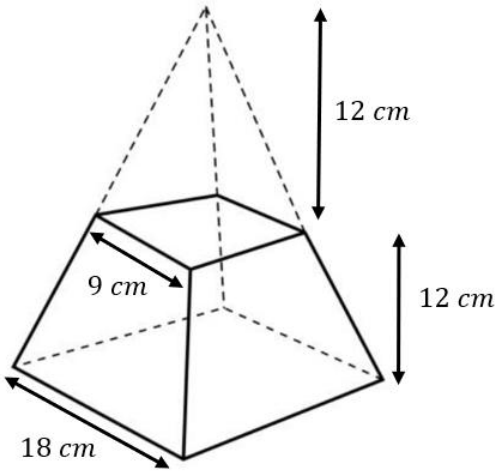
VOLUME OF FRUSTUMS

1) The diagram shows the frustum formed from a square-based pyramid. Calculate the volume of the frustum.

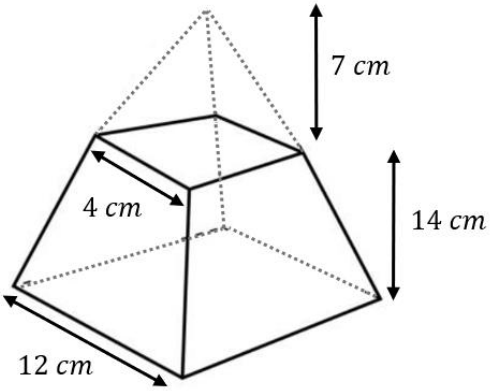


$Volume\ of\ large\ pyramid = \frac{1}{3} \times (18 \times 18) \times 24 = 2592\ cm^3$

$Volume\ of\ small\ pyramid = \frac{1}{3} \times (9 \times 9) \times 12 = 324\ cm^3$

$Volume\ of\ frustum = 2592 - 324 = \mathbf{2268\ cm^3}$

2) The diagram shows the frustum formed from a square-based pyramid. Calculate the volume of the frustum. Give your answer to 3 significant figures.

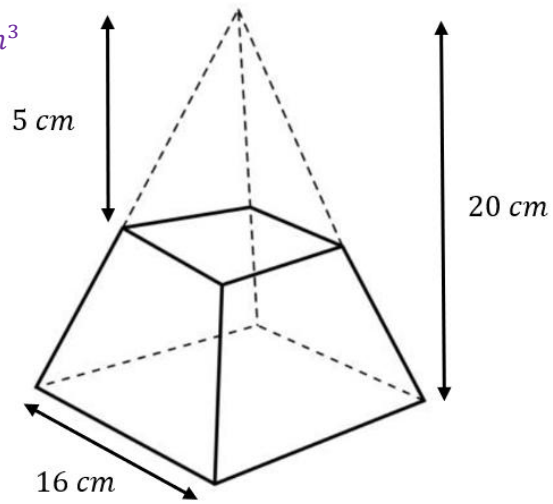


$Volume\ of\ large\ pyramid = \frac{1}{3} \times (12 \times 12) \times 21 = 1008\ cm^3$

$Volume\ of\ small\ pyramid = \frac{1}{3} \times (4 \times 4) \times 7 = \frac{112}{3}\ cm^3$

$Volume\ of\ frustum = 1008 - \frac{112}{3} = 970.66\ldots = \mathbf{971\ cm^3\ (3\ sf)}$

3) A square-based pyramid has a height of 20 cm and a base side length of 16 cm.
The top 5 cm of the pyramid is removed parallel to the base.
Calculate the volume of the frustum.



$Volume\ of\ large\ pyramid = \frac{1}{3} \times (16 \times 16) \times 20 = \frac{5120}{3}\ cm^3$

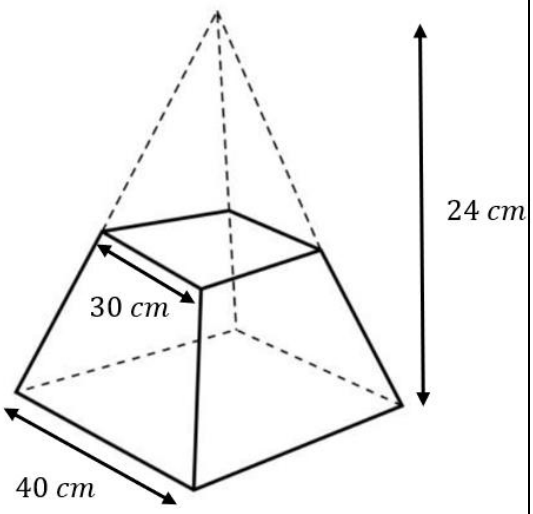
$Scale\ factor = 20 \div 5 = 4$

$Smaller\ side\ length = 16 \div 4 = 4$

$Volume\ of\ small\ pyramid = \frac{1}{3} \times (4 \times 4) \times 5 = \frac{80}{3}\ cm^3$

$Volume\ of\ frustum = \frac{5120}{3} - \frac{80}{3} = \mathbf{1680\ cm^3}$

4) **A square-based pyramid has a height of 24 cm and a base side length of 40 cm.**
The top of the pyramid is removed parallel to the base and has a base side length of 30 cm.
Calculate the volume of the frustum.



$Volume\ of\ large\ pyramid = \frac{1}{3} \times (40 \times 40) \times 24 = 12800\ cm^3$

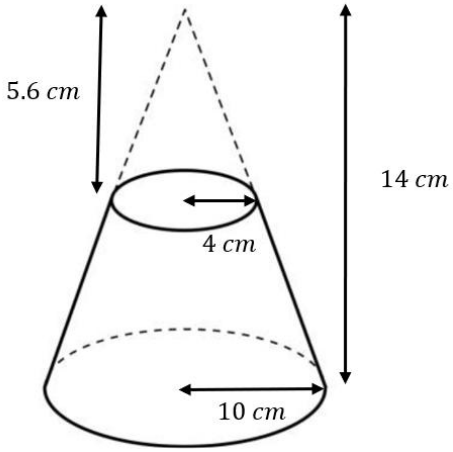
$Scale\ factor = 40 \div 30 = \frac{4}{3}$

$Smaller\ height = 24 \div \frac{4}{3} = 18\ cm$

$Volume\ of\ small\ pyramid = \frac{1}{3} \times (30 \times 30) \times 18 = 5400\ cm^3$

$Volume\ of\ frustum = 12800 - 5400 = \mathbf{7400\ cm^3}$

5) **A cone has a height of 14 cm and a radius of 10 cm.**
The frustum of a cone is formed by removing a smaller cone, parallel to the base.
The smaller cone has a height of 5.6 cm and a radius of 4 cm.
Calculate the volume of the frustum. Give your answer to 1 decimal place.



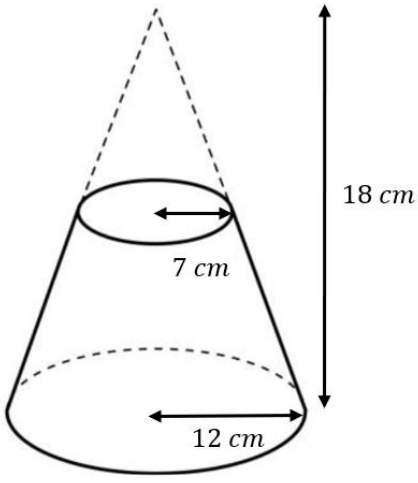
$Volume\ of\ large\ cone = \frac{1}{3} \times \pi \times 10^2 \times 14 = \frac{1400}{3}\pi\ cm^3$

$Volume\ of\ small\ cone = \frac{1}{3} \times \pi \times 4^2 \times 5.6 = \frac{448}{15}\pi\ cm^3$

$Volume\ of\ frustum = \frac{1400}{3}\pi - \frac{448}{15}\pi = 1372.2476\ldots$

$= \mathbf{1372.2\ cm^3\ (1\ dp)}$

6) **The diagram shows the frustum formed from a cone.**
Calculate the volume of the frustum. Give your answer in terms of π.



$Volume\ of\ large\ cone = \frac{1}{3} \times \pi \times 12^2 \times 18 = 864\pi\ cm^3$

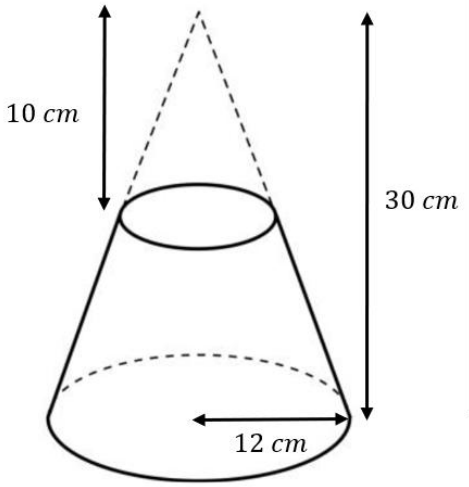
$Scale\ factor = 12 \div 7 = \frac{12}{7}$

$Smaller\ height = 18 \div \frac{12}{7} = \frac{21}{2}\ cm$

$Volume\ of\ small\ cone = \frac{1}{3} \times \pi \times 7^2 \times \frac{21}{2} = \frac{343}{2}\pi\ cm^3$

$Volume\ of\ frustum = 864\pi - \frac{343}{2}\pi = \mathbf{\frac{1385}{2}\pi\ cm^3}$

7) A cone has a height of 30 cm and a base radius of 12 cm.
The top 10 cm of the cone is removed.
Calculate the volume of the remaining frustum.
Give your answer to the nearest cubic centimetre.



$Volume\ of\ large\ cone = \frac{1}{3} \times \pi \times 12^2 \times 30 = 1440\pi\ cm^3$

$Scale\ factor = 30 \div 10 = 3$

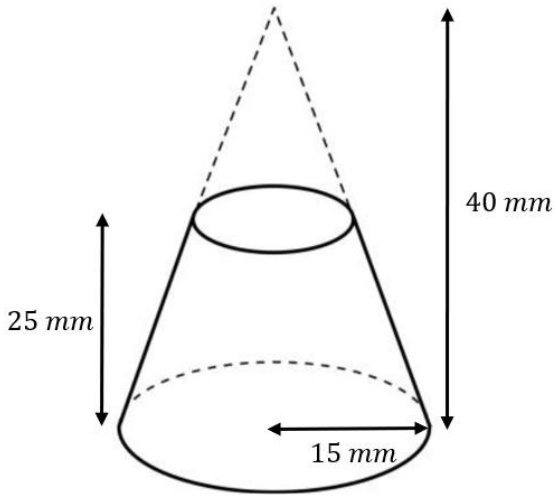
$Smaller\ radius = 12 \div 3 = 4\ cm$

$Volume\ of\ small\ cone = \frac{1}{3} \times \pi \times 4^2 \times 10 = \frac{160}{3}\pi\ cm^3$

$Volume\ of\ frustum = 1440\pi - \frac{160}{3}\pi = 4356.341\ldots$

$= \mathbf{4356\ cm^3\ (unit)}$

8) A cone has a height of 40 mm and a base radius of 15 mm.
The remaining frustum has a height of 25 mm as shown in the diagram.
Calculate the volume of the frustum.
Give your answer to the nearest cubic millimetre.



$Volume\ of\ large\ cone = \frac{1}{3} \times \pi \times 15^2 \times 40 = 3000\pi\ mm^3$

$Smaller\ height = 40 - 25 = 15\ mm$

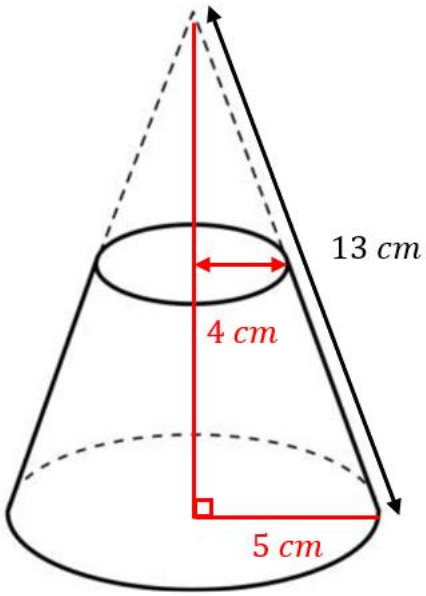
$Scale\ factor = 40 \div 15 = \frac{8}{3}$

$Smaller\ radius = 15 \div \frac{8}{3} = \frac{45}{8}\ mm$

$Volume\ of\ small\ cone = \frac{1}{3} \times \pi \times \left(\frac{45}{8}\right)^2 \times 15 = \frac{10125}{64}\pi\ mm^3$

$Volume\ of\ frustum = 3000\pi - \frac{10125}{64}\pi = 8927.7681\ldots = \mathbf{8928\ mm^3\ (unit)}$

9) A cone has a diameter of 10 cm and a slant height of 13 cm.
A frustum of a cone is formed by removing a smaller cone, parallel to the base.
The smaller cone has a diameter of 8 cm. Work out the volume of the frustum.
Give your answer to 2 decimal places.



Radius of large cone = 5 cm

Use Pythagoras' theorem to work out the perpendicular height of the large cone

$a^2 + b^2 = c^2$

$h^2 = 13^2 - 5^2 = 144$

$h = \sqrt{144} = 12\ cm$

$$Volume\ of\ large\ cone = \frac{1}{3} \times \pi \times 5^2 \times 12 = 100\pi\ cm^3$$

Radius of small cone = 4 cm

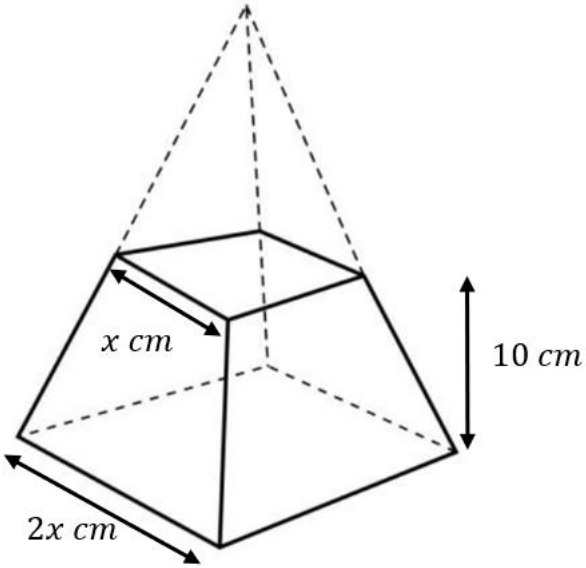
$Scale\ factor = 5 \div 4 = 1.25$

$Small\ height = 12 \div 1.25 = 9.6\ cm$

$$Volume\ of\ small\ cone = \frac{1}{3} \times \pi \times 4^2 \times 9.6 = \frac{256}{5}\pi\ cm^3$$

$$Volume\ of\ frustum = 100\pi - \frac{256}{5}\pi = 153.3097 = \mathbf{153.31\ cm^3\ (2\ dp)}$$

10) A frustum of a square-based pyramid has a height of 10 cm, a top side length of x cm and a bottom side length of $2x$ cm. The volume of the frustum is 3500 cm^3.
Work out the value of x. Give your answer to 1 decimal place.



$Scale\ factor = 2x \div x = 2$

$Large\ height = 20\ cm$

$Volume\ of\ large\ pyramid = \frac{1}{3} \times (2x \times 2x) \times 20 = \frac{80}{3}x^2\ cm^3$

$Volume\ of\ small\ pyramid = \frac{1}{3} \times (x \times x) \times 10 = \frac{10}{3}x^2\ cm^3$

$Volume\ of\ frustum = \frac{80}{3}x^2 - \frac{10}{3}x^2 = \frac{70}{3}x^2\ cm^3$

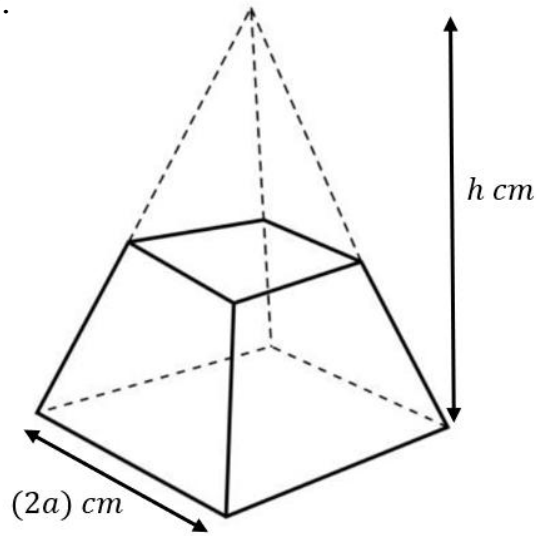
$\frac{70}{3}x^2 = 3500$

$x^2 = 3500 \div \left(\frac{70}{3}\right) = 150$

$x = \sqrt{150} = 12.2474\ldots = \mathbf{12.2}\ (\mathbf{1}\ \boldsymbol{dp})$

Challenge

11) A square-based pyramid has height h cm and base side length $(2a)$ cm.
The top half of the pyramid is removed by a plane parallel to the base.
Show that the volume of the remaining frustum is $\frac{7}{6}a^2h$ cm^3.



Top half is removed, so scale factor is 2

Smaller side length = a

Smaller height = $\frac{1}{2}h$

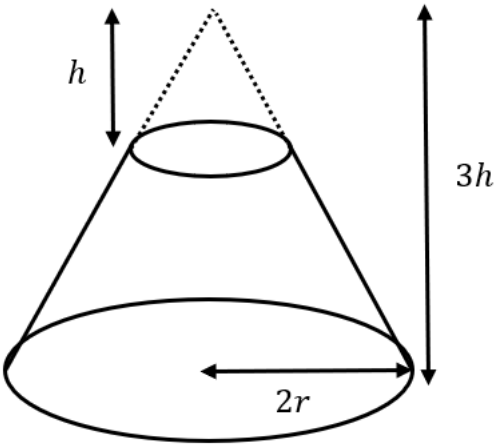
$Volume\ of\ large\ pyramid = \frac{1}{3} \times (2a \times 2a) \times h = \frac{4}{3}a^2h\ cm^3$

$Volume\ of\ small\ pyramid = \frac{1}{3} \times (a \times a) \times \frac{1}{2}h = \frac{1}{6}a^2h\ cm^3$

$Volume\ of\ frustum = \frac{4}{3}a^2h - \frac{1}{6}a^2h = \frac{7}{6}a^2h\ cm^3$

12) A cone has a height of $3h$ and a radius of $2r$.
The top cone removed has a height of h.
Work out the volume of the frustum. Give your answer in terms of π, r and h.

$Volume\ of\ large\ cone = \frac{1}{3} \times \pi \times (2r)^2 \times 3h = 4\pi hr^2$



$Scale\ factor = 3h \div h = 3$

$Small\ radius = \frac{2r}{3}$

$Volume\ of\ small\ cone = \frac{1}{3} \times \pi \times \left(\frac{2r}{3}\right)^2 \times h = \frac{4}{27}\pi hr^2$

$Volume\ of\ frustum = 4\pi hr^2 - \frac{4}{27}\pi hr^2 = \mathbf{\frac{104}{27}\pi hr^2}$