Shivam Classes
A Premier Institute JEE(Main & Adv.)
NEET/AIIMS.NTSE.OLYMPID KVPY
SHIVAM CLASSES
- By IITians

TEST
CLASS : XI

Test Date : 28-01-2025 **JEE (Advanced) - 5** **MAX. MARKS : 198**

(SOLUTION) [CHEMISTRY]

1. (C)
2. (C)
3. (D)
4. (C)
5. (A,C)
6. (B,D)
7. (A,B,C,D)

XeO_4, XeO_3F_2, XeO_6^{4-}, XeO_3

All of the given can have maximum 3, 3, 4, 3 "O" atoms are in same plane respect.

8. (A,B)
9. (C)
10. (A,B,C,D)

11. (C)

(A) $N_2^+ \Rightarrow \sigma_{1s^2}, \overset{*}{\sigma}_{1s^2}, \sigma_{2s^2}, \overset{*}{\sigma}_{2s^2}, \pi_{p_x^2} = \pi_{2p_z^2}, \sigma_{2p_z^1}$

(B) $O_2^+ \Rightarrow \sigma_{1s^2}, \overset{*}{\sigma}_{1s^2}, \sigma_{2s^2}, \overset{*}{\sigma}_{2s^2}, \sigma_{2p_x^2}, \pi_{2\pi_y^2} = \pi_{2\pi_z^2}, \pi_{2\pi_y^1} = \pi_{2\pi_z}$

(C) $O_2^- \Rightarrow \sigma_{1s^2}, \overset{*}{\sigma}_{1s^2}, \sigma_{2s^2}, \overset{*}{\sigma}_{2s^2}, \sigma_{2p_x^2}, \pi_{2p_y^2} = \pi_{2p_z^2}, \overset{*}{\pi}_{2p_y^2} = \overset{*}{\pi}_{2p_z^1}$

(D) $CN^- \Rightarrow \sigma_{1s^2}, \overset{*}{\sigma}_{1s^2}, \sigma_{2s^2}, \overset{*}{\sigma}_{2s^2}, \pi_{2p_y^2} = \pi_{2p_z^2}, \sigma_{2p_x^2}$

12. (D)

13. (D)

$F = 1s^2\ 2s^2\ 2p^5$

- E.N. of F = 4.0
- No vacant orbital in outermost shell of F
- Unpaired electrons = 2
- Number of e^- = 9
- Number of protons = 9

14. (D)
15. (3)
16. (5)
17. (5)
18. (6)

(SOLUTION) [MATHEMATICS]

19. (b) **Case I** $x \in [0,9]$

$2(3-\sqrt{x})+x-6\sqrt{x}+6=0$

$\Rightarrow \quad x-8\sqrt{x}+12=0$

$\Rightarrow \quad \sqrt{x}=4,2$

$\Rightarrow \quad x=16,4$

Case II $x \in [9,\infty)$

$2(\sqrt{x}-3)+x-6\sqrt{x}+6=0$

$\Rightarrow \quad x-4\sqrt{x}=0$

$\Rightarrow \quad x=16,0$

$\Rightarrow \quad x=16 \qquad [\because x \in [9,\infty)]$

Hence, $x=4$ and 16.

20. (d) Let the roots of the equation $x^2-6x+a=0$ be α and 4β and that of the equation $x^2-cx+6=0$ be α and 3β, then $\alpha+4\beta=6$, $4\alpha\beta=a$

and $\quad \alpha+3\beta=c, 3\alpha\beta=6$

$\Rightarrow \quad a=8$

$\therefore$ Equation becomes $x^2-6x+8=0$

$\Rightarrow \quad (x-2)(x-4)=0$

$\Rightarrow$ Roots are 2 and 4

$\Rightarrow \quad \alpha=2$ and $\beta=1$

$\therefore$ Common root is 2.

21. (a) If the three terms of the GP be $\frac{a}{r}$, a and ar, then

$$S=\frac{a}{r}+a+ar=\frac{a}{r}(1+r+r^2)$$

$$P=a^3 \text{ and } R=\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}=\frac{1}{ar}(r^2+r+1)$$

Now, $$\frac{P^2R^3}{S^3}=\frac{a^6\,\frac{1}{a^3r^3}(r^2+r+1)^3}{\frac{a^3}{r^3}(r^2+r+1)^3}=1$$

So, the required ratio is 1 : 1.

22. (a) Number of ways = (selecting 4 girls and a captain) or (selecting 3 girls, 1 boy and a captain)

$$= {}^6C_4 \times 4+{}^6C_3 \times {}^4C_1 \times 4$$

$$=380$$

23. (a, b, c) $\left(x+\frac{1}{x}+1\right)^6=\sum_{r=0}^{6} {}^6C_r\left(x+\frac{1}{x}\right)^r$

(a) $a_0=1+{}^6C_2\cdot{}^2C_1+{}^6C_4\cdot{}^4C_2+{}^6C_6\cdot{}^6C_3$

$=1+30+90+20=141$

(b) $a_5={}^6C_5 \times$ coefficient of x^5 in $\left(x+\frac{1}{x}\right)^5$

$={}^6C_5\cdot{}^5C_0=6$

(c) Putting $x=1$,

$$a_0+(a_1+b_1)+(a_2+b_2)+\ldots+(a_6+b_6)=3^6$$

$$\therefore \sum_{i=1}^{6}(a_i+b_i)=3^6-141=588$$

24. (ABD)

(A) $x^{\log_x (x+3)^2} = 16$

$\therefore\ (x+3)^2 = 16$ (as $a^{\log_a N} = N$)

$\therefore\quad x + 3 = \pm 4 \Rightarrow x = 1$ or -7

As x is used in base. Hence, $x > 0$ and $x \neq 1$

$\therefore$ No real solution.

(B) $\sqrt{\log_{10} x} + \frac{1}{2}\log_{10} x = -\frac{1}{2}$

Now, $\log_{10} x$ must be non-negative. So that square root is defined.

$\therefore\quad \log_{10} x \geq 0 \Rightarrow x \geq 1$

Hence, $\forall\, x \geq 1$ L.H.S. is non-negative but R.H.S. is negative. Hence, no solution.

(C) $x^{x+1} = x \Rightarrow x^{x+1} - x = 0 \Rightarrow x(x^x - 1) = 0$

$\therefore\ x = 0$ or $x = 1$

Hence, the equation possess real solution.

(D) $\log_2(9 - 2^x) = x - 3 \Rightarrow 9 - 2^x = 2^{x-3} = \frac{2^x}{8}$

$\therefore\quad 9 = 2^x\left(1 + \frac{1}{8}\right) \Rightarrow 2^x = 8 = 2^3$

$\therefore\quad x = 3$

but for $x = 3$, the original equation is not defined. Hence, no real solution.

25. (BD)

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a} = 3 \Rightarrow c = 3a$

$2b = a + c \Rightarrow b = 2a$

$\therefore\quad \alpha + \beta = -2$

and $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 4 - 2\cdot 3 = -2$

26. (ABD)

$p = {}^5C_1 \cdot 2 = 10,\ q = {}^5C_3 \cdot 2^3 \cdot 1 = 80$ and $r = {}^5C_5 \cdot 2^5 = 32$

$\therefore\quad 2q = 5r = 160$. Also $8p = q = 80$

and $2(p + r) = 84 > 80 = r$

COMPREHENSION (27 TO 28)

We have $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{2} \Rightarrow \alpha\beta\gamma = -2$

$\alpha^3 + \beta^3 + \gamma^3 = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) + 3\alpha\beta\gamma$

$\Rightarrow\quad \alpha^3 + \beta^3 + \gamma^3 = (2)(6+1) + 3(-2)$

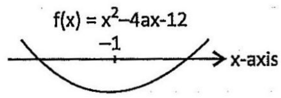
Hence, $\alpha^3 + \beta^3 + \gamma^3 = 14 - 6 = 8$

27. (D) Clearly, $(\alpha^3 + \beta^3 + \gamma^3) = 8$

28.(A) Let $f(x) = x^2 - 4ax - 12$

If both roots of $f(x) = 0$ lies on either side of $\alpha\beta + \beta\gamma + \gamma\alpha$,

so $f(-1) < 0$



$\Rightarrow\quad (-1)^2 + 4a - 12 < 0$

$\Rightarrow\quad 4a - 11 < 0 \Rightarrow a < 11/4$

So, $a_{\text{largest integral}} = 2$ **Ans.**

29.(C) (1) $f(x) = tx^2 - (2t-1)x + (5t-1)$

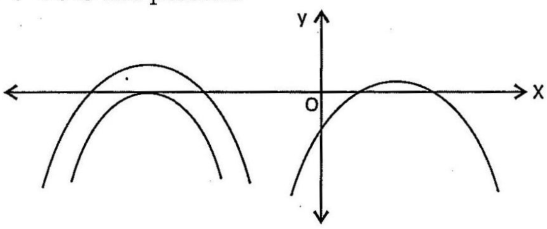
$D > 0 \quad \Rightarrow \quad (2t-1)^2 - 4t(5t-1) > 0 \quad \Rightarrow \quad 4t^2 - 4t + 1 - 20t^2 + 4t > 0$

$\Rightarrow \quad 1 - 16t^2 > 0 \quad \Rightarrow \quad 16t^2 - 1 < 0$

Hence $t \in \left(\frac{-1}{4}, \frac{1}{4}\right)$. But $t \neq 0 \quad \Rightarrow$ **(C)** **Ans.**

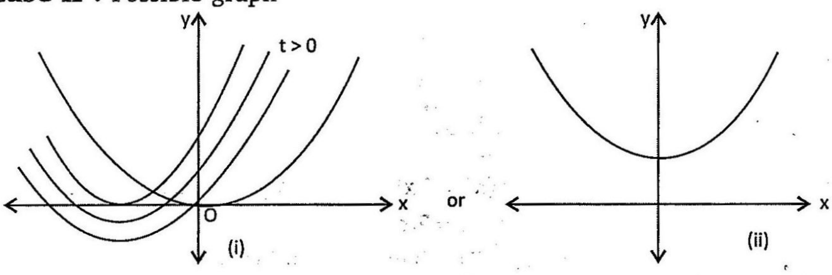
30.(D) *i.e.,* $tx^2 - (2t-1)x + (5t-1) \geq 0 \; \forall \; x > 0$

Case I : $t < 0$ is not possible



No graph is possible.

Case II : Possible graph



Now for possibility (i) the following conditions must be satisfied simultaneously.

i.e., $\left.\begin{array}{r} D \geq 0 \\ f(0) \geq 0 \\ \text{and } \frac{-b}{2a} \leq 0 \end{array}\right\}$ Intersection

$D \geq 0 \quad \Rightarrow \quad 16t^2 - 1 \leq 0 \quad \Rightarrow \quad \frac{-1}{4} \leq t \leq \frac{1}{4}$

$f(0) = 5t - 1 \geq 0 \quad \Rightarrow \quad t \geq \frac{1}{5}$

and $\frac{-b}{2a} \leq 0 \quad \Rightarrow \quad \frac{2t-1}{t} \leq 0 \quad \Rightarrow \quad 0 < t \leq \frac{1}{2}$

$\therefore \quad t \in \left[\frac{1}{5}, \frac{1}{4}\right]$ ...(1)

Also for possibility (ii), the following conditions must be satisfied simultaneously.

31. (a) A-Q ; B-S; C-S ; D-P

Put $1 + x = t$

$\therefore \quad 1 + (t-1)^4 + (t-1)^5 = \sum_{i=0}^{5} a_i t^i$

$\Rightarrow \quad 1 + (t^4 - 4t^3 + 6t^2 - 4t + 1) + (t^5 - 5t^4 + 10t^3 - 10t^2 + 5t - 1)$

$= \sum_{i=0}^{5} a_i t^i$

$\Rightarrow \quad t^5 - 4t^2 + 6t^3 - 4t^2 + t + 1$

$= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

$\therefore \quad a_0 = 1, \; a_1 = 1, \; a_2 = -4, \; a_3 = 6, \; a_4 = -4$ and $a_5 = 1$

Now, $|a_2| = 4, \; a_3 = 6, \; a_1 a_3 a_5 = 6, \; a_2 + a_3 + a_5 = 3$

32.(b) (A-Q ; B-S ; C-P; D-R

(A) $(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\ldots+(\sqrt{49}-\sqrt{48})=-1+7=6$ **Ans.**

(B) $A=12; B=4 \Rightarrow A-B=8$ **Ans.**

(C) $A=\log^2_{\sqrt{2}}\left(\frac{1}{4}\right)=\log^2_{2^{1/2}}(2^{-2})=\left(\frac{-2}{1/2}\log_2 2\right)^2=(-4)^2=16$

$B=\log^3_{2\sqrt{2}}(8)=\log^3_{2^{3/2}}(2^3)=\left(\frac{3}{3/2}\log_2 2\right)^3=(2)^3=8$

$C=-\log_5\log_3\sqrt{\sqrt[5]{9}}=-\log_5\log_3(3^{2/10})=-\log_5\left(\frac{1}{5}\log_3 3\right)=-\log_5\left(\frac{1}{5}\right)=1$

$A=16, B=8, C=1 \Rightarrow \left(\frac{A}{B}+C\right)=3$ **Ans.**

(D) $(2^{2/3}-2^{1/3}.5^{1/3}+5^{2/3})(2^{1/3}+5^{1/3})$

$=(a+b)(a^2-ab+b^2)=a^3+b^3=2+5=7$ **Ans.**

33. (3)$({}^{20}C_1+{}^{20}C_3+\ldots+{}^{20}C_9)^2=(2^{18})^6=8\times 32^{21}=8\times(33-1)^{21}$

$\therefore$ Remainder is 8 less than 11 *i.e.*. 3.

34. (3)
35. (2)
36. (1)

(SOLUTION) [PHYSICS]

37. (c) $K = K_{ring}+K_{particles}$

$=\left[\frac{1}{2}mv_0^2+\frac{1}{2}I\omega^2\right]+\left[\frac{1}{2}m(\sqrt{2}v_0)^2+\frac{1}{2}m(2v_0)^2+\frac{1}{2}m(\sqrt{2}v_0)^2+0\right]$

Also $\omega=\frac{v_0}{R}$, $I=mR^2$

$\therefore$ $K = 5mv_0^2$

38. (c) Initial position of centre of mass from $2m$

$x_0=\frac{m\times\ell}{m+2m}=\frac{\ell}{3}$.

Now $\frac{1}{2}k(x_1+x_2)^2=\frac{1}{2}m(2v_0)^2+\frac{1}{2}(2m)v_0^2$

and $mx_1=(2m)x_2$

After solving above equations, we get

$x_2=\sqrt{\frac{2mv_0^2}{3k}}$.

Thus maximum separation between centre of mass and m (it happens when m moves away from centre of mass) $=x_0+x_2$.

39. (a) Maximum chance of slipping occurs when spring is maximum compressed. At this moment, as force exerted by the spring is maximum, acceleration of the system is maximum. Hence maximum friction force is required at this moment. By W/E theorem

$\frac{1}{2}(M+m)v^2=\frac{1}{2}kx_m^2 \Rightarrow x_m=\sqrt{\frac{(M+m)v^2}{k}}$

Now for upper block $a_m=\frac{kx_m}{M+m}$

Force on upper block is provided by the friction force.

Therefore, $\mu mg \ge \frac{kx_m.m}{M+m}$

For limiting values $v=\mu g\sqrt{\frac{M+m}{k}}$

Using values $v_{maximum}=20$ cm/s

40. (b) Velocity of the bob at mean position,

$v=\sqrt{2g(\ell-\ell\cos 60^\circ)}$

Now $T-2g=\frac{mv^2}{\ell}$

and $T=\mu\times 8$ g

After solving we get $\mu=0.50$.

41. (a, b, c, d) The moment of inertia of ring can have any value greater than $MR^2/2$.

42. (a,c) $M=\frac{4}{3}\pi R^3\rho$ or $R=\left(\frac{3M}{4\pi\rho}\right)^{1/3}$

(i) $g=\frac{GM}{R^2}=\frac{GM}{(3M/4\pi\rho)^{3/2}}=GM\left(\frac{4\pi\rho}{3M}\right)^{2/3}$

i.e. $g\propto\rho^{2/3}$ $\therefore \frac{g_1}{g_2}=\left(\frac{\rho_1}{\rho_2}\right)^{2/3}=\left(\frac{1}{8}\right)^{2/3}=\frac{1}{4}$

(ii) $v_e=\sqrt{\frac{2GM}{R}}=\frac{(2GM)^{1/2}}{(3M/4\pi\rho)^{1/6}}$

or $\frac{v_1}{v_2}=\left(\frac{\rho_1}{\rho_2}\right)^{1/6}=\left(\frac{1}{8}\right)^{1/6}=\frac{1}{\sqrt{2}}$

43. (b, d) $mgh - \mu\, mg \cos\theta = \frac{1}{2}mv^2$

As h and v are same for all and so $\mu\cos\theta$ also must be same. Here θ is the inclination angle of the slope.

44. (a,b,c,d)

45. (a)

46. (d)

47. (a)

48. (b)

49.(a) A-s; B-p, r; C-q; D-q

50.(b) A-q, r; B-p; C-s; D-s

Capacitance	coulomb-volt^{-1}, coulomb^2-joule^{-1}
Inductance	ohm-sec
Magnetic Induction	newton (ampere-metre)$^{-1}$

$\because$ (A) $q = CV;\ U = \frac{1}{2}CV^2$

(B) $L = \frac{\phi_m}{I}$

(C) $F = I\ell B$

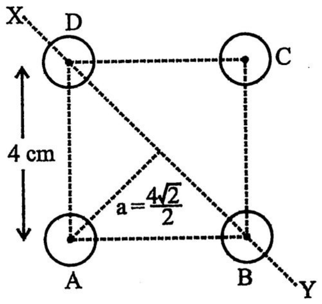
(D) Magnetic flux density $= \frac{\phi}{\text{Area}}$

51. 9

Let the four spheres be A, B, C, & D

$$I_{XY} = I_1 + I_B + I_C + I_D = 2\,I_A + 2I_B$$

$$= 2\left[\frac{2}{5}MR^2 + Ma^2\right] + 2\left[\frac{2}{5}MR^2\right]$$



$$= 4\times\frac{2}{5}MR^2 + 2Ma^2 = M\left[\frac{8}{5}R^2 + 2(a)^2\right]$$

$$= 0.5\left[\frac{8}{5}\times\left(\frac{\sqrt{5}}{2}\right)^2 + 2\times 8\right]\times 10^{-4}$$

$$= 0.5[2+16]\times 10^{-4} = 9\times 10^{-4}$$

$\therefore N = 9$

52. (3) We know that $v = \sqrt{2gR}$

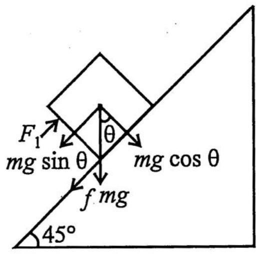
$\therefore \frac{v_p}{v} = \sqrt{\frac{g_p}{g}\times\frac{R_p}{R}}$ ...(i)

Given $\frac{g_p}{g_e} = \frac{\sqrt{6}}{11}$ ...(ii)

Also $g = \frac{4}{3}\pi G\rho R \quad \therefore \frac{g_p}{g} = \frac{\rho_p}{\rho}\times\frac{R_p}{R}$

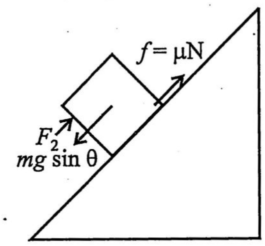
Solving we get escape velocity $v_p = 3$ km/s

53. 5



The pushing force $F_1 = mg\sin\theta + f$

$\therefore F_1 = mg\sin\theta + \mu\, mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$



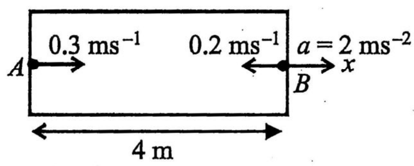
The force required to just prevent it from sliding down

$F_2 = mg\sin\theta - \mu N = mg(\sin\theta - \mu\cos\theta)$

Given, $F_1 = 3F_2$

$\therefore \sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$

54. 8



For ball A

$u_1 = 0.3\ ms^{-1},\ a_1 = -2ms^{-2},\ s_1 = x,\ t_1 = t$

$\therefore \quad s_1 = u_1t_1 + \frac{1}{2}a_1t_1^2$

$x = 0.3t - t^2$ ...(1)

For ball B

$u_2 = 0.2\ ms^{-1},\ a_2 = 2ms^{-2},\ s_2 = 4 - x,\ t_2 = t$

$\therefore \quad s_2 = u_2t_2 + \frac{1}{2}a_2t_2^2$

$4 - x = 0.2\,t + t^2$ ...(2)

From (1) and (2) $t = 8$ sec