

All of the given can have maximum

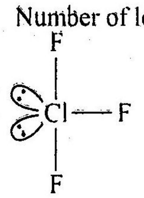
3, 3, 4, 3 "O" atoms are in same plane respect.

**26. (b,c)**

(A) 

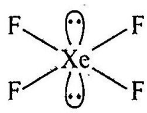

$sp^3d^2$ hybridisation square pyramidal (distorted octahedral)

Number of lone pair on centre atom = 1

(B) 

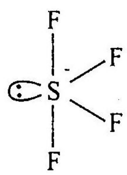

$sp^3d$ hybridisation T-shape

Number of lone pair on centre atom = 2

(C) 


$sp^3d^2$ hybridisation square planar

Number of lone pair on centre atom = 2

(D) 


$sp^3d$ hybridisation see-saw

Number of lone pair on centre atom = 1

## COMPREHENSION -1

1.0 L of 0.15 KOH = $1 \times 0.15 = 0.15$ mol

$$CO_2 + 2KOH \longrightarrow K_2CO_3 + H_2O$$

| | $CO_2$ | KOH | $K_2CO_3$ | $H_2O$ |
|---|---|---|---|---|
| Initial | 0.05 | 0.15 | 0 | 0 |
| After reaction | (0.05–0.05) | (0.15 – 0.10) | 0.05 mol | |
| Resultant | 0 | 0.05 mol | 0.05 mol | |

Thus, KOH left in 1L solution = 0.05 mol

$\therefore$ Molarity of KOH $= \frac{0.05}{1} = 0.05$ M

$K_2CO_3$ formed = 0.05 mol in 1L

$\therefore$ Molarity of $K_2CO_3$ = 0.05 M

**27. (C)**

**28. (D)**

**Comprehension - 2**

$$n = \frac{pV}{RT}$$

$$n_{NO} = \frac{\left(\frac{800}{760}\right)(0.250)}{0.0821 \times 220} = 0.0146 \text{ mol}$$

$n_{O_2}$ = 0.00437 mol (limiting reactant)

$$2NO + O_2 \longrightarrow N_2O_4(s)$$

| | 2NO | $O_2$ |
|---|---|---|
| | 2 mol | 1 mol |
| Initial | 0.0146 | 0.00437 |
| Reacted | 0.00874 | 0.00437 |
| After reaction | 0.00586 | 0 |

$N_2O_4(s)$ being in solid state does not exert pressure. Pressure is due to unreacted NO present in 0.350 L (total volume).

$$\therefore \quad p = \frac{nRT}{V} = \frac{0.00586 \times 0.0821 \times 220}{0.350}$$

= 0.302 atm

$N_2O_4(s)$ formed = $O_2$ reacted

= 0.00437 mol

= (0.00437 × 92)

= 0.402 g

**29. (d)**

**30. (a)**

**31. (a) (i) - r ; (ii) - p ; (iii) - s ; (iv) - q**

(d)

(i) p - $C_2H_4O_2$ or $(CH_2O)_2$

r- HCHO or $(CH_2O)$

t-$HCOOCH_3$ or $(CH_2O)_2$

Thus, (i) - (p, r, t)

(ii) H in $CH_3COOH$, $NH_2CONH_2$, HCHO and $HCOOCH_3$

$$= \frac{4}{60} \times 100 = 6.666\%$$

Thus, (ii) - (p, q, r, t)

(iii) C in $CH_3COOH$, HCHO, $HCOOCH_3$

$$= \frac{24}{60} \times 100 = 40\%$$

Thus, (iii) - (p, r, t)

(iv) O in p, r, t = 53.3%

Thus, (iv) - (p, r, t)

(v) H in (p) and (s) attached to O are equal

$CH_3—C(=O)—O—H$

CH, $CH_2$—O—H

Thus, (v) - (p, s)

**32. (b) (i) - q ; (ii) - s ; (iii) - p,q,r ; (iv) - p,q,r**

**33. (5)** Five (5) $\psi = \left(\frac{1}{32\pi a_0^3}\right)^{1/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$

If $r = a_0$ (Bohr's radius)

$$\psi = \left(\frac{1}{32\pi a_0^3}\right)^{1/2}\left(2 - \frac{a_0}{a_0}\right)e^{-a_0/2a_0}$$

$$\psi = \left(\frac{1}{32\pi a_0^3}\right)^{1/2} e^{-\frac{1}{2}}$$

$$\psi^2 = \frac{1}{32\pi a_0^3 e} = \left(\frac{1}{2}\right)^5 (\pi a_0^3 e)^{-1}$$

$\therefore \quad n = 5$

**34. (4)** (Kr, Ne), $(Na, Na^+)$, $(I^-, Cl^-)$, $(Li^+_{Aq.}, Na^+_{Aq.})$

**35.(2)** (d) (h) are correct.

**36.(6)** All the six terminal P — O bonds are identical.

## (SOLUTION) [ MATHEMATICS ]

**37.** (c) We have, $-4 \le x < 2$

$\Rightarrow \quad -2 \le x + 2 < 4$

$\Rightarrow \quad 0 \le |x+2| < 4$

$\Rightarrow \quad -3 \le |x+2| - 3 < 1$

$\Rightarrow \quad 0 \le ||x+2| - 3| \le 3$

**38.** (a) Let the roots of three equations be $(\alpha, \beta)$, $(\beta, \gamma)$ and $(\gamma, \delta)$, then substituting $\beta$ in first two equations, we get

$$\beta^2 + p\beta + qr = 0 \text{ and } \beta^2 + q\beta + rp = 0$$

Subtracting, we get

$$\beta(p-q) + r(q-p) = 0$$

$\Rightarrow \quad \beta = r$ if $p \ne q$

Similarly, we will get

$$\alpha = q \text{ and } \gamma = p$$

$$\therefore \quad \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{p+q+r}{pqr}$$

$$= \frac{\Sigma p}{pqr}$$

**39.** (a) We have,

$$S_n = \frac{n}{1 - \frac{1}{n+1}}$$

$$= \frac{n(n+1)}{n+1-1} = n+1$$

$\Rightarrow \quad S_n^{\ 2} = (n+1)^2$

So, $S_1^2 + S_2^2 + .... S_{2n-1}^2 = 2^2 + 3^3 + ... + (2n)^2$

$$= \frac{1}{6}(2n)(2n+1)(4n+1) - 1$$

$$= \frac{1}{3}[n(2n+1)(4n+1) - 3]$$

**40.** (d) $N_1+N_2+N_3+N_4+N_5$

= Total number of ways − $N$ number of ways when no odd number is selected

Since, only available even numbers are 2, 4, 6 and 8, the number of ways when no odd number is selected, is zero.

$\therefore N_1+N_2+N_3+N_4+N_5={}^9C_5-0=126$

**41.** (a,c)

Let person $P_i$ get $x_i$ number of things such that

$$x_1+x_2+x_3+x_4+x_5+x_6=30$$

If $x_i$ is odd or $x_i=2\lambda_i+1$, where $\lambda_i\geq 0$, then

$2(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6)+6=30$

$\Rightarrow\quad \lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6=12.$

Then, number of solutions is ${}^{12+6-1}C_{6-1}={}^{17}C_5$

If $x_i$ is even or $x_i=2\lambda_i$, where $\lambda_i\geq 1$, then

$2(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6)=30$

or $\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6=15$

Therefore, required number of ways is ${}^{15-1}C_{6-1}={}^{14}C_5$

**42.** (b, d) Let $f(x)=x^2+ax+b$, then

$$x^2+(2c+a)x+c^2+ac+b=f(x+c)$$

Thus, the roots of $f(x+c)=0$ will be 0, $d-c$.

**43.** (b, c, d)

Let $S_n$ denote the sum of first $n$ terms of AP.

According to the question,

$r=\dfrac{S_n}{S_{2n}-S_n}=\dfrac{1}{\dfrac{S_{2n}}{S_n}-1}$ is independent of $n$.

Therefore, $\dfrac{S_{2n}}{S_n}$ is independent of $n$.

Now, $\dfrac{S_{2n}}{S_n}=\dfrac{\dfrac{2n}{2}(2\alpha+(2n-1)\beta)}{\dfrac{n}{2}(2\alpha+(n-1)\beta)}=\dfrac{2(2\alpha-\beta+2n\beta)}{(2\alpha-\beta+n\beta)}$

This ratio is independent of $n$, if

$2\alpha-\beta=0$

$\therefore\quad \alpha:\beta=1:2$

$\therefore\quad r=\dfrac{1}{4-1}=\dfrac{1}{3}$

$\alpha$ and $\beta$ are the roots of the equation $ax^2+bx+c=0$

$\therefore\quad 2\alpha+\alpha=-\dfrac{b}{a}$ and $2\alpha\times\alpha=\dfrac{c}{a}$

Solving those, we get

$$2\left(-\frac{b}{3a}\right)^2=\frac{c}{a}\Rightarrow 2b^2=9ac$$

$$1+r+r^2+....=\frac{1}{1-r}=\frac{1}{1-\frac{1}{3}}=\frac{3}{2},\alpha=1,\beta=2$$

$\therefore$ Sum of 10 terms, AP $=\dfrac{10}{2}(2\times1+(10-1)\times2)=100$

**44.** (a, d) Equations $x^2+bx+c=0$ and $bx^2+cx+1=0$ have common root.

$$(bc-1)^2=(c-b^2)(b-c^2)$$

$\Rightarrow\ b^2c^2+1-2bc=bc-b^3-c^3+b^2c^2$

$\Rightarrow\ 1+c^3+b^3=3bc$

$\Rightarrow (1+c+b)(c^2+b^2+1-b-c-bc)=0$

$\Rightarrow b+c+1=0$ and $c^2+b^2+1=c+b+bc$

**45.(b).or 46.(b)**

Let roots of equation are $a$, $ar$, $ar^2$

$\Rightarrow\quad a(1+r+r^2)=\dfrac{19}{2}$ ...(i)

and $a^2r+a^2r^2+a^2r^3=\dfrac{57}{2}$

$\Rightarrow\quad a^2r(1+r+r^2)=\dfrac{57}{2}$ ...(ii)

From Eqs. (i) and (ii),

$$\frac{1+r+r^2}{r}=\frac{19\times19}{2\times57}=\frac{19}{6}$$

$\Rightarrow\quad 6r^2-13r+6=0$

$\Rightarrow\quad (2r-3)(3r-2)=0$

$$r=\frac{3}{2}\text{ or }\frac{2}{3}$$

If $r=\dfrac{3}{2}$, then $a=2$

If $r=\dfrac{2}{3}$, then $a=\dfrac{9}{2}$

Roots $\Rightarrow 2,3,\dfrac{9}{2}$

Product $=a^3r^3=\dfrac{-k}{2}\Rightarrow(-2)(3)^3=k\Rightarrow k=-54$

As given that GP is increasing, thus taking $r=\dfrac{3}{2}$

$$S_n=\frac{2\left(\left(\frac{3}{2}\right)^n-1\right)}{\frac{3}{2}-1}=4\left(\left(\frac{3}{2}\right)^n-1\right)$$

As given that GP is decreasing, thus taking

$$r=\frac{2}{3}\Rightarrow S_\infty=\frac{\frac{9}{2}}{1-\frac{2}{3}}=\frac{27}{2}$$

**47.** (c) |U|CC|E|

There are four gap to place SSS.

Hence, required no of ways $={}^4C_3\times3!=24$

**48.** (b) |U|C|C|E|

There are five available places for SSS.

$\therefore$ Total number of ways no two S's are together

$$={}^5C_3\times\frac{4!}{2!}=120$$

Hence, number of words having CC separated and SSS separated $=120-24=96$

**49.** (i) → C; (ii) → (A, B, C, D); (iii) → (B, C, D)

(i) $7^{\frac{2x^2-5x-9}{2}} = (2)^{\frac{3}{2}\log_2 7}$

$\Rightarrow \frac{2x^2-5x-9}{2} = \frac{3}{2}$

$\Rightarrow 2x^2-5x-12=0$

Hence, $x = 4, \frac{-3}{2}$

(ii) $2^{2(x+\sqrt{x^2-2})} = 5\cdot 2^{x-1+\sqrt{x^2-2}} + 6$

Let $\lambda = 2^{\sqrt{x^2-2}+x}$

$\Rightarrow \quad \lambda^2 - \frac{5\lambda}{2} = 6$

$\Rightarrow 2\lambda^2 - 5\lambda - 12 = 0$

$\Rightarrow \quad \lambda = 4, -\frac{3}{2}$

Hence, $2^{\sqrt{x^2-2}+x} = 2^2$

$\Rightarrow \quad x = \frac{3}{2}$

(iii) $\because \sqrt{|x-3|^{x-1}} = \sqrt[4]{|x-3|^{x+1}}$

$\Rightarrow |x-3|^{\frac{x-1}{2}} = |x-3|^{\frac{x+1}{4}}$

$\therefore \quad x-3=0 \Rightarrow x=3$

or $\quad x-3=1 \Rightarrow x=4$ or $x-3=-1$

$\Rightarrow \quad x=2$

Also, $\quad \frac{x-1}{2} = \frac{x+1}{4}$

$\Rightarrow \quad 2(x-1) = x+1 \Rightarrow x=3$

**50.** (i) → B, (ii) → A, (iii) → C, (iv) → D

(i) $f(x) = (x-a)(x-c) + \pi(x-b)(x-d) = 0$

$f(a) = \pi(a-b)(a-d) > 0$

$f(b) = (b-a)(b-c) < 0$

$f(c) = \pi(c-b)(c-d) < 0$

and $f(d) = (d-a)(d-c) > 0$

One root in $(a, b)$ and other root in $(c, d)$.

(ii) As, $a > 0$, $f(x) = ax^2 + bx + c$

$f(1) = a + b + c < 0$

$\Rightarrow$ 1 lies in between the roots.

(iii) $x^2 + bx + c = 0$

As difference of roots is an integer and sum is an integer.

$\Rightarrow$ This is only possible when fractional part of both is $\frac{1}{2}$ or 0.

In $\frac{1}{2}$ case, product cann't be integer.

$\Rightarrow$ Roots are integers.

(iv) $(a^2+b^2+c^2)x^2 + 2(ab+bc+cd)x + (b^2+c^2+d^2) = 0$

$\because a, b, c$ and $d$ are in GP, let common ratio be $r$.

Then, $(1+r^2+r^4)x^2 + 2(r+r^3+r^5)x + (r^2+r^4+r^6) = 0$

$\Rightarrow D = 4r^2(1+r^2+r^4) - 4r^2(1+r^2+r^4)^2 = 0$

**51.** (6) Let the roots be $a-3d$, $a-d$, $a+d$ and $a+3d$.

Sum of roots $= 4a = 0$

$\Rightarrow \quad a = 0$

Hence, roots are $-3d, -d, d$ and $3d$.

Product of roots $= 9d^4 = m^2$

$\Rightarrow \quad d^2 = \frac{m}{3} \quad$ ...(i)

Again, $\Sigma x_1 x_2 = 3d^2 - 3d^2 - 9d^2 - d^2 - 3d^2 + 3d^2$

or $\frac{10m}{3} = 3m + 2$ or $10m = 9m + 6$ or $m = 6$

**52.** (9) Given that $\frac{S_7}{S_{11}} = \frac{6}{11}$

$\Rightarrow \quad \frac{\frac{7}{2}(2a+6d)}{\frac{11}{2}(2a+10d)} = \frac{6}{11} \Rightarrow \frac{7(a+3d)}{11(a+5d)} = \frac{6}{11}$

$\Rightarrow \quad 7a + 21d = 6a + 30d \Rightarrow a = 9d$

$\Rightarrow \quad T_7 = a + 6d = 15d$

Given that $130 < T_7 < 140$

$130 < 15d < 140 \Rightarrow d = 9$

**53.** (9)

$5400 = 2^3 \times 3^3 \times 5^2$

Since, divisor is divided by 6 but not by 9, divisor must be even

i.e. 2 must occur atleast once, Also, 3 must occur exactly once.

So, number of required divisor $= 3 \times 1 \times 3 = 9$

**54.** (5) Given, $a = 22, c = 12$

and $a + b + c = 45$

$22 + 12 + b = 45$

$b = 11 = 2\lambda + 1$

$\Rightarrow \quad \lambda = 5$