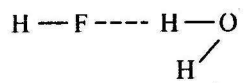
Shivam Classes
A Premier Institute JEE(Main & Adv.)
NEET/AIIMS.NTSE.OLYMPID KVPY
SHIVAM CLASSES - By IITians

TEST
CLASS : XI

Test Date : 16-12-2024 **JEE (Advanced) - 4** **MAX. MARKS : 198**

(SOLUTION) [CHEMISTRY]

1.(a)


(due to hydrogen bonding)

2. (b)
3. (c)
4. (a)
5. (c) O_2^{2-} peroxide ion is diamagnetic ion.

6. (abd)
7. (abc)
8. (ac)

9.(a) Total no. of atoms in one mole vitamin A = 51
Total no. of atoms in one mole vitamin $A_2 = 49$
$\because$ 49 atoms are present in one mole A_2
$\therefore$ 1 atom is present in $\frac{1}{49}$ moles A_2
$\therefore$ 51 atoms are present in $\frac{51}{49}$ moles A_2

10.(c) Total mass = mass of vitamin A + mass of vitamin A_2
$= 286 + 284 = 570$ gm

11. (c)
12. (b)
13. (a) A-(qrt) ; B - (st) ; C - (q,r,s) ; D -(prs)
(P) I.E.$_2$ of O, $\Delta H = +ve$
(Q) E.A. of O, $\Delta H = -ve$
(R) I.E.$_1$ + I.E.$_2$ of O, $\Delta H = +ve$
(S) EGE$_2$ of O, $\Delta H = +ve$
(T) EGE$_1$ + EGE$_2$ of O, $\Delta H = +ve$

14. (b) A - (q) ; B - (prst) ; C - (p) ; D - (q)
All the cations have higher I.E. than corresponding atom.

15. (8)
16. (4)
17.(4)
18. (8)

(SOLUTION) [PHYSICS]

19. (b) $3\hat{i}+3\hat{j}+3\hat{k} = \frac{(10+20+30)\times 0 + 40\times \vec{r}_2}{(10+20+30+40)}$

$\therefore \quad \vec{r}_2 = 7.5\hat{i}+7.5\hat{j}+7.5\hat{k}.$

20. (c) $\frac{F_0(T+\frac{T}{2})}{2} = m(0-u)$

$\therefore \quad F_0 = \frac{4mu}{3T}.$

21. (d) Given, $F = cs^{-1/3}$

Acceleration, $a = \frac{F}{m} = \frac{c}{m}s^{-1/3}$

or $v\frac{dv}{ds} = \frac{c}{m}s^{-1/3}$

or $\int_0^v vdv = \frac{c}{m}\int_0^s s^{-1/3}ds$

$\frac{v^2}{2} = \frac{c}{m}\frac{s^{2/3}}{2/3}$

or $v^2 = \frac{3c}{\alpha m}s^{2/3}$

or $v = ks^{1/3}$

Now power $= Fv$

$= cs^{-1/3} \times ks^{1/3} = s^0$

22. (b) $W_{net} = W_{gravity} + W_{spring}$

$= mg(h+d) - \frac{1}{2}kx^2.$

23. (b,d) The total momentum and total energy of (earth + ball) system remain conserved.

24. (b, c, d) $\frac{1}{2}kx^2 = mgx$, $\therefore$ $x = \frac{2mg}{k}$

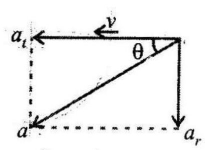
At $\frac{x}{2}$, force in the spring, $F = k\frac{x}{2} = k\frac{mg}{k} = mg$,

so, net force on the block is zero. Also at extreme position the block has net upward force and so net upward acceleration.

25. (a, b)

$a_t = \frac{dv}{dt} = 2 =$ constant

$a_r = \frac{v^2}{R}$



As v increasing, therefore a_r is increasing.

$\tan\theta = \frac{a_r}{a_t}$

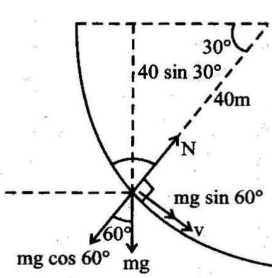
26. (b, d) By constraint

$a_B + 4a_A = 2a_C + 3a_D$

$\Rightarrow a_D = \frac{7}{3}$ m/s^2

$2T - 10 = 1 \Rightarrow T = 11/2$ N

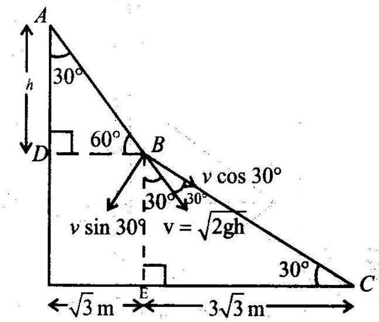
27. (a)



28. **(b)** From (2), $mv^2 = 100$ $\therefore v = 10$ ms^{-1}

29. (b) As the inclined plane is frictionless,
The K. E . at B = P.E. at A

$\frac{1}{2}mv^2 = mgh$ $v = \sqrt{2gh}$



In ΔADB, $\tan 60° = \frac{h}{\sqrt{3}}$

$\therefore h = 3$ m $\therefore v = \sqrt{6g} = \sqrt{60}$ m/s

This is the velocity of the block just before collision. This velocity makes an angle of 30° with the vertical. Also in right angled triangle BEC, $\angle EBC = 60°$. Therefore v makes an angle of 30° with the second inclined plane BC. The component of v along BC is v cos 30°.

It is given that the collision at B is perfectly inelastic therefore the impact forces act normal to the plane such that the vertical component of velocity becomes zero. The component of velocity along the incline BC remains unchanged and is equal to v cos 30°

$= \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{180}{4}} = \sqrt{45}$ m/s

30. (b) In ΔBCE, $\tan 30° = \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BE}{3\sqrt{3}} \Rightarrow BE = 3$m

Applying mechanical energy conservation .
Mechanical energy at B = Mechanical energy at C

$\frac{1}{2}M\left(\sqrt{45}\right)^2 + M \times 10 \times 3 = \frac{1}{2}Mv_c^2$

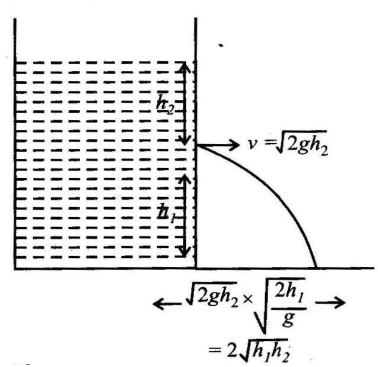
$45 + 60 = v_c^2$ $\therefore v_c = \sqrt{105}$ m/s

31. (c) A-(q,r) ; B-(p,t); C-(s)

Capacitance	coulomb-volt^{-1}, coulomb^2-joule^{-1}
Inductance	ohm-sec, volt-second (ampere)$^{-1}$
Magnetic Induction	newton (ampere-metre)$^{-1}$

Mean diameter $= \frac{0.61 + 0.57 + 0.59}{3} = 0.59$ cm

32.(a) **A → p; B → p; C → p; D → s**



If $g_{eff} > g$

$g_{eff} = g$

$g_{eff} < g$

In all the three cases $d = 2\sqrt{h_1 h_2} = 1.2$ m

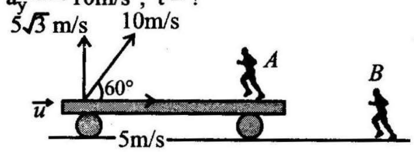
If $g_{eff} = 0$, then no water leaks out

33. 5

From the perspective of observer A, considering vertical motion of the ball from the point of throw till it reaches back at the initial height.

$U_y = +5\sqrt{3}$ m/s, $S_y = 0$

$a_y = -10 m/s^2$, t = ?



Applying $S = ut + \frac{1}{2}at^2$

$0 = 5\sqrt{3}t - 5t^2$

$\therefore$ $t = \sqrt{3}$ sec

Considering horizontal motion from the perspective of observer B. Let u be the speed of train at the time of throw.

The horizontal distance travelled by the ball = $(u + 5)\sqrt{3}$.

The horizonal distance travelled by the boy

$= \left[u\sqrt{3} + \frac{1}{2}a(\sqrt{3})^2\right] + 1.15$

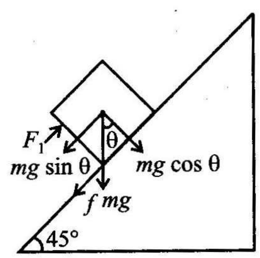
As the boy catches the ball therefore

$(u+5)\sqrt{3} = u\sqrt{3} + \frac{3}{2}a + 1.15$

$\therefore 5\sqrt{3} = 1.5a + 1.15$ $\quad \therefore 7.51 = 1.5a$

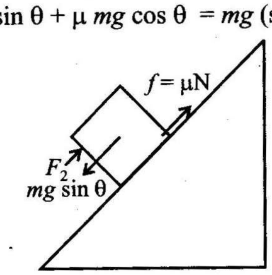
$\therefore a \approx 5 m/s^2$

34. 5



The pushing force $F_1 = mg\sin\theta + f$

$\therefore F_1 = mg\sin\theta + \mu\, mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$



The force required to just prevent it from sliding down

$F_2 = mg\sin\theta - \mu N = mg(\sin\theta - \mu\cos\theta)$

Given, $F_1 = 3F_2$

$\therefore$ $\sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$

$\therefore$ $1 + \mu = 3(1 - \mu)$ [$\because \sin\theta = \cos\theta$]

$\therefore$ $4\mu = 2$ $\quad \therefore \mu = 0.5$

$\therefore$ $N = 10\mu = 5$

35. (4) $E = A^2 e^{-0.2t}$

$\therefore$ $\log_e E = 2\log_e A - 0.2t$

On differentiating we get

$$\frac{dE}{E} = 2\frac{dA}{A} - 0.2\frac{dt}{t} \times t$$

As errors always add up therefore

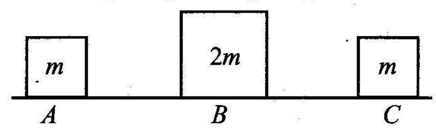
$$\frac{dE}{E} \times 100 = 2\left(\frac{dA}{A} \times 100\right) + 0.2t\left(\frac{dt}{t} \times 100\right)$$

$$\therefore \frac{dE}{E} \times 100 = 2 \times 1.25\% + 0.2 \times 5 \times 1.5\%$$

$$\therefore \frac{dE}{E} \times 100 = 4\%$$

36. 4 The velocity of B just after collision with A is

$$v_B = \frac{(m_B - m_A)u_B}{m_B + m_A} + \frac{2m_A u_A}{m_A + m_B}$$



$$= \frac{0 + 2m \times 9}{m + 2m} = 6 \text{ m/s}$$

The collision between B and C is completely inelastic.

$\therefore$ $m_B v_B = (m_B + m_c)v$

$$\therefore v = \frac{6 \times 2m}{2m + m} = 4 \text{m/s}.$$

(SOLUTION) [MATHEMATICS]

37. (B) $\log_3 x \times \log_x 2x \times \log_{2x} y = \log_3 y$

and $\log_x x^2 = 2$

$\therefore \log_3 y = 2 \Rightarrow y = 9$

38. (B) We have $2011x^3 + 2x^2 + 1 = 0 \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$ ...(1)

Put $x = \frac{1}{t}$ in equation (1), we get $\frac{2011}{t^3} + \frac{2}{t^2} + 1 = 0 \Rightarrow t^3 + 2t + 2011 = 0 \begin{matrix} 1/\alpha \\ 1/\beta \\ 1/\gamma \end{matrix}$ ...(2)

So, $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 0$ and $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = 2$ **Ans.**

39. (C) Let $\frac{1}{H_{i+1}} - \frac{1}{H_i} = k \Rightarrow \frac{H_i - H_{i+1}}{H_i H_{i+1}} = k$

$\Rightarrow (H_i - H_{i+1}) = kH_iH_{i+1}$

where k is the common difference of corresponding A.P.

$$\therefore \sum_{i=1}^{100} (-1)^i \left(\frac{H_i + H_{i+1}}{H_i - H_{i+1}}\right) = \sum_{i=1}^{100} \frac{(-1)^i}{k} \cdot \frac{(H_i + H_{i+1})}{H_iH_{i+1}} = \sum_{i=1}^{100} \frac{(-1)^i}{k}\left(\frac{1}{H_{i+1}} + \frac{1}{H_i}\right)$$

$$= \frac{1}{k}\left[\left(\frac{-1}{H_2} - \frac{1}{H_1}\right) + \left(\frac{1}{H_3} + \frac{1}{H_2}\right) + \left(-\frac{1}{H_4} - \frac{1}{H_3}\right) + \left(\frac{1}{H_5} + \frac{1}{H_4}\right) + \ldots + \left(\frac{1}{H_{101}} + \frac{1}{H_{100}}\right)\right]$$

$$= \frac{1}{k}\left(\frac{1}{H_{101}} - \frac{1}{H_1}\right) = \frac{100k}{k} = 100$$ **Ans.**

Note :

$$\frac{1}{H_2} - \frac{1}{H_1} = k$$
$$\frac{1}{H_3} - \frac{1}{H_2}$$
$$\vdots \qquad \vdots$$
$$\frac{1}{H_{100}} - \frac{1}{H_{99}}$$
$$\frac{1}{H_{101}} - \frac{1}{H_{100}}$$

$$\frac{1}{H_{101}} - \frac{1}{H_1} = 100k$$

40. (D) $T_{r+1} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}(x)^r$

For first negative term, $n - r + 1 < 0 \Rightarrow r > n + 1$

$\Rightarrow r > \frac{32}{5} \therefore r = 7. \left(\because n = \frac{27}{5}\right)$

Therefore, first negative term is T_8

41.(ACD) I → 2 L → 2 T → 2 U → 1 S → 1 R → 1 A → 1 O → 1 N → 1

(A) Total number of arrangement $= \frac{(12)!}{2!2!2!}$

(B) For vowels to be separated they must be in gap.
Hence, consonants must be arranged at first.

$\therefore$ Total number of ways $= \frac{7!}{2!2!} \times {}^8C_5 \times \frac{5!}{2!}$

$= \frac{5040}{4} \times \frac{8\times7\times6}{6} \times 60$

$= 7! \times 2\times7\times3\times5\times4 = 8! \times 105$

(C) For the order of the vowel to be unchanged the arrangement of the vowel can be done in a single way *i.e.*, IUAIO. Hence, we can treat it alike.

$\therefore$ Total ways $= \frac{(12)!}{5!\times 2!\times 2!}$

(D) After selecting one I and one L, we can divide it into 4 cases :

Case 1. None of 3 remaining letters chosen are the second I or L.

$\therefore$ Total ways $= {}^6C_3 + {}^6C_2 + {}^6C_1 = 20 + 15 + 6 = 41$

Case 2. We select one I and no L extra.

$\therefore$ Total ways $= ({}^6C_2 + {}^6C_1 + 1) = 22$

Case 3. We select one L and no I extra.

$\therefore$ Total ways = 22

Case 4. 2L + 2I and one extra letter = 7

$\therefore$ Total ways = 92

42.(AD) $\alpha x^2 - x + \alpha = 0$

$D = 1 - 4\alpha^2$

distinct real roots $D > 0$

$\Rightarrow \quad \alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ ... (i)

given $|x_1 - x_2| < 1$

$\Rightarrow \quad \frac{\sqrt{1-4\alpha^2}}{|\alpha|} < 1 \quad \Rightarrow \quad 1 - 4\alpha^2 < \alpha^2$

$\Rightarrow \quad \alpha \in \left(-\infty, \frac{-1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ ...(ii)

From (i) and (ii), $\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

43. (AC) $2^{1+\log_5 x} \cdot 4^{\log_5 x} + 5 \cdot 4^{\log_5 x} - 2^3 \cdot 2^{\log_5 x} - 3 \cdot 2^{\log_5 x} + 4 = 0$ (Using $a^{\log_b x} = x^{\log_b a}$)

Let $2^{\log_5 x} = t$

$2t^3 + 5t^2 - 11t + 4 = 0$

$\therefore \quad t = 1, \frac{1}{2}$ and -4

$\therefore \quad 2^{\log_5 x} = 1 \Rightarrow x = 1$

and $2^{\log_5 x} = \frac{1}{2} = 2^{-1} \Rightarrow x = \frac{1}{5}$

but $2^{\log_5 x} \neq -4 \therefore x_1 = 1$ and $x_2 = \frac{1}{5}$

44.(CD) H.M. < G.M. (b is the HM between a and c)

$\therefore$ $b < \sqrt{ac}$; $\therefore b^2 < ac$ **Ans.**

and a, b, c are in H.P.

$\therefore$ $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b} \Rightarrow \frac{a}{c} = \frac{a-b}{b-c}$ **Ans.**

45.(A) OR 46. (B)

$(1 + 2x + 3x^2)^{30} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{60} x^{60}$

Put $x = 1$ and -1, we get

$$a_0 + a_1 + a_2 + a_3 + \ldots + a_{60} = 6^{30}$$

$$a_0 - a_1 + a_2 - a_3 + \ldots + a_{60} = 2^{30}$$

Adding we get

$$a_0 + a_2 + a_4 + \ldots + a_{60} = \frac{6^{30} + 2^{30}}{2} = 2^{29}(3^{30} + 1)$$

To compute x^{59}, we need x^2 from 29 brackets and x from 1 bracket.

$\therefore$ $^{30}C_{29} \cdot 3^{29} \cdot 2 = 30 \cdot 2 \cdot 3^{29} = 20 \cdot 3^{30}$

To compute x^3, we need x from 3 brackets or x^2 from 1 bracket and x from 1 bracket.

$\therefore$ Coefficient of $x^3 = {}^{30}C_3 \cdot 2^3 + {}^{30}C_1 \cdot 3 \cdot {}^{29}C_1 \cdot 2$

$$= {}^{30}C_3 \cdot 8 + {}^{30}C_1 \cdot {}^{29}C_1 \cdot 6$$

$$= \frac{30 \cdot 29 \cdot 28}{6} \cdot 8 + 30 \cdot 29 \cdot 6$$

$$= 290 \cdot 2(56 + 18)$$

$$= 290 \times 148 = 42{,}920$$ **Ans.**

47.(C) OR 48. (C)

(1) We have $\alpha^2 - p\alpha + 2q = 0$

and $\alpha^2 - q\alpha + 2p = 0$

(Subtracting) ________________

$\Rightarrow$ $(q - p)\alpha + 2(q - p) = 0$

$\therefore$ $\alpha = -2$, as $p \neq q \Rightarrow \alpha$ lies in $(-3, -1)$ **Ans.**

(2) As $x = -2$ is a root of $x^2 - px + 2q = 0$, so $4 + 2p + 2q = 0$

$\Rightarrow$ $p + q = -2$

Also, p and q are roots of $4x^2 + mx + n = 0$ (Given)

$\therefore$ Sum of roots $= p + q = \frac{-m}{4} = -2 \Rightarrow m = 8$

Now, discriminant $= m^2 - 16n > 0$ (As roots are distinct and real)

$\Rightarrow$ $64 - 16n > 0 \Rightarrow 0 < n < 4$

$\Rightarrow$ $n = 1, 2, 3$ (As $n \in N$)

$\therefore$ Possible ordered pairs of (m, n) are $(8, 1), (8, 2), (8, 3)$

$\Rightarrow$ 3 ordered pairs. **Ans.**

49. a (A) -S ; (B) - Q ; (C) - S ; (D) - P

$12600 = 2^3 \times 3^2 \times 5^2 \times 7^1$

(A) Hence, divisible by $70 = 3 \times 3 \times 2 \times 1 = 18$

(B) If KN is perfect square then $K = 14$

(C) Total number of divisors $= 4 \times 3 \times 3 \times 2 = 72$

$\therefore$ Required ways $= \frac{72}{2} = 36 = 2l \Rightarrow l = 18$

(D) Total number of ways $= 2^{n-1}$ (where n is number of primes involved in prime factorization of N.)

50. b (A) -P ; (B) - PRS ; (C) - PR ; (D) PQR

(A) Let $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}} = y$

$\Rightarrow \quad \sqrt{6+y} = y \Rightarrow 6+y = y^2 \Rightarrow y^2 - y - 6 = 0$

$\Rightarrow \quad y = 3$ or $y = -2$ (rejected)

$\therefore \quad \log_2 \log_9 3 = -1 \Rightarrow x = -1$

(B) $(\log_2 3)(\log_3 4)\ldots(\log_{99} 100)$

$$= \frac{\log 3}{\log 2}\cdot\frac{\log 4}{\log 3}\cdot\frac{\log 5}{\log 5}\ldots\frac{\log 99}{\log 98}\cdot\frac{\log 100}{\log 99} = \log_2 100$$

$\therefore 2^{(\log_2 3.\log_3 4.\log_4 5 \ldots \log_{99}100)} = 2^{\log_2 100} = 100 \Rightarrow P, R, S$

(C) $\log_3 5 + \log_3 6 - \log_3 10 = \log_3\left(\frac{30}{10}\right) = \log_3 3 = 1$

(D) $\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$

$= \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+3-\sqrt{5}}}$ (as $14-6\sqrt{5} = (3-\sqrt{5})^2$)

51. (3) (A) $7^{\frac{2x^2-5x-9}{2}} = (2)^{\frac{3}{2}\log_2 7}$

$\frac{2x^2-5x-9}{2} = \frac{3}{2}$

$\Rightarrow \quad 2x^2 - 5x - 12 = 0$

Hence, $x = 4, x = -\frac{3}{2}$

(B) $2^{2\left(\sqrt{x^2-2}+x\right)} = 5.2^{x-1+\sqrt{x^2-2}} + 6$

Let $2^{\sqrt{x^2-2}+x} = \lambda$

$\Rightarrow \quad \lambda^2 - \frac{5\lambda}{2} = 6$

$\Rightarrow \quad 2\lambda^2 - 5\lambda - 12 = 0$

$\Rightarrow \quad \lambda = 4, -\frac{3}{2}$

Hence, $2^{\sqrt{x^2-2}+x} = 2^2 \Rightarrow x = \frac{3}{2}$

(C) $\sqrt{|x-3|^{x-1}} = \sqrt[4]{|x-3|^{x+1}} \Rightarrow |x-3|^{\frac{x-1}{2}} = |x-3|^{\frac{x+1}{4}}$

$\therefore \quad x - 3 = 0 \Rightarrow x = 3$

$x - 3 = 1 \Rightarrow x = 4$ or $x - 3 = -1 \Rightarrow x = 2$

Also $\frac{x-1}{2} = \frac{x+1}{4} \Rightarrow 2(x-1) = x+1 \Rightarrow x = 3]$

52. (1) Given $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P.

$\Rightarrow \quad b - \frac{a+b}{1-ab} = \frac{b+c}{1-bc} - b$

$\Rightarrow \quad \frac{-a(b^2+1)}{1-ab} = \frac{c(b^2+1)}{1-bc} \Rightarrow a + c = 2abc$

Now, given quadratic equation is

$$2ac\,x^2 + 2abc\,x + 2abc = 0$$

(Substituting $a + c = 2abc$ and then cancelling $2ac$)

$\Rightarrow \quad x^2 + bx + b = 0 \langle^{\alpha}_{\beta}$

As $\alpha + \beta = -b, \; \alpha\beta = b$

$\therefore \quad (1+\alpha)(1+\beta) = (\alpha+\beta) + (\alpha\beta) + 1 = -b + b + 1 = 1.$ **Ans.**

53. (5) Let the number of sides is n, then

$n = {}^nC_2 - n$ i.e., $2n = {}^nC_2$

i.e., $2n = \frac{n(n-1)}{2}$ i.e., $4n = n(n-1)$

i.e., $n = 5$ **Ans.**

54. (2) For constant term $T_{r+1} = {}^9C_r a^{9-r} x^{9-r} (x)^{-r/2}$

∴ For constant term

$9 - r - \frac{r}{2} = 0 \Rightarrow 9 - \frac{3r}{2} = 0 \Rightarrow r = 6$

∴ ${}^9C_6 \times a^3 = -\frac{21}{2} \Rightarrow \frac{9 \cdot 8 \cdot 7}{6} \times a^3 = -\frac{21}{2}$

$\Rightarrow a = -\frac{1}{2} \Rightarrow 2a + 1 = 0$

$\Rightarrow 2a + 3 = 2$