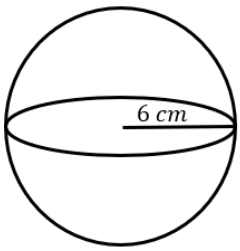
SURFACE AREA OF COMPLEX SHAPES

1) The sphere shown in the diagram has a radius of 6 cm. Work out the surface area of the sphere. Give your answer to 2 decimal places.



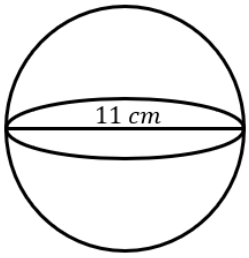
$A = 4\pi r^2$

$= 4 \times \pi \times 6^2$

$= 452.389 \ldots$

$= \mathbf{452.39\ cm^2\ (2\ dp)}$

2) The sphere shown in the diagram has a diameter of 11 cm. Work out the surface area of the sphere. Give your answer to 1 decimal place.



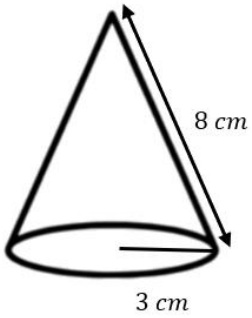
$A = 4\pi r^2$

$= 4 \times \pi \times 5.5^2$

$= 380.132 \ldots$

$= \mathbf{380.1\ cm^2\ (1\ dp)}$

3) The cone shown in the diagram has a radius of 3 cm and a slant height of 8 cm. Work out the total surface area of the cone. Give your answer in terms of π.



$Curved\ surface\ area = \pi rl$

$= \pi \times 3 \times 8$

$= 24\pi\ cm^2$

$Area\ of\ circle = \pi r^2$

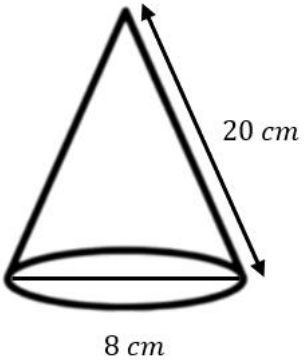
$= \pi \times 3^2$

$= 9\pi\ cm^2$

$A = 24\pi + 9\pi$

$= \mathbf{33\pi\ cm^2}$

4) The cone shown in the diagram has a diameter of 8 cm and a slant height of 20 cm. Work out the total surface area of the cone. Give your answer to 1 decimal place.



$Curved\ surface\ area = \pi rl$

$= \pi \times 4 \times 20$

$= 80\pi\ cm^2$

$Area\ of\ circle = \pi r^2$

$= \pi \times 4^2$

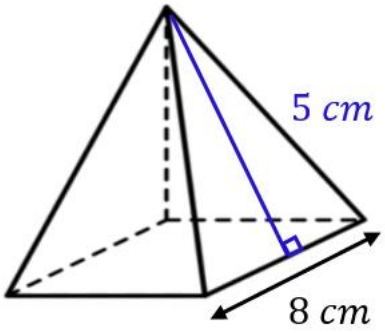
$= 16\pi\ cm^2$

$A = 80\pi + 16\pi$

$= 301.592 \ldots$

$= \mathbf{301.6\ cm^2\ (1\ dp)}$

5) A square-based pyramid is shown in the diagram. The side length of the base of the pyramid is 8 cm. Each triangular face has a height of 5 cm. Work out the total surface area of the pyramid.

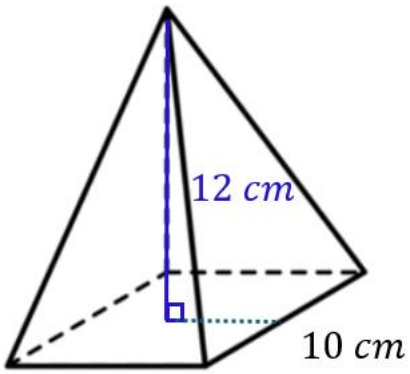


$$Total\ area\ of\ the\ four\ triangles = 4 \times \frac{b \times h}{2}$$
$$= 4 \times \frac{8 \times 5}{2}$$
$$= 80\ cm^2$$

$$Area\ of\ square\ base = 8 \times 8 = 64\ cm^2$$

$$A = 80 + 64 = \mathbf{144\ cm^2}$$

6) A square-based pyramid is shown in the diagram. The perpendicular height from the apex to the centre of the square base is 12 cm. Work out the total surface area of the pyramid.



Use Pythagoras' theorem to find the height of each triangular face

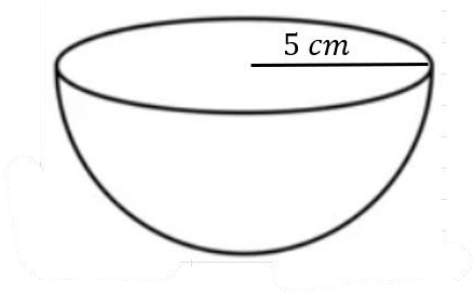
$a^2 + b^2 = c^2$
$5^2 + 12^2 = h^2$
$h^2 = 169$
$h = \sqrt{169} = 13\ cm$

$$Total\ area\ of\ the\ four\ triangles = 4 \times \frac{b \times h}{2}$$
$$= 4 \times \frac{10 \times 13}{2}$$
$$= 260\ cm^2$$

$$Area\ of\ square\ base = 10 \times 10 = 100\ cm^2$$

$$A = 260 + 100 = \mathbf{360\ cm^2}$$

7) The diagram shows a hemisphere with a circular base of radius 5 cm. Work out the total surface area of the hemisphere. Give your answer in terms of π.



$$Curved\ surface\ area\ of\ hemisphere = \frac{1}{2} \times 4\pi r^2$$

$$= \frac{1}{2} \times 4 \times \pi \times 5^2$$

$$= 50\pi\ cm^2$$

$$Area\ of\ circle = \pi r^2$$

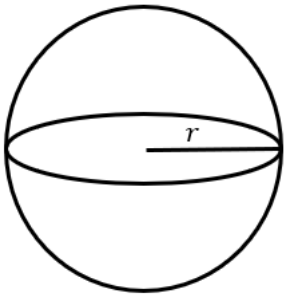
$$= \pi \times 5^2$$

$$= 25\pi\ cm^2$$

$$Total\ surface\ area = 50\pi + 25\pi$$

$$= \mathbf{75\pi\ cm^2}$$

8) The sphere shown in the diagram below has a surface area of 1000 mm^2. Work out the missing radius. Give your answer to 2 decimal places.



$$A = 4\pi r^2$$

$$4 \times \pi \times r^2 = 1000$$

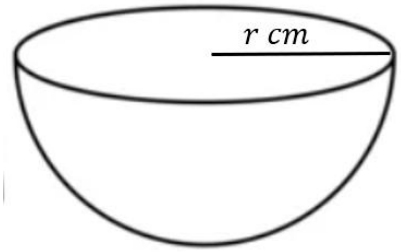
$$\pi r^2 = 250$$

$$r^2 = \frac{250}{\pi}$$

$$r = \sqrt{\frac{250}{\pi}} = 8.9206\ldots$$

$$= \mathbf{8.92\ mm\ (2\ dp)}$$

9) The diagram shows a hemisphere with a circular base of radius r cm. The total surface area of the hemisphere is $\frac{147}{4}\pi$ cm^2. Work out the value of r.

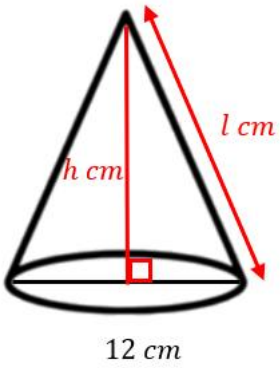


$$Curved\ surface\ area\ of\ hemisphere = \frac{1}{2} \times 4\pi r^2$$
$$= \frac{1}{2} \times 4 \times \pi \times r^2$$
$$= 2\pi r^2$$

$$Area\ of\ circle = \pi r^2$$

$$Total\ surface\ area = 2\pi r^2 + \pi r^2 = 3\pi r^2$$
$$3\pi r^2 = \frac{147}{4}\pi$$
$$3r^2 = \frac{147}{4}$$
$$r^2 = \frac{147}{12}$$
$$r = \sqrt{\frac{147}{12}} = \mathbf{3.5}$$

10) The cone shown in the diagram below has a diameter of 12 cm, a perpendicular height of h cm, and a curved surface area of 60π cm^2. Work out the value of h.



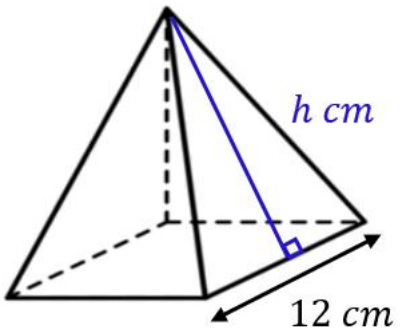
$$Curved\ surface\ area = \pi rl$$
$$= \pi \times 6 \times l$$
$$= 6l\pi$$

$$6l\pi = 60\pi$$
$$6l = 60$$
$$l = 10$$

Use Pythagoras' theorem to solve for h

$$h^2 + 6^2 = 10^2$$
$$h^2 = 10^2 - 6^2 = 64$$
$$h = \sqrt{64} = \mathbf{8}$$

11) A square-based pyramid is shown in the diagram below. The side length of the base of the pyramid is 12 cm. Each triangular face has a height of h cm. Given that the total surface area of the solid is 396 cm^2, work out the value of h.



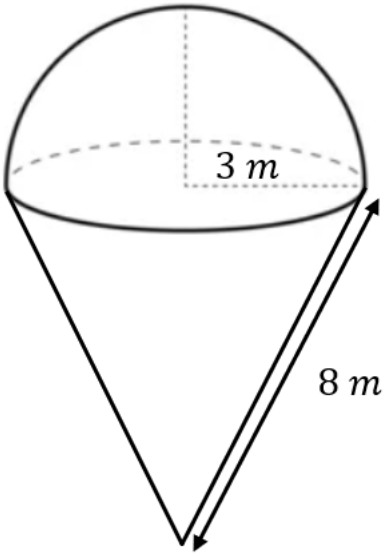
$$Total\ area\ of\ the\ four\ triangles = 4 \times \frac{b \times h}{2}$$
$$= 4 \times \frac{12 \times h}{2}$$
$$= 24h\ cm^2$$

$$Area\ of\ square\ base = 12 \times 12 = 144\ cm^2$$

$$24h + 144 = 396$$
$$-144 \quad -144$$
$$24h = 252$$
$$\div 24 \quad \div 24$$
$$\boldsymbol{h = 10.5}$$

12) A solid consists of a hemisphere of radius 3 m joined to a cone with a radius of 3 m and a slant height of 8 m.

Two coats of paint will be applied to the outside of the solid.
The paint comes in tins. Each tin costs £14.50 and will cover 25 m^2 of the solid.
Work out the total cost of the paint. You must demonstrate all your working.



$$Curved\ surface\ area\ of\ hemisphere = \frac{1}{2} \times 4\pi r^2$$
$$= \frac{1}{2} \times 4 \times \pi \times 3^2$$
$$= 18\pi\ m^2$$

$$Curved\ surface\ area\ of\ cone = \pi rl$$
$$= \pi \times 3 \times 8$$
$$= 24\pi\ m^2$$

$$Total\ surface\ area = 18\pi + 24\pi$$
$$= 42\pi\ m^2$$

$$Surface\ area\ for\ two\ coats = 84\pi\ m^2$$

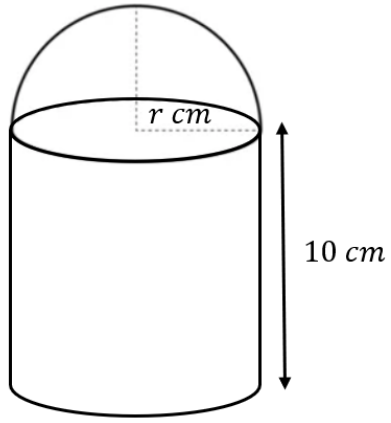
$$Number\ of\ tins\ needed = 84\pi \div 25 = 10.555\ldots$$

11 tins needed

$$Total\ cost = £14.50 \times 11 = \boldsymbol{£159.50}$$

Challenge

13) **A solid is made by joining a hemisphere to a cylinder. The cylinder and the hemisphere both have a radius of r cm. The height of the cylinder is 10 cm. The bottom of the solid is a circle and the inside of the solid is hollow.**



The total surface area of the solid is A cm^2.

Show that:

$$A = \pi(3r^2 + 20r)$$

$Curved\ surface\ area\ of\ hemisphere = \frac{1}{2} \times 4\pi r^2$

$= 2\pi r^2\ cm^2$

$Curved\ surface\ area\ of\ cylinder = 2\pi r \times h$

$= 2 \times \pi \times r \times 10$

$= 20\pi r\ cm^2$

$Area\ of\ bottom = \pi r^2$

$Total\ surface\ area = 2\pi r^2 + 20\pi r + \pi r^2$

$= 3\pi r^2 + 20\pi r$

$= \boldsymbol{\pi(3r^2 + 20r)}$

Higher Tier Only

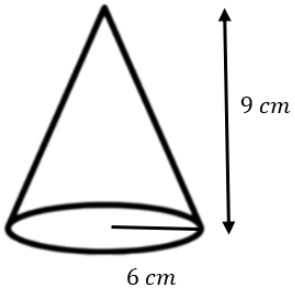
14) A cone has a radius of 6 cm and a perpendicular height of 9 cm.
A pyramid has a square base of side length x cm and the height of each triangular face is 8 cm.
Given that the total surface area of the two shapes is equal, work out the value of x.
Give your answer to 1 decimal place.

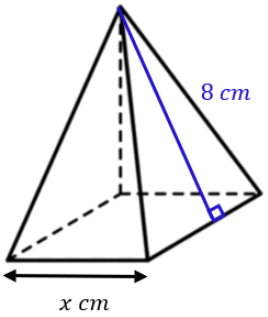
Use Pythagoras' theorem to work out the slant height of the cone

$9^2 + 6^2 = l^2$

$l^2 = 117$

$l = \sqrt{117}$





$Curved\ surface\ area\ of\ cone = \pi rl$

$= \pi \times 6 \times \sqrt{117}$

$= 6\pi\sqrt{117}$

$Area\ of\ circle = \pi r^2$

$= \pi \times 6^2$

$= 36\pi$

$Total\ surface\ area\ of\ cone = 36\pi + 6\pi\sqrt{117}$

$Total\ area\ of\ the\ four\ triangles = 4 \times \frac{b \times h}{2}$

$= 4 \times \frac{x \times 8}{2}$

$= 16x\ cm^2$

$Area\ of\ square\ base = x \times x = x^2\ cm^2$

$x^2 + 16x = 36\pi + 6\pi\sqrt{117}$

$x^2 + 16x - 36\pi - 6\pi\sqrt{117} = 0$

Use quadratic formula:

$$x = \frac{-16 \pm \sqrt{16^2 - (4 \times 1 \times (-36\pi - 6\pi\sqrt{117})}}{2(1)}$$

$x = -27.5188\ldots$ or $x = 11.5188\ldots$

$\boldsymbol{x = 11.5\ (1\ dp)}$