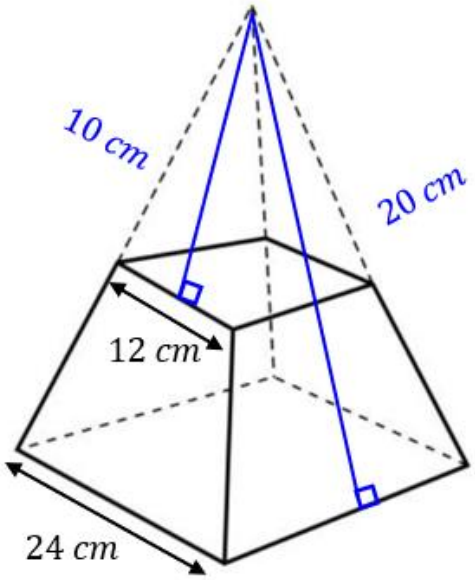
SURFACE AREA OF FRUSTUMS

1) A frustum is formed by removing the top of a square-based pyramid that was cut parallel to the base. The dimensions are as follows:
   - Large square base side length = 24 cm
   - Small square top side length = 12 cm
   - Height of each large triangular face = 20 cm
   - Height of each small triangular face = 10 cm



Calculate the total surface area of the frustum.

$$Area\ of\ four\ large\ triangular\ faces = 4 \times \frac{b \times h}{2}$$

$$= 4 \times \frac{24 \times 20}{2}$$

$$= 960\ cm^2$$

$$Area\ of\ large\ square\ base = 24 \times 24$$

$$= 576\ cm^2$$

$$Area\ of\ four\ small\ triangular\ faces = 4 \times \frac{b \times h}{2}$$

$$= 4 \times \frac{12 \times 10}{2}$$

$$= 240\ cm^2$$

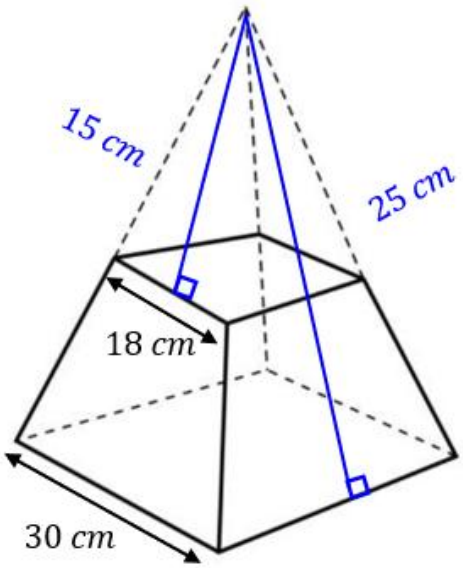
$$Area\ of\ small\ square\ base = 12 \times 12$$

$$= 144\ cm^2$$

$$Total\ surface\ area\ of\ frustum = 960 + 576 - 240 + 144 = \mathbf{1440\ cm^2}$$

2) A frustum is formed by removing the top of a square-based pyramid that was cut parallel to the base. The dimensions are as follows:

- Large square base side length = 30 cm
- Small square top side length = 18 cm
- Height of each large triangular face = 25 cm
- Height of each small triangular face = 15 cm



Calculate the total surface area of the frustum.

$$Area\ of\ four\ large\ triangular\ faces = 4 \times \frac{b \times h}{2}$$

$$= 4 \times \frac{30 \times 25}{2}$$

$$= 1500\ cm^2$$

$$Area\ of\ large\ square\ base = 30 \times 30$$

$$= 900\ cm^2$$

$$Area\ of\ four\ small\ triangular\ faces = 4 \times \frac{b \times h}{2}$$

$$= 4 \times \frac{18 \times 15}{2}$$

$$= 540\ cm^2$$

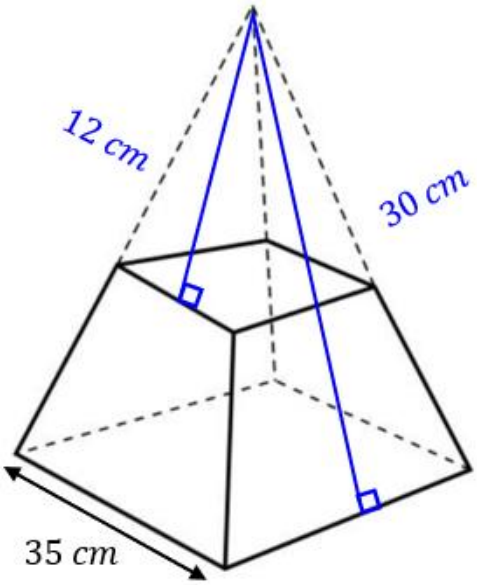
$$Area\ of\ small\ square\ base = 18 \times 18$$

$$= 324\ cm^2$$

$$Total\ surface\ area\ of\ frustum = 1500 + 900 - 540 + 324 = \mathbf{2184\ cm^2}$$

3) A frustum is formed by removing the top of a square-based pyramid that was cut parallel to the base. The dimensions are as follows:

- Large square base side length = 35 cm
- Height of each large triangular face = 30 cm
- Height of each small triangular face = 12 cm



Calculate the total surface area of the frustum.

$$Area\ of\ four\ large\ triangular\ faces = 4 \times \frac{b \times h}{2}$$

$$= 4 \times \frac{35 \times 30}{2}$$

$$= 2100\ cm^2$$

$$Area\ of\ large\ square\ base = 35 \times 35$$

$$= 1225\ cm^2$$

$$Scale\ factor = 30 \div 12 = 2.5$$

$$Small\ square\ base\ side\ length = 35 \div 2.5 = 14\ cm$$

$$Area\ of\ four\ small\ triangular\ faces = 4 \times \frac{b \times h}{2}$$

$$= 4 \times \frac{14 \times 12}{2}$$

$$= 336\ cm^2$$

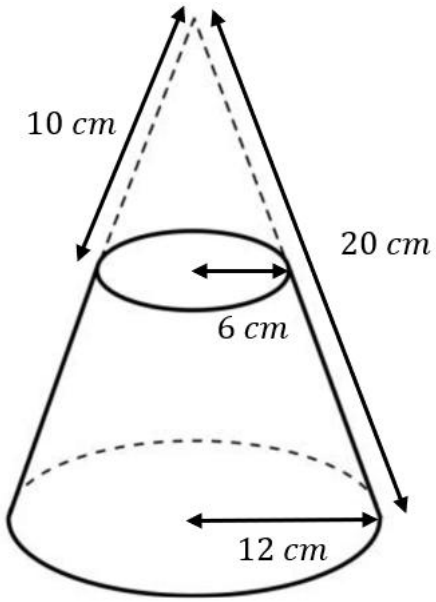
$$Area\ of\ small\ square\ base = 14 \times 14$$

$$= 196\ cm^2$$

$$Total\ surface\ area\ of\ frustum = 2100 + 1225 - 336 + 196 = \mathbf{3185\ cm^2}$$

4) A frustum is formed by removing the top of a cone. The dimensions are as follows:

- Large radius = 12 cm
- Small radius = 6 cm
- Large slant height = 20 cm
- Small slant height = 10 cm



Calculate the total surface area of the frustum.

Give your answer in terms of π.

$Large\ curved\ surface\ area = \pi rl$

$= \pi \times 12 \times 20$

$= 240\pi\ cm^2$

$Area\ of\ large\ circle = \pi r^2$

$= \pi \times 12^2$

$= 144\pi\ cm^2$

$Small\ curved\ surface\ area = \pi rl$

$= \pi \times 6 \times 10$

$= 60\pi\ cm^2$

$Area\ of\ small\ circle = \pi r^2$

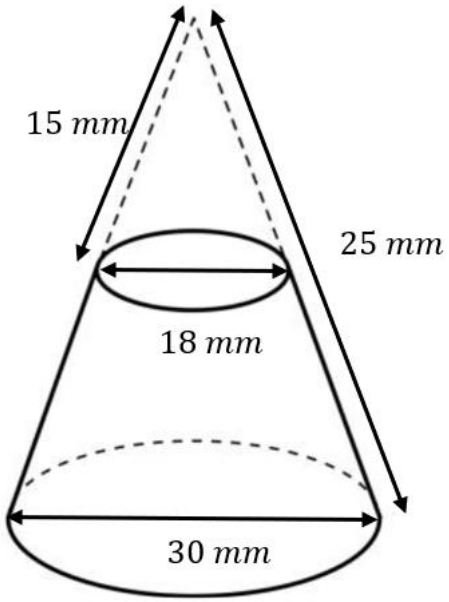
$= \pi \times 6^2$

$= 36\pi\ cm^2$

$Total\ surface\ area\ of\ frustum = 240\pi + 144\pi - 60\pi + 36\pi = \mathbf{360\pi\ cm^2}$

5) A frustum is formed by removing the top of a cone. The dimensions are as follows:

- Large diameter = 30 mm
- Small diameter = 18 mm
- Large slant height = 25 mm
- Small slant height = 15 mm



Calculate the total surface area of the frustum.

Give your answer to 1 decimal place.

$Large\ curved\ surface\ area = \pi rl$

$= \pi \times 15 \times 25$

$= 375\pi\ mm^2$

$Area\ of\ large\ circle = \pi r^2$

$= \pi \times 15^2$

$= 225\pi\ mm^2$

$Small\ curved\ surface\ area = \pi rl$

$= \pi \times 9 \times 15$

$= 135\pi\ mm^2$

$Area\ of\ small\ circle = \pi r^2$

$= \pi \times 9^2$

$= 81\pi\ mm^2$

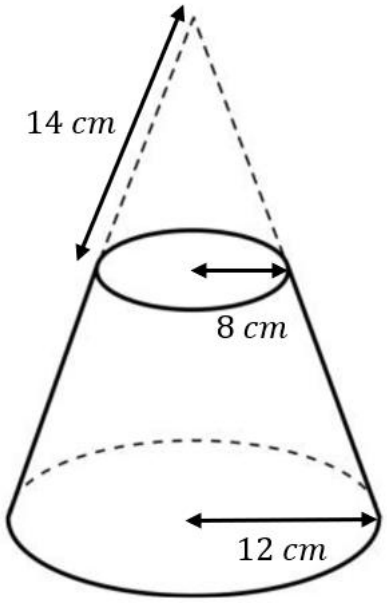
$Total\ surface\ area\ of\ frustum = 375\pi + 225\pi - 135\pi + 81\pi = 546\pi$

$= 1715.3095\ldots$

$= \mathbf{1715.3\ mm^2\ (1\ dp)}$

6) A frustum is formed by removing the top of a cone using a plane parallel to the base. The dimensions are as follows:

- Large radius = 12 cm
- Small radius = 8 cm
- Small slant height = 14 cm



Calculate the total surface area of the frustum.

Give your answer to the nearest square centimetre.

$Scale\ factor = 12 \div 8 = 1.5$

$Large\ slant\ height = 14 \times 1.5 = 21\ cm$

$Large\ curved\ surface\ area = \pi rl$

$= \pi \times 12 \times 21$

$= 252\pi\ cm^2$

$Area\ of\ large\ circle = \pi r^2$

$= \pi \times 12^2$

$= 144\pi\ cm^2$

$Small\ curved\ surface\ area = \pi rl$

$= \pi \times 8 \times 14$

$= 112\pi\ cm^2$

$Area\ of\ small\ circle = \pi r^2$

$= \pi \times 8^2$

$= 64\pi\ cm^2$

$Total\ surface\ area\ of\ frustum = 252\pi + 144\pi - 112\pi + 64\pi = 348\pi$

$= 1093.274\ldots$

$= \mathbf{1093\ cm^2\ (unit)}$

7) A cone has a radius of 8 cm and a perpendicular height of 15 cm.
A frustum is formed by removing the top of the cone using a plane parallel to the base.
The smaller cone has a radius of 4 cm.
Work out the total surface area of the frustum.
Give your answer to 2 decimal places.

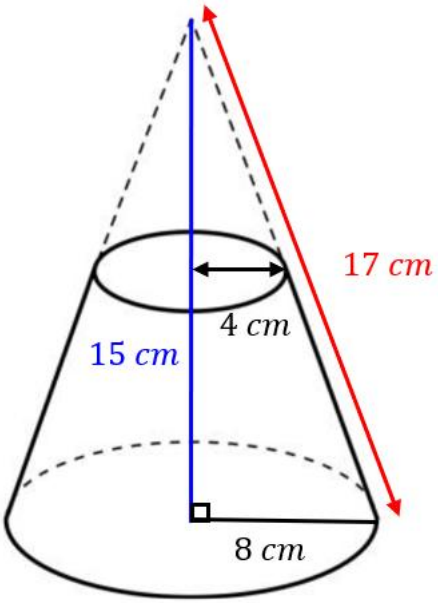
Use Pythagoras' theorem to work out the slant height of the large cone

$a^2 + b^2 = c^2$

$8^2 + 15^2 = l^2$

$l^2 = 289$

$l = \sqrt{289} = 17\ cm$



$Scale\ factor = 8 \div 4 = 2$

$Small\ slant\ height = 17 \div 2 = 8.5\ cm$

$Large\ curved\ surface\ area = \pi rl$

$= \pi \times 8 \times 17$

$= 136\pi\ cm^2$

$Area\ of\ large\ circle = \pi r^2$

$= \pi \times 8^2$

$= 64\pi\ cm^2$

$Small\ curved\ surface\ area = \pi rl$

$= \pi \times 4 \times 8.5$

$= 34\pi\ cm^2$

$Area\ of\ small\ circle = \pi r^2$

$= \pi \times 4^2$

$= 16\pi\ cm^2$

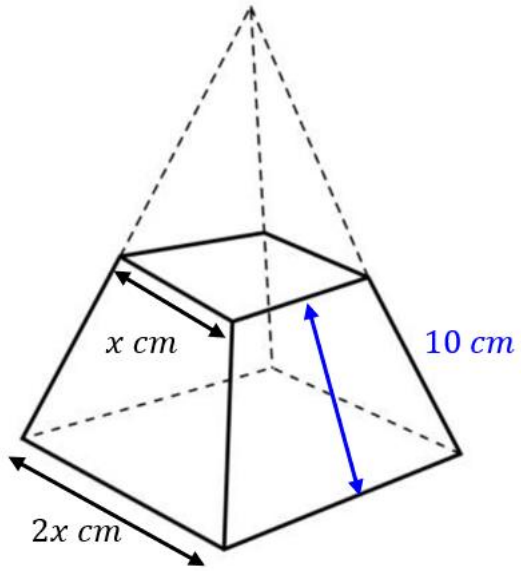
$Total\ surface\ area\ of\ frustum = 136\pi + 64\pi - 34\pi + 16\pi = 182\pi$

$= 571.7698\ldots$

$= \mathbf{571.77\ cm^2\ (2\ dp)}$

Challenge

8) **The frustum of a square-based pyramid has a slant height of 10 cm, a top side length of x cm and a bottom side length of $2x$ cm. The total surface area of the frustum is 320 cm^2.**
Work out the value of x.



$Scale\ factor = 2x \div x = 2$

$Large\ slant\ height = 10 \times 2 = 20\ cm$

$Area\ of\ four\ large\ triangular\ faces = 4 \times \frac{b \times h}{2}$

$= 4 \times \frac{2x \times 20}{2}$

$= 80x\ cm^2$

$Area\ of\ large\ square\ base = 2x \times 2x$

$= 4x^2\ cm^2$

$Area\ of\ four\ small\ triangular\ faces = 4 \times \frac{b \times h}{2}$

$= 4 \times \frac{x \times 10}{2}$

$= 20x\ cm^2$

$Area\ of\ small\ square\ base = x \times x$

$= x^2\ cm^2$

$80x + 4x^2 - 20x + x^2 = 320$
$5x^2 + 60x = 320$
$5x^2 + 60x - 320 = 0$
$x^2 + 12x - 64 = 0$
$(x + 16)(x - 4) = 0$
$x = -16 \quad or \quad x = 4$

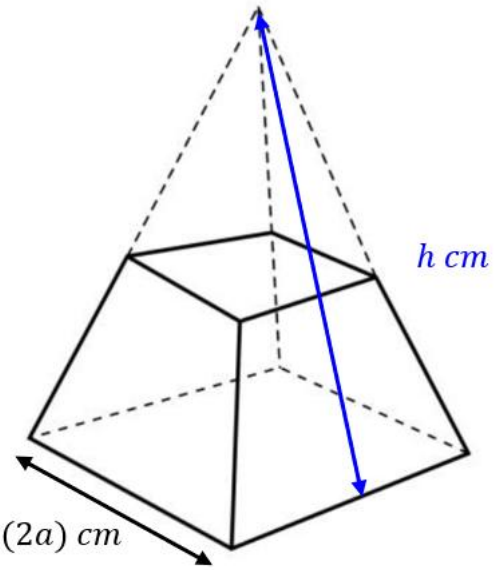
$\boldsymbol{x = 4}$

9) A square-based pyramid has slant height h cm and base side length $(2a)$ cm.
The top third of the pyramid is removed by a plane parallel to the base.
Show that the total surface area of the remaining frustum is $\left(\frac{40}{9}a^2 + \frac{32}{9}ah\right)$ cm².

Top third is removed, so scale factor is 3

Smaller side length = $\frac{2}{3}a$

Smaller slant height = $\frac{1}{3}h$



$$Area\ of\ four\ large\ triangular\ faces = 4 \times \frac{b \times h}{2}$$
$$= 4 \times \frac{2a \times h}{2}$$
$$= 4ah\ cm^2$$

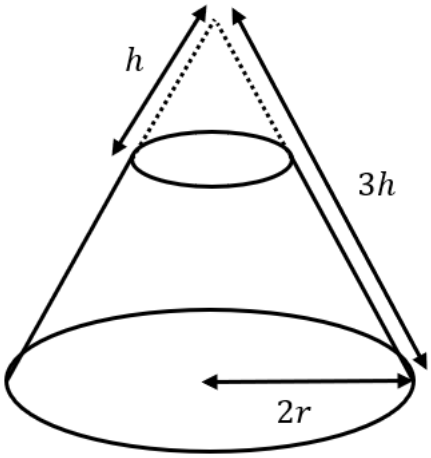
$$Area\ of\ large\ square\ base = 2a \times 2a$$
$$= 4a^2\ cm^2$$

$$Area\ of\ four\ small\ triangular\ faces = 4 \times \frac{b \times h}{2}$$
$$= 4 \times \frac{\frac{2}{3}a \times \frac{1}{3}h}{2}$$
$$= \frac{4}{9}ah\ cm^2$$

$$Area\ of\ small\ square\ base = \frac{2}{3}a \times \frac{2}{3}a$$
$$= \frac{4}{9}a^2\ cm^2$$

$$Total\ surface\ area\ of\ frustum = 4ah + 4a^2 - \frac{4}{9}ah + \frac{4}{9}a^2 = \left(\frac{40}{9}a^2 + \frac{32}{9}ah\right)\ cm^2$$

10) A cone has a slant height of $3h$ and a radius of $2r$.
A frustum is formed by removing the top of a cone that was cut by a plane parallel to the base.
The small cone has a slant height of h.
Work out the total surface area of the frustum. Give your answer in terms of π, r and h.



$Large\ curved\ surface\ area = \pi rl$

$= \pi \times 2r \times 3h$

$= 6\pi hr\ cm^2$

$Area\ of\ large\ circle = \pi r^2$

$= \pi \times (2r)^2$

$= 4\pi r^2\ cm^2$

$Scale\ factor = 3h \div h = 3$

$Small\ radius = \frac{2r}{3}$

$Small\ curved\ surface\ area = \pi rl$

$= \pi \times \frac{2r}{3} \times h$

$= \frac{2}{3}\pi hr\ cm^2$

$Area\ of\ small\ circle = \pi r^2$

$= \pi \times \left(\frac{2r}{3}\right)^2$

$= \frac{4}{9}\pi r^2\ cm^2$

$Total\ surface\ area\ of\ frustum = 6\pi hr + 4\pi r^2 - \frac{2}{3}\pi hr + \frac{4}{9}\pi r^2 = \mathbf{\frac{40}{9}\pi r^2 + \frac{16}{3}\pi hr\ cm^2}$