Analyzing the Serve of The Average Professional Tennis Player and Comparing It to A Recreational Tennis Player

Mathematics Internal Assessment

Introduction:

Sports involving rackets have been prominent since the Neolithic period. In 1873, Welsh inventor Major Walter Wingfield first invented a sport called Shpairistrikè, which later evolved into one of the most popular sports: tennis[1]. It was widespread for men and first played at a tournament in Wimbledon in 1877. In recent years, the tennis industry has expanded to numerous demographics, with over 87 million tennis players globally[12] and USD 3.77 billion in revenue as of 2021[7].

I was amongst these 87 million players. My parents bought me my first racket when I was seven, and I quickly developed a passion for the sport. Throughout the years, tennis has become a coping mechanism for me, helping me feel stronger and healthier, leaving me with positivity throughout the day, and saving me from living a painfully sedentary lifestyle during the 2020 COVID-19 pandemic. In addition, tennis taught me how to be patient, predict outcomes before they occur and wait for the right opportunity.

However, my rationale for choosing this topic was that I wanted to enhance my tennis skills, specifically my serves. Tennis services remain something that I constantly struggle with. When I first started overhand serving, it enchanted me how much accuracy was involved in something seemingly simple. To improve, I watched matches to see how professionals served. It stood out how consistent they were in their techniques– tossing, jumping, and hitting the ball in the same manner. Through my interest in physics, I learnt that sports all involve mathematics. This made me wonder if there was mathematical reasoning for why professional tennis serves are much better than a regular player.

In my exploration, I aim to analyze what the serve of an average tennis player looks like, and what distinguishes it from the serve of a recreational tennis player. In conducting this analysis, I hope to become a better tennis player and correct my errors.

I will start by determining what an average tennis player physically looks like, using correlation and statistics. I will then use a video of the serve of this player, and model aspects like the toss of their serve, and what angle they hit the ball. I will then use calculus and differentiation to find the velocity and acceleration of the ball, and to find out how well the serve is. Using this information, I am going to compare the serve to my serve, and evaluate. Unless stated otherwise, I will round my final answers to 3 significant figures for accuracy. *Figure 1* below shows the parts and dimensions of a tennis court. This will help me with my calculations throughout this investigation.

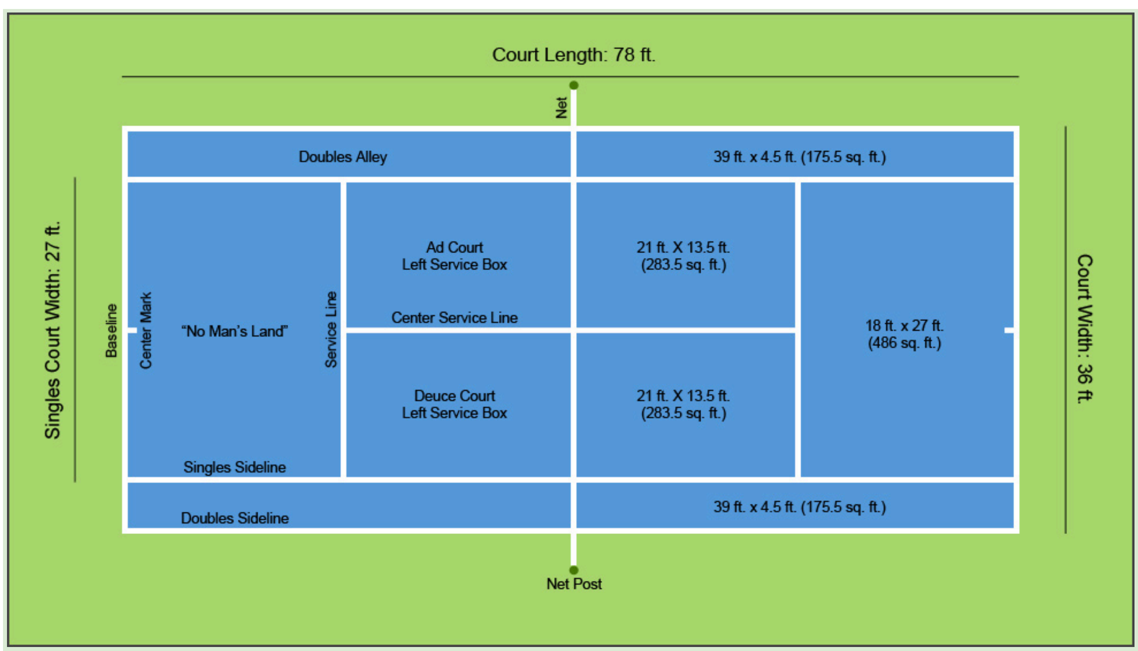


Figure 1: Parts and Dimensions of a Tennis Court[13]

Part 1: Testing the Correlation Between Height and Serving Speed

Prominently, I will see if the height of tennis players affects their serving speed. The average height of a male tennis player is 188cm and the average height of a female is 174cm, which is taller compared to the average height of a human. Serving speed refers to the speed that the ball travels upon being hit by the racket. High serving speeds are valuable as they show dominance during a game and make it difficult for their opponent to win.

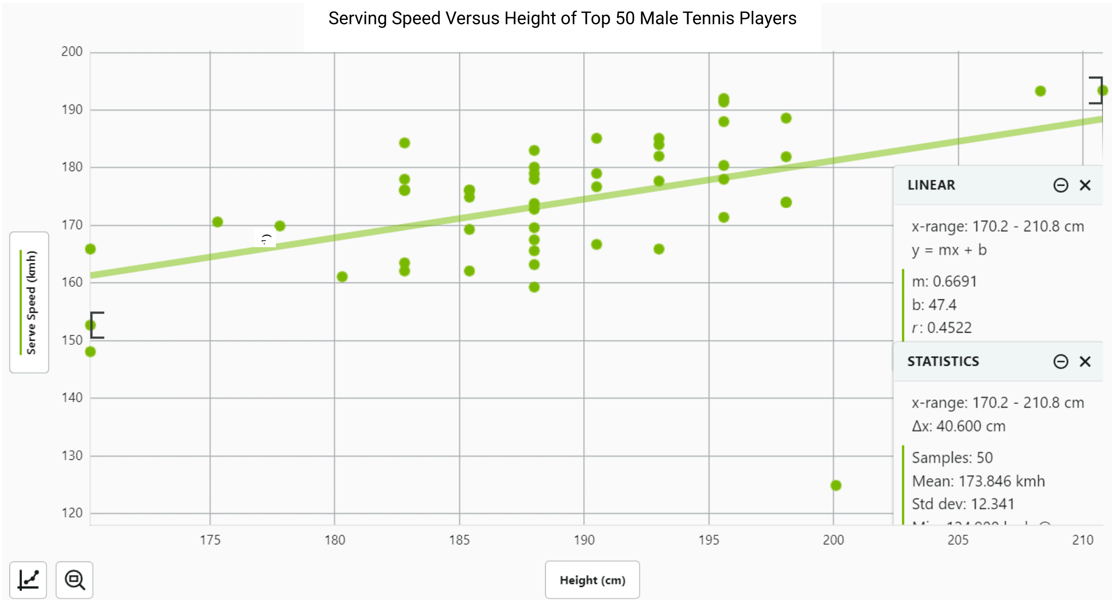
To do this, I used correlation graphs. This is because, in comparison to histograms or box whisker plots, correlation graphs can most clearly and visually tell me if there is a relationship between two distinct variables. In physics, I learnt about torque– a rotational force that causes an object to rotate about a point or axis. In serving, your arms, torso and legs create this torque, which propels your body to rotate when serving[2]. Based on this, I predict that taller players will have a greater torque, giving them a higher serving speed.

For my test, I found the heights and serving speeds of the top 50 ranked male and female players respectively. *Table 1A* below shows the data for male tennis players (please refer to *Appendix A.1* for the full dataset). I recorded the heights to 4 significant figures because I converted my units from mph to $km\,h^{-1}$ and from ft to cm, and rounded my answers to one decimal place to ensure some accuracy in the heights.

Height (cm)	service Speed ($km\,h^{-1}$)	Height (cm)	service Speed ($km\,h^{-1}$)	Height (cm)	service Speed ($km\,h^{-1}$)	Height (cm)	service Speed ($km\,h^{-1}$)
182.8	178.0	188.0	169.6	193.0	182.0	182.8	176.1
185.4	169.3	175.3	170.6	188.0	167.5	200.1	124.9
193.0	177.7	190.5	176.7	188.0	173.8	185.4	176.1

Table 1A: Height and Corresponding Serving Speed of Top 50 Male Tennis Players

I then plotted this data in *Graph 1A*



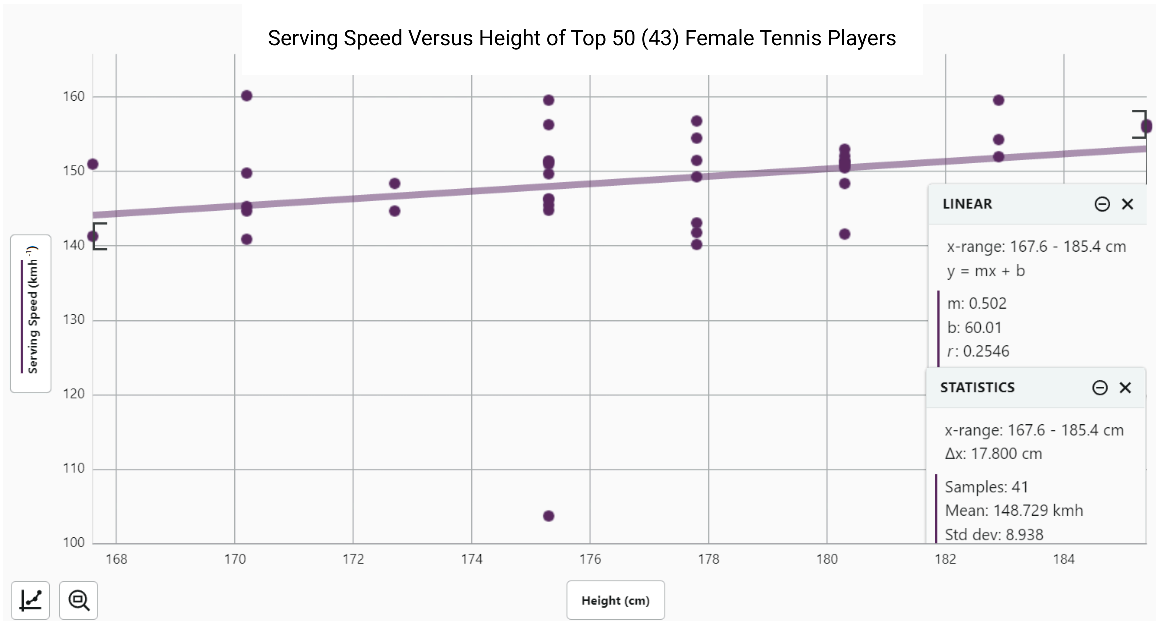
Graph 1A: Correlation Graph Representing Height and Serving Speed Of 50 Male Tennis Players[18]

$Mean \approx 174\ km\, h^{-1}$ $\qquad$ $Standard\ Deviation \approx 12.3$ $\qquad$ $Correlation(r) \approx 0.452$

I repeated the same steps, this time using the heights and serving speeds of 50 female tennis players. I could not get exactly 50 sample spaces because there is less documentation on top-ranked female biographical data than on men, as many cultures find it disrespectful to disclose a woman's age, height, or weight. My dataset is reported in *Table 1B* (please refer to *Appendix B* for the full dataset) and is visually graphed in *Graph 1B*.

Height (cm)	Serve Speed ($km\, h^{-1}$)	Height (cm)	Serve Speed ($km\, h^{-1}$)	Height (cm)	Serve Speed ($km\, h^{-1}$)	Height (cm)	Serve Speed ($km\, h^{-1}$)
145.5	175.3	151.5	177.8	151.0	167.6	155.9	185.4
141.3	167.6	152	182.9	151.0	175.3	154.3	182.9
145.3	170.2	150.8	180.3	151.1	180.3	153.0	180.3

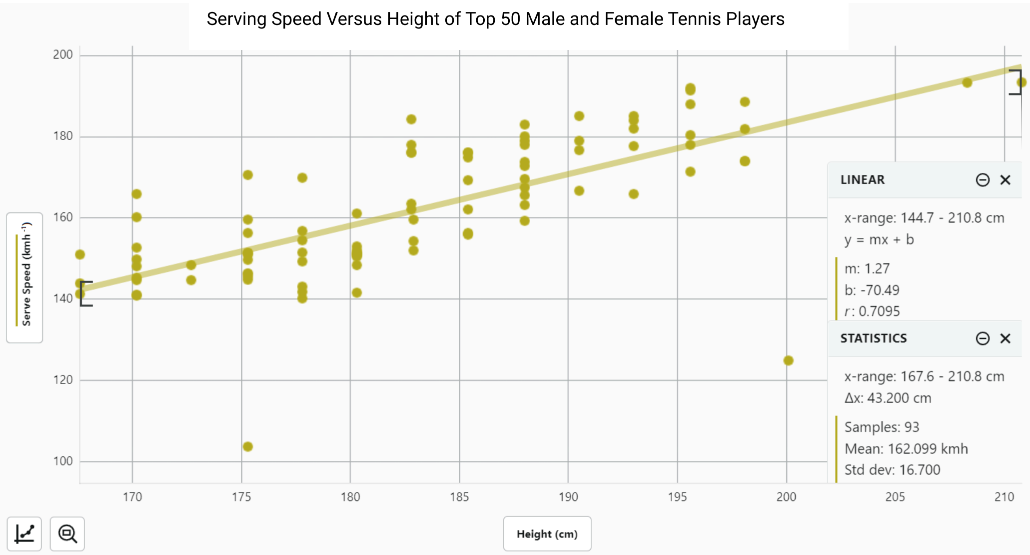
Table 1B: Height and Corresponding Serving Speed of Top 50 (43) Female Tennis Players



Graph 1B: Correlation Graph Representing Height and Serving Speed Of 43 Female Tennis Players[18]

$Mean \approx 149\ km\,h^{-1}$ $\quad$ $Standard\ Deviation \approx 8.94$ $\quad$ $Correlation(r) \approx 0.255$

Since $r \leq 0.5$, There is a weak correlation, meaning that height does not impact serving speed. This was opposite from what I predicted, so I revisited my graph. I realized this was because of similar heights. For example, in *Graph 1A*, there were huge vertical clusters of data around certain heights, which represented the mode of the graph. The mode shows what the most common height is, which in *Graph 1A* was 188cm. This was also the mean male height. This was the same case for the female heights. However, the serving speed did not have a clear cluster. I assume this is because serving speed is a measure of strength and technique, which is slightly different for everyone. I realized that since males and females have different heights, I thought that if I combined the heights of all the players into one graph (*Graph 3)*, then my correlation would be stronger and more accurate. This is seen on *page 6.*



Graph 2: Correlation Between Heights and Serving Speed of the Top 93 tennis players[18]

$$Mean \approx 162\ kmh^{-1} \qquad Standard\ Deviation \approx 16.7 \qquad Correlation(r) \approx 0.710$$

In *Graph 2* the correlation was much higher. However, since r does not equal to 1, this is not a strong correlation. On the contrary, because $r > 0.5$, this is not a weak correlation, either. Therefore, there is a moderate correlation between height and serving speed. This signifies that while there is a big relationship between these two variables, they are not completely reliant on each other. This makes sense because while most tennis players are tall, factors like weight, and experience can also influence their serve speed.

After seeing the results, I considered analyzing someone that can match this correlation, and represent the services of an average professional tennis player. I did this by using z-scores. Z-scoring is a statistical calculation that uses mean and standard deviation. The mean is the average of all the data points, and the standard deviation tells you how scattered the data is from the mean. These are used in Z-scores to find how many standard deviations a point is from the mean and will tell me if a player is a good delegate for my analysis, or if they are an outlier.

I knew the mean serving speed is $162\ km\, h^{-1}$, and the mean height for men. Upon researching, I decided to analyze Russian tennis player Andrey Rublev because he matched this average height, and his serves stood out to me. I used the formula below for my calculations:

$$z = \frac{X - \mu}{\sigma}$$

where

$z = standard\ score$

$X = testing\ value\ (168\ km\, h^{-1})$

$\mu = mean\ value\ (162\ km\, h^{-1})$

$\sigma = standard\ deviation\ (16.7)$

$$z = \frac{168 - 162}{16.7}$$

$$z = \frac{6}{16.7}$$

$$z \approx 0.359$$

Since the z-score is below 0.5, Andrey Rublev is an accurate physical representation of what a male tennis player looks like, and I will be using him for my analysis.

Part 2: The Analysis of Andrey Rublev's Serve

I will first look at the toss of the ball. The toss is significant because it builds a foundation for how the ball will be hit. Ideally, the ball should be thrown 2 to 3 feet (60-90cm) above the maximum height reached by the servicer and their racket[14]. This ensures that the servicer has enough time to hit the ball properly. Andrey Rublev uses a Wilson Six-One 95, which is almost 70cm (2 feet 3 inches)[17]. Correspondingly, his height is 6 feet (188cm). Based on this, I predict that the ball would be tossed 305-335 cm (10-11 feet) high. To test this, I used

Andrey Rublev's serves during the 2021 Australian Open and made image sequences to show the gradual motion of the ball. I then inserted a "png grid" above each image and drew a cartesian plane, to help me make coordinates. *Figures 2A-2B* represent the toss of the ball.

Figure 2A: The Initial Height of the Tennis Ball[11]

Figure 2B: The Maximum Height Reached by the Tennis Ball[11]

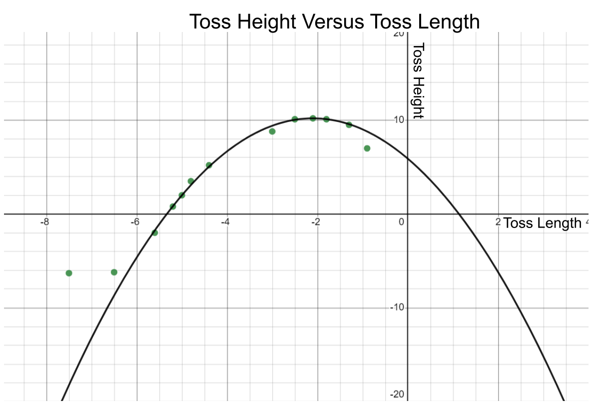
For my scale, I counted the number of grids from the baseline to the service line, which was 29 grids. In *Figure 1,* the distance between the baseline and the service line is 548cm (≈ 18ft), so I divided 548cm by 29 grids and let every grid be approximately 18.9 $\pm$0.1 cm. I put a slight uncertainty of 0.1 because when finding the exact placement of the ball, the decimal was subjective to who was viewing the grid. However, this will not significantly impact my analysis

Table 2 below are the coordinates representing the motion of the ball.

Toss Length(l) $\pm$ 0.1	Toss Height(h) $\pm$ 0.1	Toss Length(l) $\pm$ 0.1	Toss Height(h) $\pm$ 0.1
-7.5	-6.3	-4.5	5.2
-6.5	-6.2	-2.5	10.1
-5.5	-2	-2.1	10.2
-5.2	0.8	-1.8	10.1
-5	2	-1.3	9.5
-4.8	3.5	-0.9	7

Table 2: The Motion of the Tennis Ball

Using the coordinates, I modelled the ball's motion through a graph and found its equation. This will tell me whether a ball should be thrown in a smooth, symmetrical curve, a linear line or in another way. Upon seeing the maximum height and seeing that the adjacent points had the same height, I instinctively assumed that the motion of the ball followed a quadratic function. I knew I could have used second differences to prove this, but I chose not to.

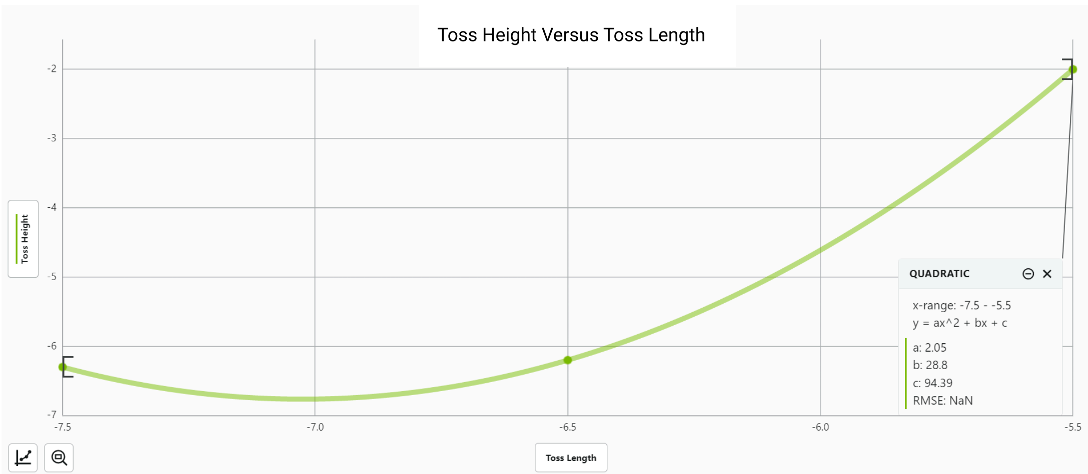


Graph 3: Motion of the Tennis Ball During the Toss[6]

This is because since this is a real-life application, environmental factors could have impacted the motion and path of the ball. Therefore, I decided to plot the coordinates instead (seen in *Graph 3)*. While this graph is a good fit during certain intervals, I did not like how the curve did not account for the beginning and end of the toss. This made me want to find another equation that could make a better fit.

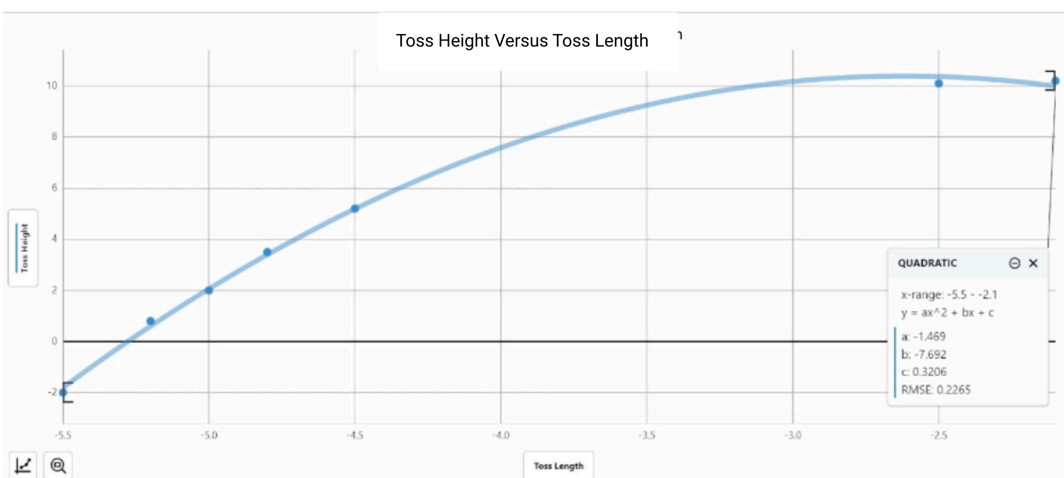
I knew this was not a linear function because the graph was not straight, and this could not be a trigonometric function because sinusoidal functions must be periodic and symmetrical, which my graph was not. I noticed that the shape of my graph changed from different intervals. This prompted me to use a piecewise function to graph.

The first three points are when the ball is in his hand. Since there are three points, there will only be one second-difference value, resulting in a quadratic function (seen in *Graph 4)*. The equation was $H(l) = 2.05l^2 + 28.8l + 94.39$. The domain is $\{l| -7.5 \leq l \leq -5.5,\ l\varepsilon R\}$.



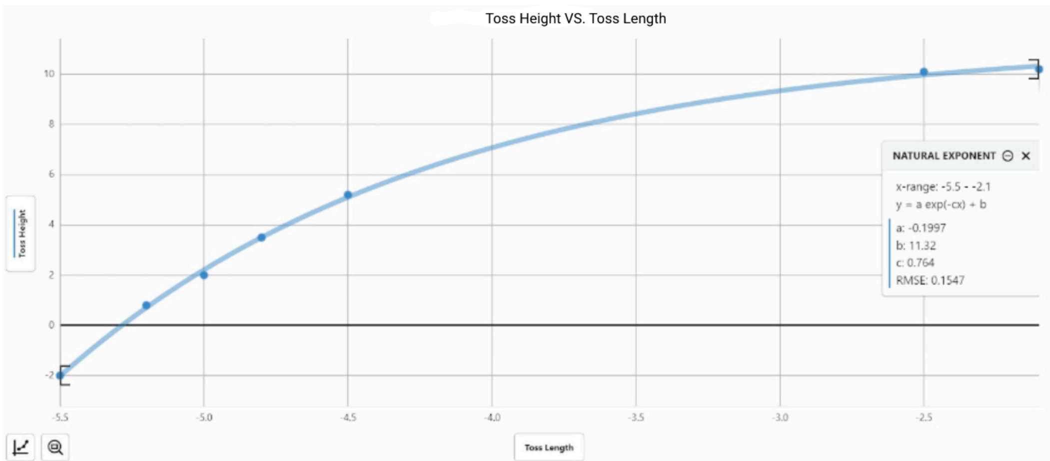
Graph 4: Equation of the graph when the tennis ball is still partially in Andrey Rublev's hand[18]

The second equation represents when the ball is released from Rublev's hands (at $l =-5.5$) and upon approaching the maximum height ($at\ l =-2.1$). I first applied a quadratic function for my curve of best fit, which is seen in *Graph 5A*.



Graph 5A: Quadratic Curve of Best Fit During the Upward Motion of the Ball[18]

While it was a good fit, I decided to compare it to a natural exponential fit, as they have similar structures, and I wanted to see which one made a better fit with my data. *Graph 5b* is seen below.



Graph 5B: Natural Exponential Curve of Best Fit During Upward Motion of the Ball[18]

The natural exponential curve of is more accurate than the quadratic function.

The form for an exponential function is:

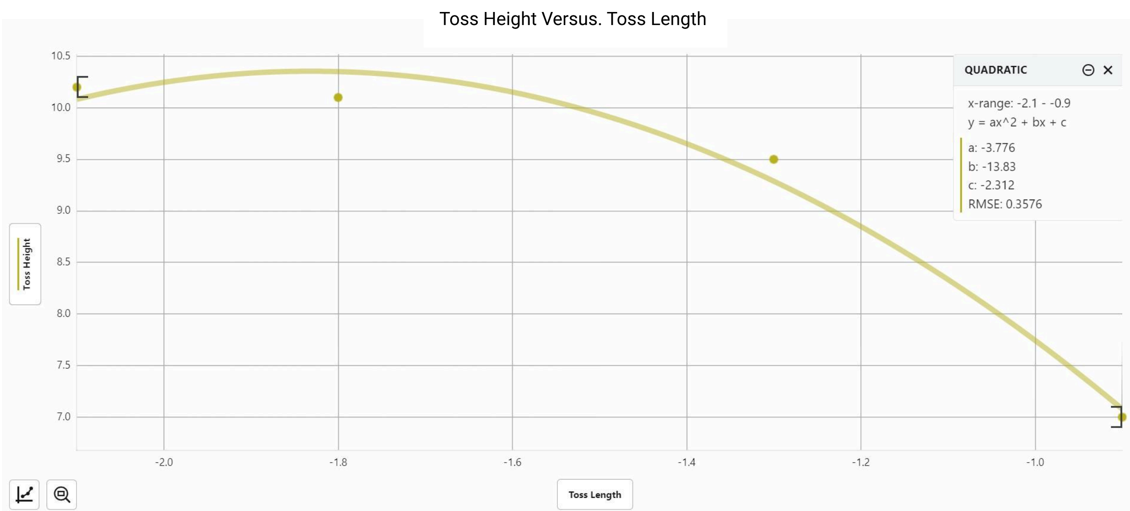
$$H(l) = ae^{(-cl)} + b$$

where:

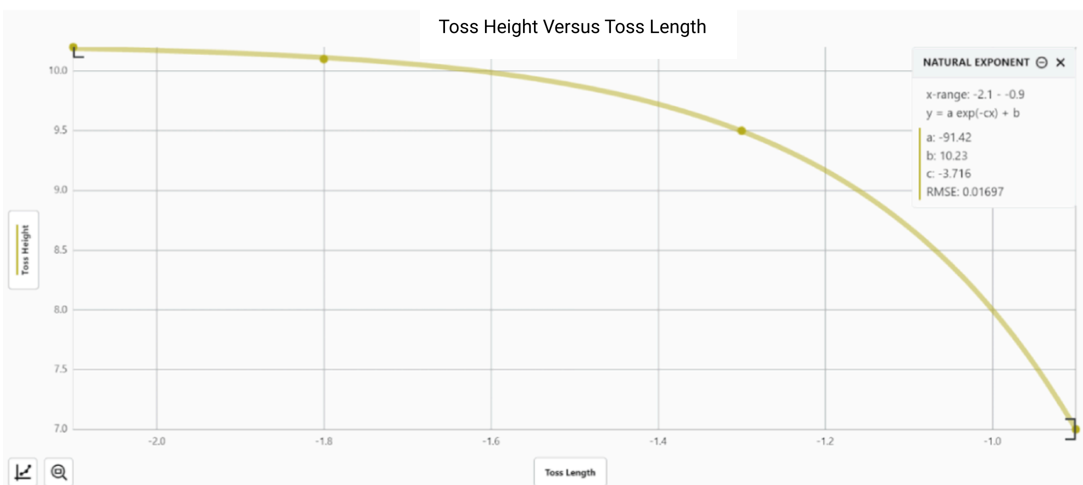
a	The exponential growth/decay factor. When $a > 0$, then as l increases, so does $H(l)$. When $a < 0$, then as l increases, $H(l)$ decreases.
b	A vertical translation.
c	A horizontal stretch or compression. When $c > 0$, there is a horizontal stretch by a factor of $\frac{1}{c}$. When $0 < c < 1$, there is a horizontal stretch by a factor of c

Table 3: The Parameters of an Exponential Function

The equation of the graph is $H(l) =- 0.1997e^{(-0.764l)} + 11.32$. The domain is $\{l| - 5.5 \leq l \leq - 2.1, l\varepsilon R\}$. I then repeated the comparison above, this time to find the curve of best fit for the downward motion of the toss, ranging from $l =- 2.1$, until the ball's intersection with the racket at $l =- 0.9$. The results can be seen in *Graphs 6A-6B*.



Graph 6A: Quadratic Curve of Best During the Downward Motion of the Ball[18]



Graph 6B: Natural Exponential Curve of Best Fit During the Downward Motion of the Ball[18]

Graph 6B clearly hit more points than *Graph 6A, proving that* a natural exponential function made a better fit. The equation of the graph is $H(l) =- 91.42e^{(3.716l)} + 10.23$. The domain is $\{l| - 2.1 \leq l \leq - 0.9,\ l\varepsilon R\}$.

This proves that a piecewise function best graphs this data. The equation of the function is:

$$H(l) = \begin{cases} -0.997e^{(-0.764l)} + 11.32 & -7.5 < l < -5.5 \\ 2.05l^2 + 28.8l + 94.39 & -5.5 < l < -2.1 \\ -91.42e^{(3.716l)} + 10.23 & -2.1 < l < -0.9 \end{cases}$$

To determine how high the ball was thrown, I used the distance formula below.

$$d = \sqrt{(l_2 - l_1) + (h_2 - h_1)^2}$$

d = distance/height between two points

l = horizontal/length coordinate of points 1 and 2(toss length)

h =vertical/height coordinate of points 1 and 2 (toss height)

To isolate and the vertical distance, I ignored the l-coordinate, so that the formula looks like this:

$$d = \sqrt{(h_2 - h_1)^2}$$

To find the height, I used the minimum value, (-7.5, -6.3), where h_1= -6.3, and the maximum value, (-2.1, 10.2), where h_2=10.2

$$d = \sqrt{[10.2 - (-6.3)]^2}$$

$d = \sqrt{16.5^2}$ (the square root and the exponent two cancel each other out)

$d = 16.5$ grid squares

$16.5\ x\ 18.9 = 311.85\ rounded\ to\ 312cm\ (10.3\ ft)$

From these calculations, the ball was tossed 312cm, which matches the prediction I made, proving that this is an ideal toss.

As a physics student, these equations immediately made me wonder what the velocity and acceleration of the object would be. In physics, velocity is an object's speed in relation to its time and displacement. It can be found by finding the derivative, which is the rate of change, of displacement. Acceleration is the rate of change in velocity, and if something is speeding up or slowing down. It is found by getting the second derivative of displacement.

The velocity and acceleration will tell me how fast the ball travels before being hit, and whether it was slowing down or speeding up. For this calculation, I only need the vertical displacement. This is represented by the toss height because the height from the origin is how far the ball travelled. However, a problem I ran into was that toss length and toss height could not be negative. In addition, displacement needs to start from a point of origin, which I did not have. I decided to solve this by translating the points from *Table 2* 7.5 units to the right and 6.3 units upward, such that the first point represents the origin of the object, and then multiplying them by 0.189m (18.9cm), because displacement is expressed in meters. This is seen in *Table 2A.*

Horizontal Displacement (m)	Vertical Displacement (m)	Horizontal Displacement (m)	Vertical Displacement (m)
0	0	0.567	2.1735
0.189	0.0189	0.945	3.0996
0.378	0.8127	1.0206	3.1185
0.4347	1.3419	1.0773	3.0996
0.4725	1.5687	1.1718	2.9862
0.5103	1.8522	1.2474	2.5137

Table 4A: Horizontal and Vertical Displacement (m)

I did not round my answers to 3 significant digits because I knew I was going to need them for further calculations and rounding them would change my answer.

I originally planned to use derivatives, as they are the easiest way to find the velocity and acceleration of a position time graph. However, because I did not have a position-time graph or time, I wan not sure how to mathematically solve for velocity.

This then led me to think of this problem from a physics perspective. I recalled in physics class that we frequently used work and energy to find the velocity of objects.

Part 2.1: Using Physics to Find Velocity and Acceleration

Physically, when you do work, you apply forces on an object, and that causes the object to be displaced. For example, when you walk, you are exerting a frictional force on the floor. This is what causes you to move forward. Energy is related to work because whenever work is done, the total energy changes. This is why when you are walking, you are using energy. Energy is measured in Joules (J) and comes in different forms. The most common are gravitational potential energy (E_p) and kinetic energy (E_K).

Gravitational potential energy is the amount of energy needed to do work against gravity. In tennis, when we throw the tennis ball, the ball works against gravity to move from one point to another. As the higher the ball travels, the more force it exerts to fight against the gravitational constant ($9.8m\,s^{-2}$). This results in an increase in work and gravitational potential energy to keep the ball moving. Because of this concept, an object has the most gravitational potential energy when it is at its maximum height because that is when all the energy the ball has is allocated towards keeping the ball's upward motion. After that, there is insufficient energy to keep the ball up, which is why the ball eventually descends. The equation for gravitational potential energy is.

$$E_P = mgh$$

Where

$E_P = \ gravitational\ potential\ energy$

$m = \ mass\ of\ the\ object$

$g = \ gravitational\ constant\ (9.8m\ s^{-2})$

$h = \ height$

According to the conservation of energy, energy cannot be created nor destroyed, it can only be transformed from one form to another The concept states that:

$$\Sigma E_{TOTAL} = \ \Sigma E'_{TOTAL}$$

This reiterates that the energy before action must equal the energy after it. My favourite example of this is a car collision. Before a crash, the car uses kinetic energy to drive forward. However, when the car is forced to stop during a car crash, the energy that was once used for kinetic energy becomes converted to other forms of energy like heat energy, light energy, and sound energy, which is why we hear sounds when a car crash occurs.

In tennis, when the ball descends from its maximum height, the gravitational potential energy decreases. However, to conserve this total energy, the gravitational potential gets transformed into kinetic energy. Kinetic energy is what causes an object to be in motion, which is seen when the ball descends. The formula for kinetic energy is

$$E_K = \frac{1}{2}mv^2$$

Where:

$E_K = \ kinetic\ energy$

$m = \ mass$

$v = velocity$

An increase in gravitational potential energy causes a proportional decrease in kinetic energy. Therefore, when E_P is at a maximum, $E_G = \ 0$,as no other energies are involved here.

$$E_P + E_K = E_{TOTAL}$$

We can use these concepts to find velocity. The mass of an average tennis ball is $51.8g$ Since I know the height of the ball at its maximum height, we can find E_{TOTAL} by finding the maximum gravitational potential energy ($9.8mh$) when the maximum height is $h = 3.12$.

$$E_{TOTAL} = 58.1 \times 9.8 \times 3.1185 + 0$$

$$E_{TOTAL} \approx 1776\text{J}$$

This means that the energy will have to be 1776J no matter what. Using this information, we can find out what the kinetic energy would have to be when the ball is not at maximum height. We can do this by using the conservation of energy equation.

$$E_P + E_K = E_{TOTAL}$$

$$mgh + \frac{1}{2}mv^2 = E_{TOTAL}$$

Rearrange the equation to solve for v.

$$v = \frac{\sqrt{2(E_{TOTAL} - mgh)}}{m}$$

Since I need to find the velocity when the ball intersects the racket, the height at that point is 2.5137.

$$v = \frac{\sqrt{2[1776 - (58.1)(9.8)(2.5137)]}}{58.1}$$

$$v \approx 3.44m\,s^{-1}.$$

Therefore, the velocity of the ball is $v \approx 3.44m\,s^{-1}$. I did not know how fast this quantity was exactly, but as someone who is learning to drive, I usually drive at $20km\,h^{-1}$ ($5.56m\,s^{-1}$), and I find it fast. Therefore, I believe that the ball travels at a fast pace.

Since velocity is the derivative of displacement-time graphs, the formula for velocity is

$$Velocity(v) = \frac{Displacement(s)}{Time\ (t)}$$

To find t, I rearranged the formula and divided s by v.

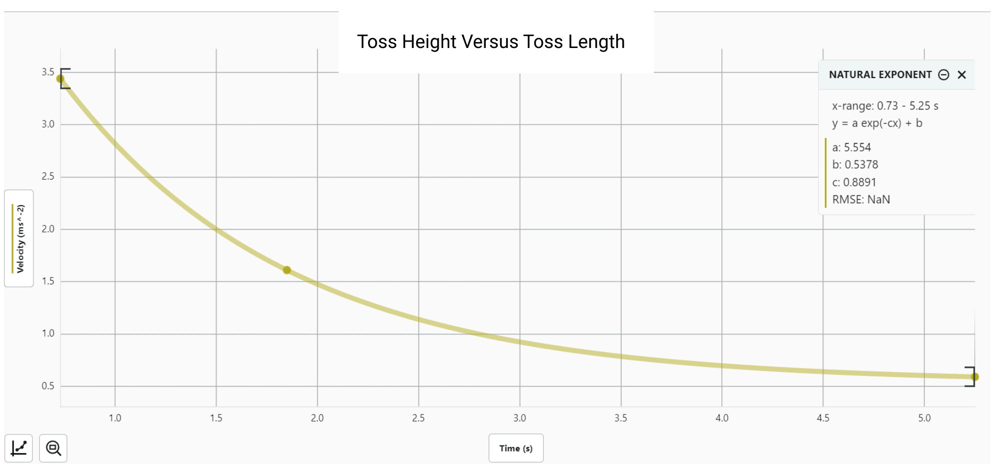
$$t = 0.730\ seconds.$$

I repeated the same calculation for all my heights that represent the downward motion to find the velocity and time. My results are seen below in *Table 3.*

Displacement (s)	E_p (J)	$Velocity\ (v)$	Time (t)
3.1185	1776	0	0
3.0996	1764	0.59	5.25
2.9862	1700	1.61	1.85
2.5137	1431	3.44	0.730

Table 5: Velocity and Time(t) During Different Heights

I then graphed the following points to make a velocity-time graph. This is seen in *Graph 7.*



Graph 7: Velocity- Time Graph of the Downward Motion of the Ball

The equation of the graph is $v(t) = 5.554e^{(-0.8891t)} + 0.5378$

Using the velocity time graph, I can find the acceleration by taking the first derivative of the graph. I chose to use derivatives as opposed to formulas because it is much more efficient to substitute a time value and get my answer, and it limits the chances of making errors. I know that acceleration can be found by getting the first derivative of velocity. I am. To find the derivative, I used the chain rule formula. The chain rule is used to find the derivative of a function that composes another function. The rule is described below.

$$a(t) = \frac{dv}{dt} = \frac{dv}{du}\frac{du}{dt}$$

Where

$\frac{dv}{dt}$ $is\ a(t)$, which represents acceleration.

$\frac{dv}{du}$ is the derivative of v in terms of variable u.

$\frac{du}{dt}$ is the derivative of u in terms of t.

$$v'(t) = 5.554e^{(-0.8891t)} + 0.5378 \text{ (the derivative of a constant is zero)}$$

$$v'(t) = 5.554e^{(-0.8891t)} \times (-\ 0.8891)$$

$$v'(t) = a(t) - 4.94e^{(-0.889t)}$$

I substitute $t = 0.73$ into the equation to find the acceleration at that time.

$$a(t) =- 4.94e^{[-0.8891(0.73)]}$$

$$a(t) \approx\ - 2.5m\,s^{-2}.$$

To find out if the object is accelerating or decelerating, you can take the product of the acceleration and the velocity. If $V(t)A(t) > 0$, the object speeds up. If $V(t)A(t) < 0$, the object slows down. Since acceleration is negative, but velocity is positive, the product of the two will be negative. This means that the object is slowing down.

From this analysis, I can conclude that an ideal toss involves the ball being thrown with a reasonable amount of force so that the ball is not very slow while descending but is also thrown gently so that it does come down with great force. When I first saw the negative acceleration, I was confused because I assumed that the object would speed up while descending. However, in my equations and calculations, I did not account for any air resistance, even though it could have had an impact on the motion of the ball. If the air resistance were strong enough, the ball could have slowed down while descending.

This made me wonder what angle the ball made with the racket during its point of intersection. The angle of incidence, which is the angle at which something is hit, reflects the angle and the motion at which the object is then reflected. It is important that the ball passes the opponent's service line but is not too wide that the ball goes past the opponent's baseline.

From *Figure 3A,* you can see that the angle is not perpendicular to Point C, which is horizontal. This means that the angle was not hit from a right angle.

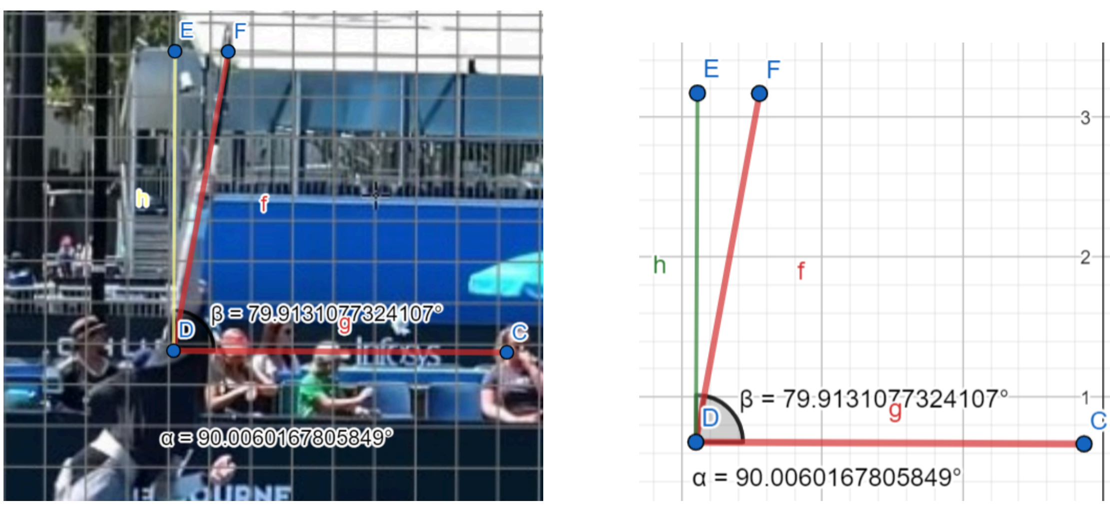


Figure 3: The Angle at Which the Racket Intersects the Tennis Ball[9]

To find the angle, I inserted an image on GeoGebra, and using the angle tool, added points C, D and F, to create angle ∠CDE, which was 79.9º. I know that it is best to hit an angle at an acute angle, so that it can clear the net at a lower height, making it hard for the player to receive the ball. The optimal angle to hit a tennis ball is between 60º to 80º, or 10-30º to the perpendicular. Since this angle is approximately 80º, this would be considered an optimal angle[5].

From this exploration, I noticed that tennis players have a very consistent style of serving. However, I realized that one of the main reasons we consider professionals so superior is that we never compare them to regular people, and just see the career they make out. To put how good these players' services are, I am going to be looking at a prominent aspect that determines service success: aces. A tennis ace is a legal service in which your opponent cannot receive the service because it is far too difficult to get it. They are hard to get because of the large amount of speed that you must put in, and only some individuals can master them. To determine the true success of Rublev's services, I am going to find the probability of him getting two aces in a row.

The probability of him getting one ace is 8.6% (0.086)[8]. I let $P(A)$ represent the probability of him serving an ace. I know that the probability of something cannot exceed 1, so I subtracted 0.086 from 1 to get the probability of Rublev failing to hit an ace, represented by $P(A')$, which was 0.914.

I then made a tree diagram to visualize the probability of him hitting two aces in a row, which is seen in *Figure 4.*

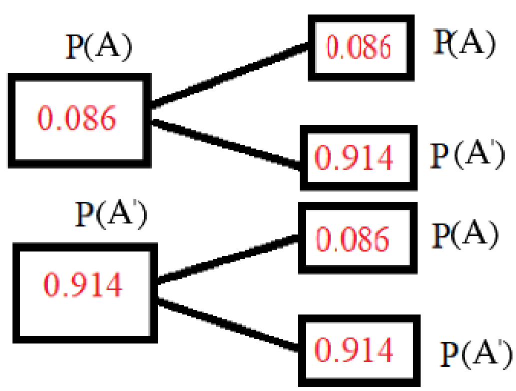


Figure 4: A Tree Diagram Representing the Probability of Getting an Ace.

I know that the probability of him getting one ace does not directly impact his ability to get another ace because they both require the same energy and the same format. Due to this, I used the independent events probability formula below.

$$P(A \cap A) = P(A)P(A)$$

Where

$P(A \cap A)$ is the probability of getting $P(A)$ twice.

$= (0.086)(0.086)$

≈ 0.007396

The probability of getting two aces in a row is approximately 0.007 or 0.740%.

On average, the average percentage of aces is 7%.[2] From this information, the first aces are similar in percentages, and this means that Andrey Rublev is an above-average servicer in tennis and has a better probability of hitting two aces in a row than many other players.

Part C: Comparing This To The Serve of A Recreational Tennis Player

The second part of my aim wants to briefly compare the services of a professional tennis player to a recreational tennis player. From all my knowledge, I compared the service of Andrey Rublev to the service of my father. He is 178cm tall, a recreational tennis player, and my coach. I chose to analyze him instead of myself because I realized that biologically, I am shorter and have a smaller mass than him, so these techniques may not be as ideal to me as they would be for my father, which is why he is replicating the services.

I first started by using png grids to find the height of the toss of the tennis ball. *Figures 4A-B* show the height of the toss of the tennis ball. For my scale, I saw that the length of the center service line was 17.9 grids. Using *Figure 1,* the center service line is 640 cm (21ft), meaning that each grid was 35.8cm (1.17ft). This number might be slightly inaccurate because the video was taken at an angle, and the perspective you look at the court slightly changes its dimensions.

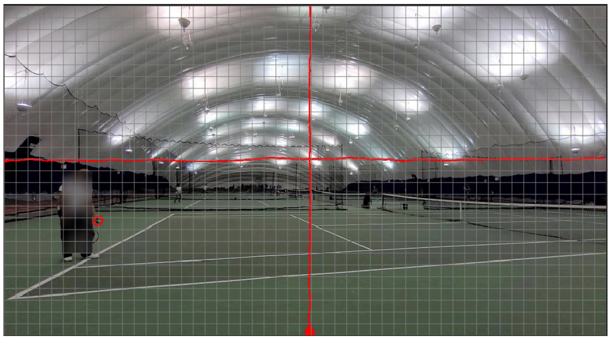


Figure 4A: The Initial Position of the Tennis Ball

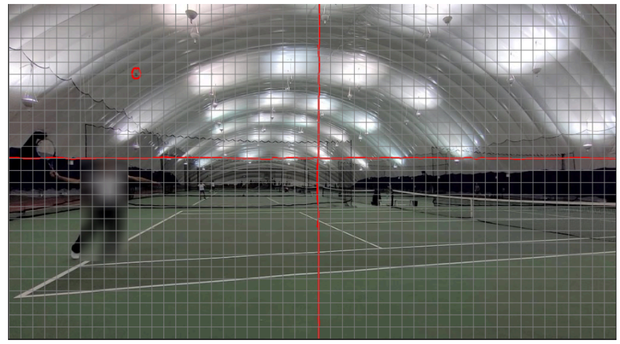


Figure 4B: Maximum Height of the Tennis Ball

In *Figure 4A,* the initial position of the ball is at $(-18, -5.5)$ and in *Figure 4B,* it is at $(-15, 7)$. Using the altered distance formula from *Page 9,* I calculated the vertical distance between the heights. My answer was 12.5 units, which is approximately 447 cm (14.6ft). This is much higher than how high Rublev tossed the ball. Since my dad is shorter than him and the Wilson Ultra Power 105 Tennis Racket he used is 69cm (2.26ft), his toss should be lower. This can be detrimental to a serve as it impacts the angle hit the ball. I guessed that my dad's angle of intersection was bigger than ideal. *Figure 5* is the angle of intersection for my dad's serve.

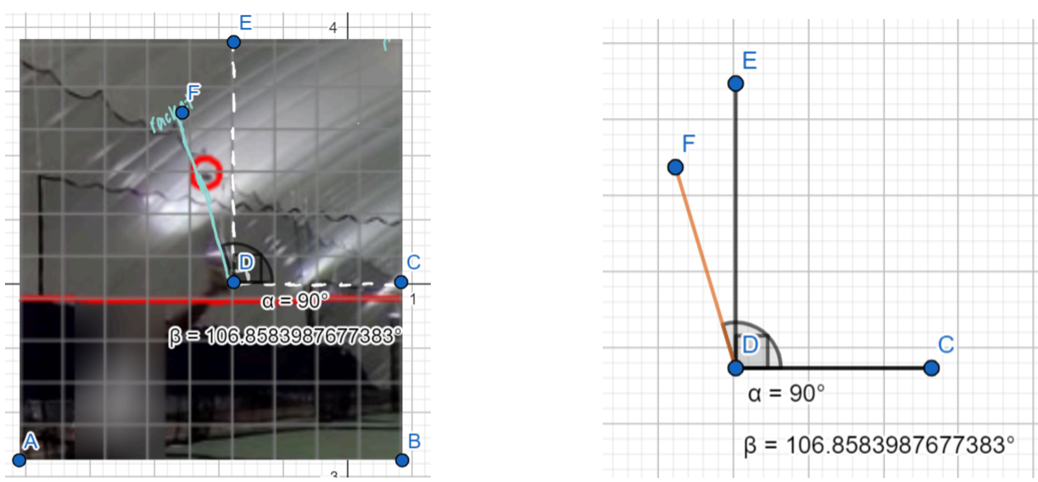


Figure 5: Angle of Intersection Between My Father's Racket and the Tennis Ball[9]

Figure 5 says that the ball was hit at a 107° angle. As predicted, the angle is obtuse and greater than the angle that was hit by Andrey Rublev, Proceeding with this investigation, I briefly looked at the path that the tennis ball made, which is seen in *Figures 6A-B*

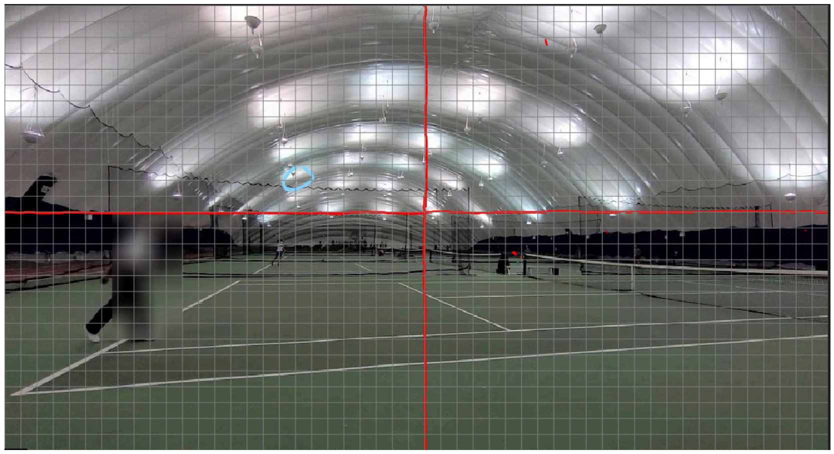


Figure 6A: The Motion of the Ball Just After Intersecting the Racket

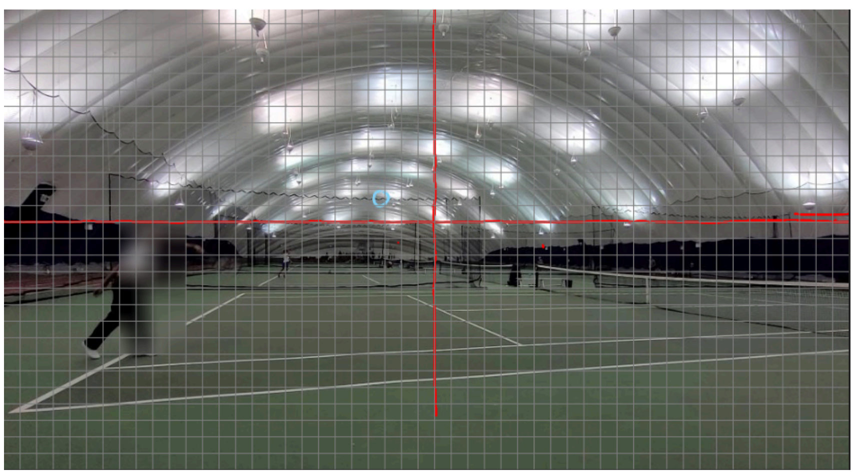


Figure 6B: Motion of the Tennis Ball

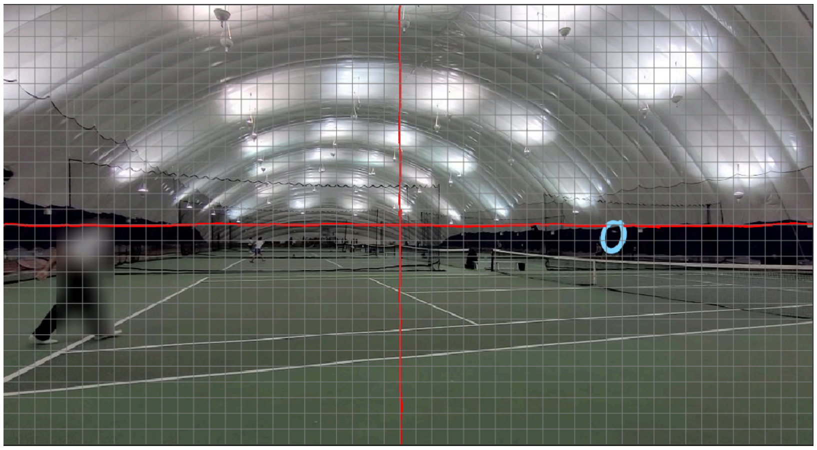


Figure 6C: The Tennis Ball Right Above the Tennis Net

From *Figure 6*, I sawthat the path of the tennis ball is horizontally stretched. While I cannot compare this to the path of Andrey Rublev, I interpreted that this occurred because of the angle of the intersection. In physics, when we learn about projectile motion, we talk about how the angle has a huge impact on the motion that the path takes on. Because my dad hit the ball at an obtuse angle, the ball would have followed a wider path. However, from *Figure 6C,* I noticed that the ball is still close to the net, and that the ball did not go outside the boundaries. This might be because of the height difference between my father and Rublev. Since Rublev is taller, if he hit the ball at an obtuse angle, the ball would have gone out. However, if my dad hit the ball at an acute angle, similar to Rublev, his ball would have hit the net. This means that along with the toss of the ball, the height can also impact the way you hit the ball.

Conclusion

This intensive investigation clearly answered my question as to what the serve of an average professional tennis player looks like. As someone who is below average height, the moderate correlation between height and serving speed taught me that while height does play a factor in greater serving speed, countless other factors also influence it, and I can do better if I put in the effort. Besides that, the piecewise function cleared the myth that all tosses travel in a quadratic equation, and instead, they follow distinct paths depending on the time and the circumstances. My velocity and acceleration taught me that it is ideal to gently toss the ball, rather than throw it upwards so that the ball does not travel too fast and speed up or down. Based on my comparison to my father's service, I learnt the importance of planning out every move, and the importance of tossing the ball at the right angle, which is something that professionals pay closer attention to, as it sets the groundwork for a service.

When carrying out this experiment, some limitations that impacted my investigation are

1. Only one tennis serve of Andrey Rublev was analyzed, and this could not have been one of his best serves, for it to be an accurate representation of how a good service looks like.
2. The ideal height of a toss and angle of incidence were recommended by a secondary source, which may have gathered its data from another source. The outside source may have been subjected to bias or an incorrect opinion, which could change my findings. I could have solved this problem by using a range of projectiles to find the optimal angle or using trial and error to find the optimal height of the toss.

A problem with my investigation was the inability to analyze the path that the ball made while serving. The video that I used to find this service did not properly highlight the motion and could have provided me with much concrete evidence as to how these factors impacted the serve.

As someone who also plays other sports like badmintonthis exploration made me realize more prominently why the services in each sport are different. For example, in badminton, you are not allowed to use an overhand service. This makes sense because the net is much higher than in tennis, making it extremely difficult to clear the ball over the net.

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Appendix A:

Height and Serving Speed of Top 50 Male Tennis Players

Height (cm)	service Speed ($km\ h^{-1}$)	Height (cm)	service Speed ($km\ h^{-1}$)	Height (cm)	service Speed ($km\ h^{-1}$)
182.8	178.0	188.0	169.6	193.0	182.0
185.4	169.3	175.3	170.6	188.0	167.5
193.0	177.7	190.5	176.7	188.0	173.8
182.8	184.3	185.4	162.1	195.6	171.4
198.1	174.0	182.8	176.1	188.0	183.0
195.6	180.4	170.2	148.10	188.0	163.2
198.1	188.6	195.6	192.0	188.0	165.6
188.0	179.0	170.2	165.9	198.1	181.9
195.6	191.4	188.0	180.1	182.8	162.1
198.1	174.0	208.3	193.3	193.0	184.0
185.4	174.9	193.0	185.1	185.4	176.1
188.0	172.8	177.8	169.9	182.8	163.5
170.2	152.7	190.5	166.7	182.8	176.1
200.1	124.9	195.6	178.0	210.8	193.4
185.4	176.1	190.5	179.0	188.0	159.3
195.6	188	180.3	161.1	193.0	165.9
190.5	185.1	188.0	178.0		

Appendix B:

Height and Serving Speed of Top 50 Female Tennis Players

Height (cm)	service Speed ($km\ h^{-1}$)	Height (cm)	service Speed ($km\ h^{-1}$)	Height (cm)	service Speed ($km\ h^{-1}$)
145.5	175.3	151.5	177.8	151.0	167.6
141.3	167.6	152.0	182.9	151.0	175.3
145.3	170.2	150.8	180.3	151.1	180.3
156.8	177.8	143.1	177.8	154.3	182.9
159.6	182.9	151.4	175.3	141.6	180.3
148.4	172.7	141.8	177.8	160.2	170.2
103.7	175.3	148.4	180.3	151.4	175.3
143.9	167.6	159.6	175.3	146.3	175.3
141.1	170.2	149.3	177.8	140.9	170.2
156.3	175.3	151.4	180.3	144.7	170.2
154.5	177.8	146.3	175.3	144.7	172.7
144.8	175.3	156.3	185.4	149.8	170.2
150.5	180.3	152.1	180.3	149.7	175.3
140.2	177.8	151.4	180.3	153.0	180.3
155.9	185.4	153.0	180.3	140.2	177.8
154.3	182.9	151.4	180.3		