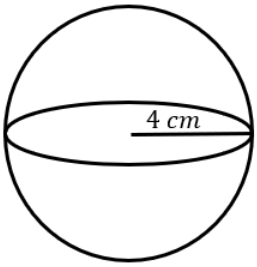
VOLUME OF COMPLEX SHAPES

1) The sphere shown in the diagram has a radius of 4 cm. Work out the volume of the sphere. Give your answer to 2 decimal places.



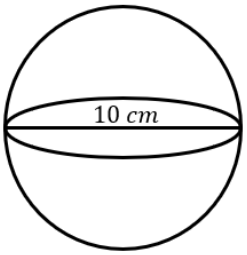
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 4^3$

$= 268.0825 \ldots$

$= \mathbf{268.08\ cm^3\ (2\ dp)}$

2) The sphere shown in the diagram has a diameter of 10 cm. Work out the volume of the sphere. Give your answer to 2 decimal places.



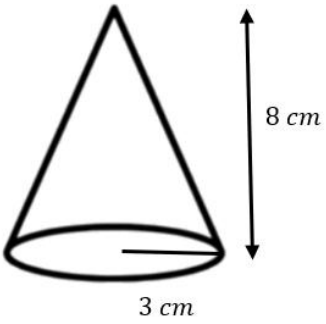
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 5^3$

$= 523.5987 \ldots$

$= \mathbf{523.60\ cm^3\ (2\ dp)}$

3) The cone shown in the diagram has a radius of 3 cm and a perpendicular height of 8 cm. Work out the volume of the cone. Give your answer to 1 decimal place.



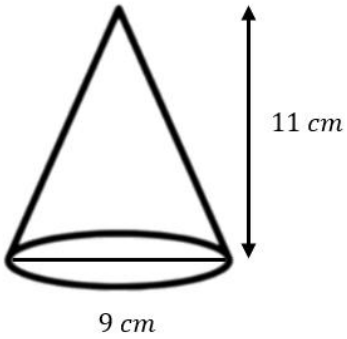
$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3^2 \times 8$

$= 75.3982 \ldots$

$= \mathbf{75.4\ cm^3\ (1\ dp)}$

4) The cone shown in the diagram has a diameter of 9 cm and a height of 11 cm. Work out the volume of the cone. Give your answer in terms of π.

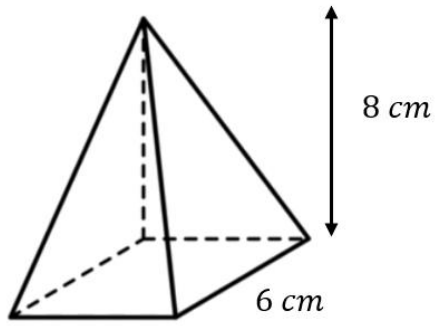


$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 4.5^2 \times 11$

$= \mathbf{\frac{297}{4}\pi\ cm^3}$

5) A square-based pyramid is shown in the diagram. Work out the volume of the pyramid.

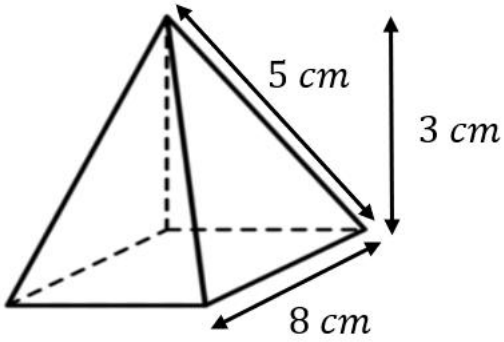


$V = \frac{1}{3} \times a \times h$

$= \frac{1}{3} \times (6 \times 6) \times 8$

$= \mathbf{96\ cm^3}$

6) The dimensions of a square-based pyramid are shown in the diagram below. Work out the volume of the pyramid.

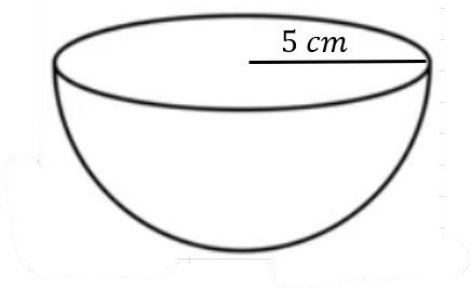


$V = \frac{1}{3} \times a \times h$

$= \frac{1}{3} \times (8 \times 8) \times 3$

$= \mathbf{64\ cm^3}$

7) The hemisphere shown in the diagram has a radius of 5 cm. Work out the volume of the hemisphere. Give your answer in terms of π.

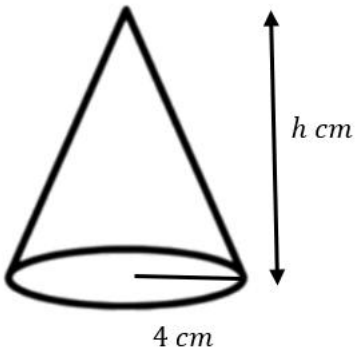


$V = \frac{1}{2} \times \frac{4}{3} \pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$

$= \mathbf{\frac{250}{3} \pi\ cm^3}$

8) The cone shown in the diagram below has a radius of 4 cm and a volume of 100.5 cm^3. Work out the value of h. Give your answer to the nearest unit.



$V = \frac{1}{3}\pi r^2 h$

$\frac{1}{3} \times \pi \times 4^2 \times h = 100.5$

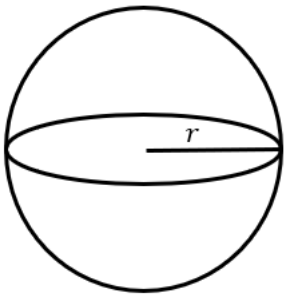
$\frac{16}{3}\pi h = 100.5$

$h = 100.5 \div \left(\frac{16}{3}\pi\right)$

$= 5.9981 \ldots$

$= \mathbf{6}\ (\boldsymbol{unit})$

9) The sphere shown in the diagram below has a volume of 288π cm^3. Work out the missing radius.



$V = \frac{4}{3}\pi r^3$

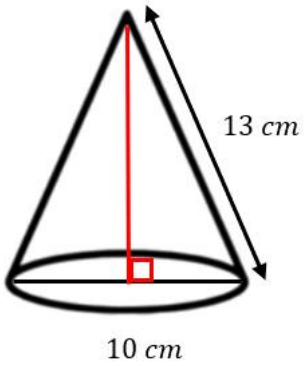
$\frac{4}{3} \times \pi \times r^3 = 288\pi$

$\frac{4}{3} r^3 = 288$

$r^3 = 216$

$r = \sqrt[3]{216} = \mathbf{6\ cm}$

10) A cone is shown in the diagram below. The cone has a diameter of 10 cm and a slant height of 13 cm. Work out the volume of the cone. Give your answer to 3 significant figures.



Use Pythagoras' theorem to find the missing perpendicular height

$a^2 + b^2 = c^2$

$h^2 + 5^2 = 13^2$

$h^2 = 13^2 - 5^2$

$h^2 = 144$

$h = \sqrt{144} = 12\ cm$

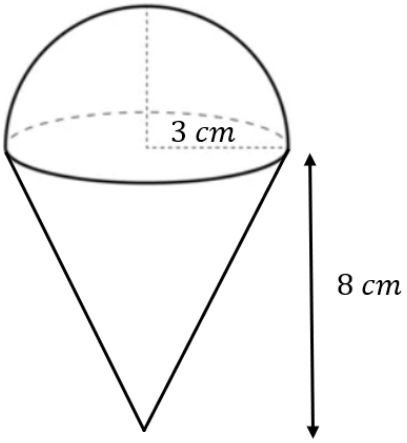
$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 5^2 \times 12$

$= 314.1592\ldots$

$= \mathbf{314\ cm^3\ (3\ sf)}$

11) A solid consists of a hemisphere of radius 3 cm joined to a cone with radius 3 cm and height 8 cm. Work out the total volume of the solid. Give your answer to 3 significant figures.



Volume of hemisphere

$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3$

$= 18\pi\ cm^3$

Volume of cone

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3^2 \times 8$

$= 24\pi\ cm^3$

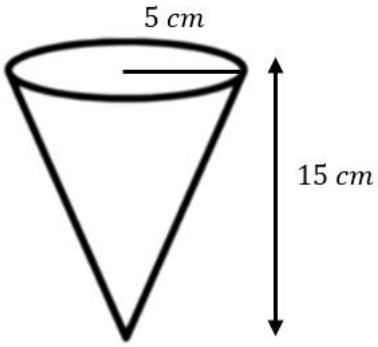
Total volume

$18\pi + 24\pi$

$= 131.9468\ldots$

$= \mathbf{132\ cm^3\ (3\ sf)}$

12) A conical container has radius 5 cm and height 15 cm. Water is poured in at a rate of 20 cm^3 per second. How long does it take to fill the cone completely? Give your answer to the nearest second.



$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 5^2 \times 15$

$= 125\pi\ cm^3$

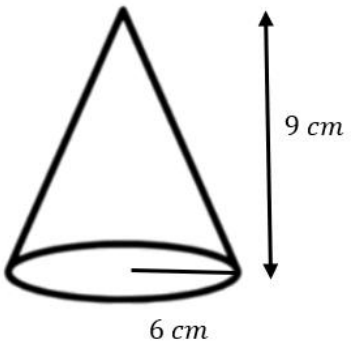
$= 392.69908 \ldots cm^3$

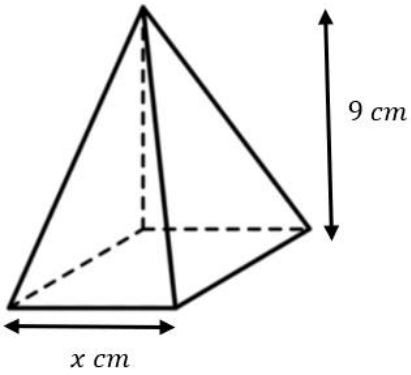
Time taken

$392.69908 \ldots \div 20 = 19.6349 \ldots$

$20\ seconds$

13) A cone and a pyramid have the same height of 9 cm.
The cone has a radius of 6 cm. The pyramid has a square base of side length x cm.
Given the volume of the two shapes are equal, work out the value of x.
Give your answer to 1 decimal place.





Volume of cone

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6^2 \times 9$

$= 108\pi\ cm^3$

Volume of pyramid

$V = \frac{1}{3} \times a \times h$

$= \frac{1}{3} \times (x^2) \times 9$

$= 3x^2\ cm^3$

$3x^2 = 108\pi$

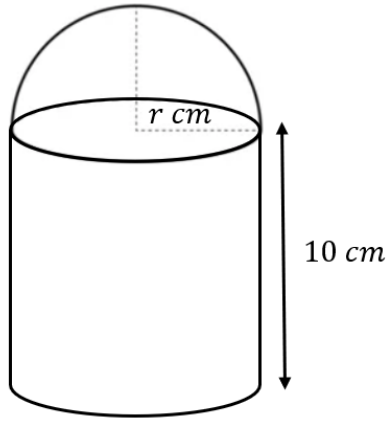
$x^2 = \frac{108\pi}{3}$

$x = \sqrt{\frac{108\pi}{3}} = 10.6347 \ldots$

$x = 10.6\ (1\ dp)$

Challenge – Higher Tier Only

14) A solid toy is made by joining a hemisphere to a cylinder. The cylinder and the hemisphere both have a radius of r cm. The height of the cylinder is 10 cm.



The total volume of the toy is V cm^3.

Show that:

$$V = \frac{2\pi r^3 + 30\pi r^2}{3}$$

Volume of hemisphere

$\frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

Volume of cylinder

$\pi r^2 \times h$

$= \pi r^2 \times 10$

$= 10\pi r^2$

Total volume:

$V = \frac{2\pi r^3}{3} + 10\pi r^2$

$V = \frac{2\pi r^3}{3} + \frac{30\pi r^2}{3}$

$V = \frac{2\pi r^3 + 30\pi r^2}{3}$

15) The radius of a sphere is increased by 2 cm.
The new sphere has twice the volume of the original sphere.
Show that $r^3 - 6r^2 - 12r - 8 = 0$.

Original radius = r

New radius = r + 2

New sphere volume = twice original sphere volume

$$\frac{4}{3} \times \pi \times (r+2)^3 = 2 \times \frac{4}{3} \times \pi \times r^3$$

$$\div \left(\frac{4}{3}\pi\right) \qquad \div \left(\frac{4}{3}\pi\right)$$

$$(r+2)^3 = 2r^3$$

$$(r+2)(r+2)(r+2) = 2r^3$$

$$(r+2)(r^2+2r+2r+4) = 2r^3$$

$$(r+2)(r^2+4r+4) = 2r^3$$

$$r^3+4r^2+4r+2r^2+8r+8 = 2r^3$$

$$r^3+6r^2+12r+8 = 2r^3$$

$$r^3-6r^2-12r-8 = 0$$