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Introduction:

The **research question** in investigation is:

To what extent does the average string tension (lbs.) of seven different zones on a tennis racket correlate to the height (in.) of the tennis ball's bounce (aka. bounce height)?

As of 2022, the global tennis racket market is valued at US\$ 392.34 million, and is expected to increase to US\$525.90 million by 2030 (The Insight Partners, 2023). In this booming market, rackets are becoming innovative and better equipped with features to enhance one's performance.

During my Internal Assessment in "IB SL Mathematics: Analysis and Approaches," I analyzed the serve of the average professional tennis player and compared it to the serve of my father. I learnt that the initial toss of the ball into the air prior to serving impacts the angle and location, the ball gets hit, which ultimately changes the direction, velocity, and momentum of the ball. Hence, it is the most crucial step in serving and hitting the ball. However, until grasping a thorough knowledge of forces through the IB SL Physics curriculum, I never questioned why hitting a tennis ball in designated places like "the sweet spot" was considered ideal. In investigating this reasoning, I aim to improve my tennis skills and learn where to hit a tennis ball.

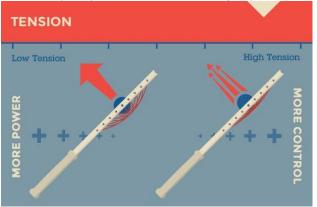
However, the question "Where on a tennis racket can one obtain the highest height of a tennis ball's bounce? does not help understand the scientific anatomy of racket. Thus, to graph and physically prove why various parts of a tennis racket have different bounce heights, the question focuses on the impact string tension has on bounce height. This is done by creating zones on the tennis racket and comparing the bounce heights at these zones to the corresponding tension.

Variables like friction experienced by a ball at different zones can vary based on the day someone plays tennis and their technique. Similarly, sound frequency and vibration of different parts of tennis racket does not provide much useful qualitative information when playing the sport (you can measure tension by touching a string, whereas you can only figure out vibration by hitting a string with a racket during a game). Meanwhile, tension considers the material of the racket (strings), making each zone naturally different and easy to measure, while keeping other variables constant, making it an ideal independent variable.

Background Theory and Hypothesis:

Tension is characterized as a pulling force through strings or ropes and can show how much the string is being stretched. The International System (SI) unit for tension is Newtons (N) but is often measured in pounds (lbs.) in sports. A larger tension results in a greater force and means more of the string is being stretched (Merriam-Webster, n.d). According to *Tennis Warehouse*, having shorter strings (located at the corners) "under lower tension play similar to longer strings under higher tension" (TimothyO, 2012). This is largely related to the inverse relationship between tension and power. **Power** is the rate of energy conversion, and the SI unit is Watts (W). The lower tension results in high power, but the short length helps maintain control and lower the power. Contrastingly, higher tension strings have lower power, but the length of the string gives it less control and increases its power. This is because as the tension of an object decreases, there is less force holding the strings firmly in place, so the strings are more vulnerable to stretching and have more energy. This causes ΔE from the equation $P = \frac{\Delta E}{\Delta t}$ to increase, leading to more power, as noted in *Figure 1*.

Figure 1: Theoretical Impact of Tension on the Bounce Height of a Tennis Ball



Source: (Tennis Strings Explained)

Increased energy (gravitational potential energy) represented as $E_p = mgh$, where m = mass(kg), $g = 9.8 \, ms^{-2}$ (acceleration due to gravity), and h = vertical displacement, should give a higher h value, proving that $Bounce\ Height = \frac{1}{Tension}$ and $Power \propto bounce\ height$.

Figure 2 shows the parts of a racket. The sweet spots include "the center of percussion, vibration node, and the best bounce" areas (Kaya, 2021).

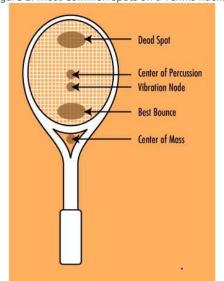


Figure 1: Most Common Spots on a Tennis Racket

Source: (Chang, the science behind tennis racquet performance and choosing the right racquet 2017)

From a tennis scope, the best bounce has the highest **coefficient of restitution**, since the ratio between the rebound height and the initial height of the ball is the largest. Contrastingly, the dead spot has no bounce height, and is situated near the center and top of the racket, proving a negative correlation between tension and bounce height.

Hypothesis: The center of a tennis racket (high tension), will lead to lower bounce heights than those near the corner, creating an inverse relationship between tension and bounce height.

Variables:

| Independent Variable | How it is changed |
|---------------------------|--|
| The racket zones and | To determine changes in tension, without manipulating the strings, |
| their respective tensions | zones were created on the racket. Each zone has different tension |
| Dependent Variable | How it should be affected: |
| The bounce height of | The ball will be dropped in each of these bounce heights, and depending |
| the tennis ball | on the tension of the zone, the bounce height will reflect the results. To |
| | ensure redundancy, each variable is tested with five trials. |
| Controlled Variables | Reason and how are they controlled: |
| Force and pressure | - Placing your arm closer to the middle than to the end of the grip |
| tennis racket | can cause imbalance in the center of mass and forces. |
| experiences | - <i>Image 1</i> shows demonstrates how the racket was held. |
| | Image 1: Demonstration of How the Tennis Racket Was Held |
| | |
| | |
| | |
| | |
| | |
| Temperature and | - Weather can cause more forces to act on the racket and the ball. |
| weather conditions | - All trials were done under room conditions. |
| Type of Equipment | - Tension usually decreases with the age and usage of the racket. |
| Used | - The same tennis ball and racket are used throughout all the trials |
| | and a new Penn tennis ball is used for the experiment. |
| Height the ball is | - E_p increases with height. |
| dropped | - There is tape indicating where the ball is dropped, and the distance |
| | between the two tape is 19in. |

Apparatus:

- Coloured tape
- Measuring Tape: $(\pm 0.5in)$
- 1 tennis racket (5-year-old Wilson Ultra Power Pro 105)
- 1 tennis ball (Penn tennis ball) Tourna Stringmeter $\pm 0.5 lbs$
- Recording Device
- Erasable marker

Image 2: Experiment Setup

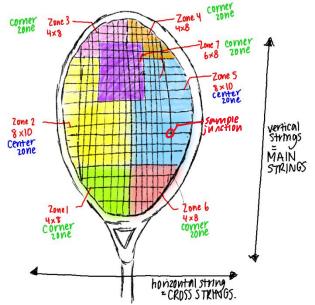




Procedure:

- 1. Stick two pieces of coloured tape on the wall, so that the top of the two tapes is 19in apart.
- 2. Using a marker, divide the racket into six zones. The dimensions of the corner "zones" are 4×8 squares, where each square is a junction of a horizontal and vertical string, while the middle zones are 8×10 squares. The middle zones are larger because most balls get hit in the middle of the tennis racket.
- **3.** Add a seventh zone in the center of the racket (4 squares away from the top and 4 square from the right-most side. The zone should be is 8 squares long and 9 squares wide). *Figure 3* is a diagram of the tennis racket and its different zones.
- **4.** Using a stringmeter¹, collect the string tensions of each junction in that zone, and calculate the average string tension of that zone.
- **5.** Drop the tennis ball from the top of the first tape, while holding the racket at the top of the second tape. Ensure that the tennis ball is dropped, and no large force is added. The ball must hit the respective zone. This may take time and practice prior to starting the five trials.
- **6.** Repeat steps 4-6 again for the six other zones.

Figure 3: Accurate Representation of the Different Zones and Parts on the Tennis Racket



¹ A string meter is a tool that measures the tension of the strings on a tennis racket. It is placed on the junction of the main and cross strings. Initially, a smaller string meter was bought, but the scale only went up till 45lbs. While a scaling could have been made after that number, it was unknown whether the scaling of the string meter was still the same as before. This could have large problems with the uncertainty of the tension. Consequently, the string meter was returned, and a 70lbs Tourna string meter is used.



Source: Amazon.ca

The rationale behind adding a seventh zone in the middle is due to the common belief that a tennis ball should be hit from the center of the racket, as it has greater force increases the momentum of the ball. In addition, the middle of a tennis racket is far more complex than any other part of a racket, as all sweet spots are there, and two center zones may not adequately represent this. For instance, it may be more effective to hit the leftmost part of a center zone rather than the middle, where that center zone ends. However, these zones do not account for the corners and the center separately, which might cause inaccuracies in the average; adding a seventh zone allows for a thorough analysis to be made.

To process the data, videos of the trials were recorded (failed or successful) and were later filtered based on whether they complied with the variables that were held constant. Using the software *Video Pad*, image sequences were created on the video of the successful trials. Using *Photopea*, a coordinate grid of these images was placed.

Image 3 shows how the data was collected and processed.

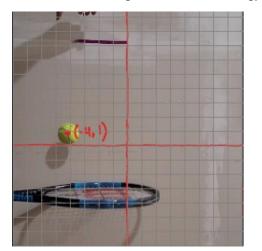
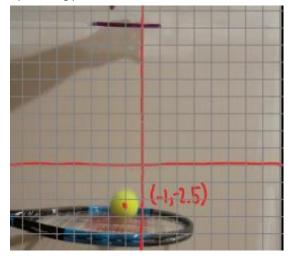


Image 2: How the methodology and data processing procedure looks like



Safety and Ethical Considerations:

There are little to no safety and ethical considerations as these resources are recyclable and can be readily available, while these materials are non-toxic. Nonetheless, shoes were worn to ensure safety when dropping the ball.

Raw Data:

| Table 1: Tension o | f Junctions in Racke | t (<u>+</u> 0.5 lbs. _. |)—Zone 1 |
|--------------------|----------------------|------------------------------------|----------|
| | | | |

| 39.0 | 41.0 | 40.0 | 39.0 | 38.0 |
|------|------|------|------|------|
| 39.0 | 41.0 | 39.0 | 37.0 | 39.0 |
| 42.0 | 41.0 | 38.0 | 36.0 | 40.0 |
| 42.0 | 41.0 | 38.0 | 40.0 | 41.0 |
| 41.0 | 39.0 | 40.0 | 39.0 | 38.0 |

These observations are recorded for Zones 2-6, and can be found in *Appendix A*. These numbers are organized based on the junction that was measured first. However, since the junctions were not measured in a particular pattern, the organization of these numbers do not propose anything.

| | Zone 1 | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|---------|------------|------------|---------|-------------|------------|--------------|------------|
| Trial 1 | (-4, -2) | (-2, -1) | (3, -8) | (0.5, -7) | (-2.5, -7) | (-3, -3.5) | (-0.5, -5) |
| Trial 2 | (-4, -2) | (-1, -1) | (3, -7) | (1, -5) | (-4, -7) | (-4, -5) | (-1, -4.5) |
| Trial 3 | (-4, -2.5) | (-1, -2.5) | (3, -8) | (3, -6) | (-3.5, -6) | (-4.5, -5) | (-2, -7) |
| Trial 4 | (-5, -1.5) | (-1, -2) | (4, -6) | (2, -4.5) | (-4, -6) | (-6.5, -4.5) | (-2, -7) |
| Trial 5 | (-4, -1.5) | (-1.5, -1) | (4, -7) | (1.5, -4.5) | (-4, -6) | (-7, -4.5) | (-0.5, -5) |

Table 3: Coordinates of Max Height the Tennis Ball Reaches ± 0.5 grid squares

| | Zone 1 | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|---------|-------------|------------|-----------|-----------|--------------|--------------|-----------|
| Trial 1 | (-4, 1.5) | (-2,0) | (3, -6) | (1, -6) | (-2.5, -4.5) | (-2,0) | (0, -3.5) |
| Trial 2 | (-4,1) | (-1,0) | (3, -6.5) | (1, -4.5) | (-4, -5) | (-4, -1) | (-1, -2) |
| Trial 3 | (-4,1) | (-0.5, -1) | (3, -7.5) | (3, -5.5) | (-4, -4) | (-3.5, -1.5) | (-1, -4) |
| Trial 4 | (-4, 1.5) | (0, -0.5) | (3.5, -5) | (2, -4) | (-3.5, -4) | (-6, -2) | (-1, -4) |
| Trial 5 | (-4.5, 1.5) | (-1,0) | (3.5, -6) | (2, -3.5) | (-4, -4.5) | (-7, -1.5) | (0, -3.5) |

Observations:

- Surprisingly, although the tennis racket was bisected and the dimensions of Zones 1,3,4 and 6 were the same, Zone 1 had a slightly higher tension than Zone 6, and Zone 4 had a slightly higher tension than Zone 3.
- The tennis ball had a smaller bounce height at Zones 2 and 5 and even smaller bounces near Zones 3 and 4, as they were closest to the edge and would be stopped by it.
- There was a loud vibrational sound whenever the ball was hit from the bottom zones of the racket as opposed to the top.
- There were more consecutive bounces after the initial bounce height with the bottom zones than the top zones.
- Zone 1 and Zone 6 had larger bounce height values than Zone 7.

Creating A Legend:

These values are interpreted using grid squares, and this legend displays how each square corresponds to a designated number of inches. To lower the variability between trials, a legend is made for every zone, and is noted in *Table 4*. This is calculated by dividing 19.0 ± 0.5 in (2.63%) by the number of squares between the pieces of tape.

Table 4: Legend, where one 1 grid represents x in.

| | Zone 1 | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|-----|--------|--------|--------|--------|--------|--------|------------|
| in. | 1.73 | 1.73 | 1.06 | 1.11 | 1.11 | 1.11 | 1.58 |
| | ± 0.05 | ± 0.05 | ± 0.03 | ± 0.03 | ± 0.03 | ± 0.03 | ± 0.04 |

Processed Data:

The average tension of each zone is calculated to see if there is a variation in the tensions of the strings, and a sample calculation, using Zone 1 data, is created.

Sample Calculation of the Average Tension (using Zone 1)

| Calculation | Uncertainty |
|---|------------------------------------|
| $= [36 + 37 + (38.0 \times 4) + (39.0 \times 7) +$ | 0.5×25 (number of trials) |
| $(40.0 \times 4) + (41.0 \times 6) + (42.0 \times 2)] \div$ | $={25 (number of trials)}$ |
| 25 | |
| $= 39.48 (\approx 39.5 lbs)$ | = 0.5 |

Using standard deviation, the average tension of Zone 1 is 39.5 ± 1.5 lbs. Standard deviation is used instead of average uncertainties because the averages are the same for each trial; it does not give an accurate representation as to the actual spread of data, which might show the total uncertainty in the data. This calculation is repeated for six other zones, and the results are summarized below.

Table 5: Average Tensions of Each Zone (lbs.)

| | Zone 1 | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Average | 39.5 ± 1.5 | 41.2 ± 2.5 | 38.6 ± 1.4 | 40.0 ± 2.2 | 40.0 ± 1.7 | 38.7 ± 2.7 | 45.0 ± 2.3 |
| Tension | | | | | | | |
| (lbs.) | | | | | | | |

Sample Calculation of Average Vertical Displacement of the Tennis Ball (Max Height - Initial Height)

| Calculation | Uncertainty |
|---|---------------------------------|
| =1.5-(-2) | 0.5 + 0.5 = 1 |
| = 3.5 (Zone 1, trial 1) | 1×5 (number of trials) |
| Repeat for all other Zone 1 trials and find the | $={5 (number of trials)}$ |
| average vertical displacement. | = 1 |
| = 3.5 + 3 + 3.5 + 3 + 3 | Or using standard deviation |
| = 3.2 grid squares | = 0.2 |

The mean is taken instead of the mode since it incorporates the spread of data, whereas modes are used to show repeated or skewed data points, which would disregard any outliers.

The averages of the other zones are calculated the same way and are summarized below.

Table 6: Average Vertical Height of the Tennis Ball per zone (measured in grid squares)

| | Zone 1 | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|----------------|---------------|---------------|---------------|---------------|-----------------|---------------|---------------|
| Average | 3.2 ± 0.2 | 1.2 ± 0.2 | 1.0 ± 0.5 | 0.7 ± 0.2 | $2.0_{\pm} 0.3$ | 3.3 ± 0.5 | 2.3 ± 0.7 |
| Height | | | | | | | |
| (grid squares) | | | | | | | |

Grid squares are not useful in understanding the actual bounce height, which is why they are converted into inches using the legend (page 7). A sample calculation is written on page 9.

Sample Calculations of Converting Grid Square Measurements into Inches (using Zone 1)

| Calculation | Uncertainty |
|---------------------|---|
| $= 3.2 \times 1.73$ | $= 6.25\% \left(\frac{0.2}{3.2} \times 100\right) + 2.89\% \left(\frac{0.05}{1.73} \times 100\right)$ |
| = 5.576 (5.6 in) | = 9.14% (0.5 <i>in</i>) |

The results of the calculations are listed in *Table 7*.

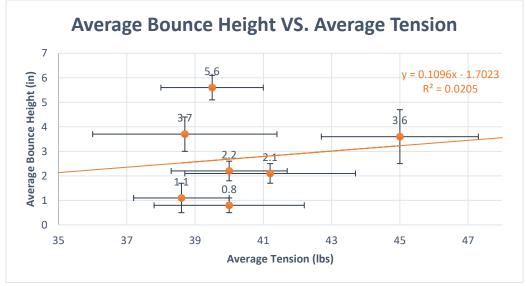
Table 7: Average Vertical Height Reached by the Tennis Ball per zone (converted to inches)

| | Zone 1 | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|-------------|-----------|---------------|---------------|---------------|-----------|---------------|---------------|
| Height (in) | 5.6 ± 0.5 | 2.1 ± 0.4 | 1.1 ± 0.6 | 0.8 ± 0.3 | 2.2 ± 0.4 | 3.7 ± 0.7 | 3.6 ± 1.2 |

Analysis:

Using these averages, a graph comparing the average tension of the zones and their average horizontal displacements is developed. The error bars represent the standard deviation of the average tension and bounce heights. To assess the correlation and dictate any outliers, a line of best fit was plotted for the average tensions.

Graph 1: Comparison Between Bounce Height (in) and Average Tension (lbs.) Based on Corresponding Zones



Here, tension and bounce height have a direct correlation, which is contrary to the proposed hypothesis. However, due to the small correlation of 0.0205, it is inferred that there may be outliers. Moreover, due to the large error bars, it is worth considering whether the line of best fit has an uncertainty associated with it when comparing it to other lines, and whether it is a valid best fit. The uncertainties are obtained by creating a steepest possible line of fit that oversees as many error bars as possible. Since the original line of best fit cannot go through all data points, the possible line of fit is exempt from having to go through all error bars as well. Besides that, an inverse life of bit is also plotted to see if more points would be touched if the line was showing an inverse correlation (which was the hypothesized result). The results are displayed in *Graph 2* below. The slopes of these are discovered using trial and error.

Average Bounce Height VS. Average Tension y = -1.255x + 51.427 Average Bounce Height (in) 5.63 y = 0.4458x - 163 v = 0.1096x - 1.7023 2 $R^2 = 0.0205$ 0.81 35 37 39 41 45 47 Average Tension (lbs) line of best fit possible line of fit inverse line of fit

Graph 2: Finding the Uncertainty of the Line of Best Fit by Plotting a Possible Life of Fit and an Inverse Line of Fit.

% Uncertainty =
$$\left| \frac{Steepest Worst Gradient - Best Gradient}{Best Gradient} \right| \times 100$$

= $\left| \frac{0.4458 - 0.1096}{0.1096} \right| \times 100$
= 307%

Additionally, using the LINEST function on Microsoft Excel, which discovers the properties of lines, the uncertainty of the line of best fit is found; the y values (B1:B7) are the bounce height values, and the x values (A1:A7) are tension values. The "const." value of 1 means that the line of best fit is not forced through the origin (Oxford, 2014) not have to go through the origin. This is helpful because these values are independent of each other and going through the origin will change the y-intercept. The stat. value is also 1 will give the uncertainty values of the slope (Moyle, 2017). The input and uncertainty values are stated below.

Image 3: Excel LINEST input to determine the uncertainty of the line of best fit and results.

| =LINEST(B1:B7,A1:A7,1,1 | | | |
|--|-------------|--------|--|
| | Slope | 0.1069 | |
| LINEST(known_ys, [known_xs], [const], [stats]) | Uncertainty | 0.3385 | |
| | | | |

A percent uncertainty of 307% in comparison to the possible line of fit, along with an uncertainty of the line of best fit show that there is a large inaccuracy and discrepancy. In addition, the inverse line above hits five points while the line of best fit only hits two. This confirms the belief that there are outliers in this experiment. Mathematically, z-scores are the best way to find these outliers, as they measure the discrepancy of a point from the average using standard deviation

and mean, whereas tools like Interquartile Range use medians, which do not skew the data. The formula used to calculate Z-scores, along with the results are stated below.

$$Z = \left| \frac{X - \mu}{\sigma} \right|$$

Where X = test bounce height, $\mu = \text{average}$ bounce height, $\sigma = \text{standard}$ deviation

Sample Calculation Used to Assess Whether a Zone's Bounce Height is an Outlier or Not

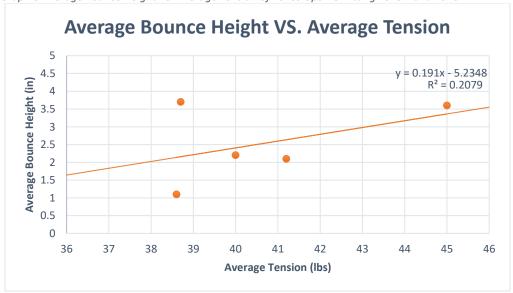
| Calculation | Uncertainty |
|--|--|
| $Z = \frac{5.6 - 2.7}{1.6}$ = 1.81 Since the z score is greater than 1, it is deemed | Standard deviations are standard uncertainties. Since z-scores use this on to find the spread of data, they account for uncertainties and propagating more is not necessary. |

Appendix B has the results for the z-score values of bounce heights in Zones 2-7.

Table 7: Determining Whether a Zone is an Outlier or Not (based on Z-score Calculations recorded in Appendix B)

| Zone | Zone 1 | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|---------|--------|--------|--------|--------|--------|--------|--------|
| Outlier | yes | no | no | yes | no | no | no |

Graph 3:Average Bounce Height VS. Average Tension of Zones Upon Omitting Zone 1 and Zone 4



Here, the variance (0.2079) is only slightly greater than before meaning that the two zones may have been outliers. However, this too is not an accurate justification as the magnitude of variance is still considerably low.

Conclusion and Discussion:

There is a variance of 0.2079, showing a weak correlation between the average tension of a tennis racket and the bounce height. The hypothesis states that string tension and average bounce height would have an inverse correlation, but the positive slope of 0.1069 ± 0.3385 , contradict

this idea. According to literature, stating that there is a strong inverse correlation between the two variables (magnitude close to -1), the value obtained is much lower than expected.

The are random errors associated in finding the average tensions for each zone range from 1.4-2.7, and from 0.2-1.1 for the vertical displacement of the ball. This can occur due to the change in precision when processing the data and using standard deviations to calculate the averages. This large uncertainty also derives from the fact that there is a legend correlating a digital distance to one that is in person. The coordinate grids have an uncertainty of ± 0.5 , which is quite high. However, a coordinate grid method is the best way to carry out this experiment upon attempting and failing to execute these trials using a motion sensor, where the distance was far too large to accurately measure the horizontal and vertical displacement of the ball. In addition, it was too difficult to track the bounce of the ball manually with the use of just a meter stick, which is why measuring tape and videos were taken instead, making this a strength of this exploration rather than a weakness.

At first glance, the systematic errors appear to be quite large due to the complete difference in correlation. However, upon evaluating the averages of these variables using *Table 2* and *Table 4*, you realize that there is a trend between tension and bounce height that fits in with literature.

- From Zone 1 (corner) to Zone 2 (center), the tension increases by 1.7lbs while bounce height decreases by 3.5in.
- From Zone 3 (corner) to Zone 4 (corner), the tension increases by 1.4lbs, while bounce height decreases by 0.3in.
- From Zone 5(center) to Zone 6 (corner), the tension decreases by 1.3lbs, while bounce height increases by 1.5in.

This means that there are other factors influencing the strings of a tennis racket, and possibly resulting in the outliers. To verify, research is conducted on the characteristic's sweet spots, and compared to previous knowledge.

Based on oscillations and waves, an increase in tension leads to a decrease in **amplitude** (Swaminathan, 2006), where amplitude is the "maximum extent of a vibration or oscillation" from the equilibrium (Oxford languages and google – English). This makes sense as less tension allows for more flexibility and oscillation of the string. This idea is reiterated in *Figure 1*, where the racket experiencing more power (left) has a larger arrow, showing that the bounce height is going to be greater than the racket on the right. However, a "vibration node (*Figure 2*)" is a sweet spot for its speed and bounce. A node is a point on a wave with zero amplitude. According to research, vibration nodes have negligible vibrations. This means that there is an increase in tension. They also have increased power since less energy is transferred to sound energy. While the center in general has a low bounce height, a vibration node is an exception.

Similarly, with "best bounce, "it is located near the "center of mass" (*Figure 2*), and the handle of the racket. This causes the racket to be stiff and give it more tension (not as much as the tension in the center), allowing for less elastic potential energy to be lost, and more of it going towards gravitational potential energy, leading to greater power and bounce height. This shows that factors like vibrations, waves, and the place the tennis ball was hit on the racket also make a huge difference and can be a limitation. If this experiment were to be redone, vibration dampeners would have been put on the top of the racket (near the dead spot, because it is believed the dead spot has the most vibrations), and near the corners, so tension could be better isolated.

Other limitations to this experiment include:

| Limitations/ | Impacts of Results | Suggested Improvements |
|---|--|--|
| Weaknesses The spacing of the tennis strings were not equal (seen in Figure 3). | Collective strings act with a combined force, increasing the power of the zone, while decreasing the tension it faces on its own, leading to different bounce heights and placement of dead spots and nodes. This may have caused Zone 3's bounce height to be less than Zone 4's. | While it is hard to change the location of the strings, this should be considered while making the zones. Instead of having zones regulated by the number of "squares" and junctions, they were measured based on length. |
| When the ball hit the racket, the racket did not remain straight. | It is inevitable that the racket will stay stagnant because the force of the tennis ball makes the force of gravity larger than the normal force. Depending on the point where the ball hits the racket, the Bounce height may be larger by one grid point. | Hawk eye software, which look at the trajectory of the ball along with sound distribution to determine the exact time the ball hit the racket could have been used to ensure a precise timing. Furthermore, the videos could have been recorded in slow motion so more image sequences were available and so more detailed motion was given. |
| Where on the racket the ball was hit (due to the large zones) | Upon analyzing <i>Figure 3</i> and <i>Figure 2</i> , a dead spot can be in the upper half of Zone 7, along with the sides of Zone 3 and 4. However, since Zone 7 is so large, the average bounce heights are skewed based on if the ball was hit at the borders of Zone 7 and Zone 2/5, or Zone 3 or 4. (this can also result in outliers because Zone 7 may include a dead spot and a center of percussion) | Initially, 9 zones were created instead of 7, dividing the center zones (seen in <i>Appendix C</i>). However, this did not work because smaller zones make it harder to ensure that the ball falls into the zones. Due to the lack of symmetry in the middle, the zones should be split into three there (<i>Appendix D</i>). This would allow for more equal zones and would provide further understanding as to why the tension of symmetrical zones (Zones 3 and 4) are not equal in tension and bounce height. |

Some strengths of this exploration include:

| Strength | Impact on the Experiment | | | | |
|---------------------------------|--|--|--|--|--|
| Measuring tension of | Main strings have less tension than cross strings, making it comfortable | | | | |
| main strings rather | to hit with a lot of power. Since the focus was regarding bounce height | | | | |
| than of cross strings | of the ball, measuring main strings made more sense as they are heavily | | | | |
| (indicated in <i>Figure 3</i>) | involved in power. This makes the inverse relationship between the | | | | |
| | power and tension more apparent. | | | | |
| All the junctions were | Initially, it was assumed that the tension of one string would be same | | | | |
| measured and | throughout the racket. However, despite having the same string, | | | | |
| recorded (in relation | tension was different depending on its junction, resulting in the | | | | |
| to the main string) | measurements of each junction rather than each string. | | | | |

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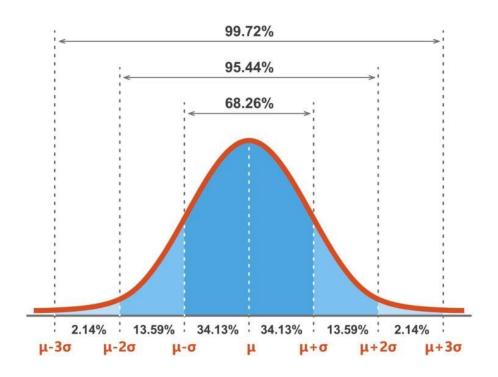
 $Appendix\ A$ Tension of the Junctions in the Tennis Rackets of Zones 2-7 (based on Main String measurements) using a stringmeter $(\pm 0.5 lbs)$

| ZONE 2: | | | | | | | | | |
|---------|------|------|------|------|-------|------|------|------|------|
| 41.0 | 39.0 | 43.0 | 47.0 | 42.0 | 41.0 | 37.0 | 36.0 | 41.0 | 40.0 |
| 42.0 | 40.0 | 41.0 | 42.0 | 39.0 | 41.0 | 39.0 | 37.0 | 44.0 | 46.0 |
| 42.0 | 39.0 | 43.0 | 44.0 | 40.0 | 42.0 | 42.0 | 38.0 | 45.0 | 44.0 |
| 41.0 | 40.0 | 44.0 | 41.0 | 39.0 | 39.0 | 43.0 | 40.0 | 45.0 | 39.0 |
| 43.0 | 42.0 | 45.0 | 42.0 | 41.0 | 38.0 | 36.0 | 42.0 | 44.0 | 38.0 |
| | | | | ZO | NE 3: | | | | |
| 38.0 | 38.0 | 38.0 | 41.0 | 37.0 | 39.0 | 36.0 | 39.0 | 40.0 | 38.0 |
| 39.0 | 40.0 | 39.0 | 41.0 | 40.0 | 38.0 | 36.0 | 40.0 | 39.0 | 39.0 |
| 37.0 | 36.0 | 39.0 | 40.0 | 38.0 | | | | | |
| | | | | ZO | NE 4: | | | | |
| 38.0 | 40.0 | 41.0 | 39.0 | 41.0 | 37.0 | 39.0 | 42.0 | 38.0 | 41.0 |
| 39.0 | 39.0 | 42.0 | 39.0 | 44.0 | 39.0 | 36.0 | 41.0 | 40.0 | 45.0 |
| 37.0 | 38.0 | 40.0 | 43.0 | 42.0 | | | | | |
| | | | | ZO | NE 5: | | | | |
| 42.0 | 42.0 | 40.0 | 42.0 | 40.0 | 40.0 | 39.0 | 37.0 | 40.0 | 37.0 |
| 41.0 | 42.0 | 40.0 | 38.0 | 40.0 | 40.0 | 39.0 | 37.0 | 37.0 | 35.0 |
| 45.0 | 41.0 | 41.0 | 39.0 | 42.0 | 38.0 | 37.0 | 39.0 | 37.0 | 36.0 |
| 42.0 | 42.0 | 42.0 | 39.0 | 38.0 | 40.0 | 38.0 | 38.0 | 38.0 | 36.0 |
| 46.0 | 47.0 | 43.0 | 38.0 | 37.0 | 37.0 | 37.0 | 36.0 | 41.0 | 35.0 |
| | | | | ZO | NE 6: | | | | |
| 42.0 | 37.0 | 40.0 | 42.0 | 39.0 | 43.0 | 38.0 | 38.0 | 42.0 | 40.0 |
| 42.0 | 39.0 | 37.0 | 41.0 | 41.0 | 41.0 | 38.0 | 41.0 | 41.0 | 40.0 |
| 38.0 | 39.0 | 40.0 | 42.0 | 38.0 | | | | | |
| ZONE 7: | | | | | | | | | |
| 47.0 | 44.0 | 44.0 | 43.0 | 49.0 | 49.0 | 46.0 | 48.0 | 48.0 | 43.0 |
| 45.0 | 43.0 | 45.0 | 45.0 | 48.0 | 48.0 | 46.0 | 42.0 | 42.0 | 42.0 |
| 43.0 | 43.0 | 43.0 | 43.0 | 47.0 | 50.0 | 47.0 | 41.0 | 41.0 | 45.0 |
| 44.0 | 44.0 | 45.0 | 44.0 | 45.0 | 49.0 | 48.0 | 41.0 | 45.0 | 45.0 |
| 43.0 | 44.0 | 46.0 | 46.0 | 43.0 | 49.0 | 49.0 | 43.0 | 44.0 | 45.0 |

Appendix B
Z-score Calculation Values and Meaning

| | Zone 2 | Zone 3 | Zone 4 | Zone 5 | Zone 6 | Zone 7 |
|-------|--------|--------|--------|--------|--------|--------|
| Value | 0.375 | 1.00 | 1.20 | 0.625 | 0.313 | 0.563 |

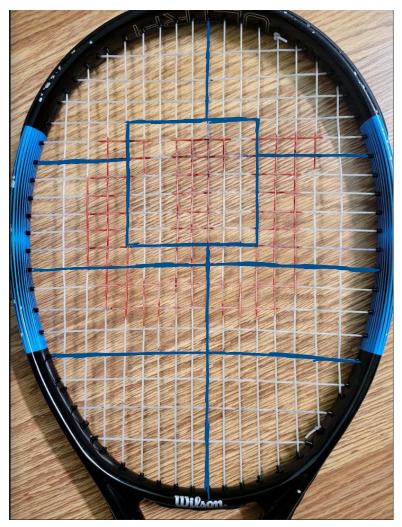
Table 2: This Graph Is Used to Help Understand Whether a Point is an Outlier or Not



Source: (Mcleod, 2023)

Since 68.26% of the data lies within 1 standard deviation of the mean, anything above 1 standard deviation is considered an outlier.

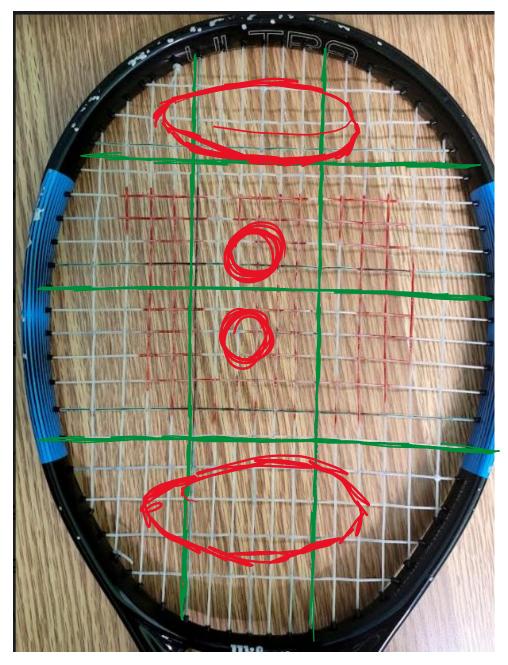
Appendix C
Image of Previous Zone That Was Used for the Experiment



This design was made because all the zones would be equal. However, upon trying this by dropping the ball, some zones were too small and difficult to drop a ball onto.

Appendix D

An Idea for A Zone in the Future



This design of zones is will look at the tensions of each zone much more separately, as there are spaces for the ball to be hit, while the sweet spots and parts of a tennis racket are more focused in one zone rather than two or three, and will allow for the standard deviation of average tensions to be lower, and the bounce heights to be more accurate. The red circles represent the dead spot, center of percussion, the vibration node and the best bounce (in order of up to down). Each spot will be approximately isolated in one zone, encouraging a much comprehensive investigation next time regarding the behaviours of these zones, and the impact tension plays on them.