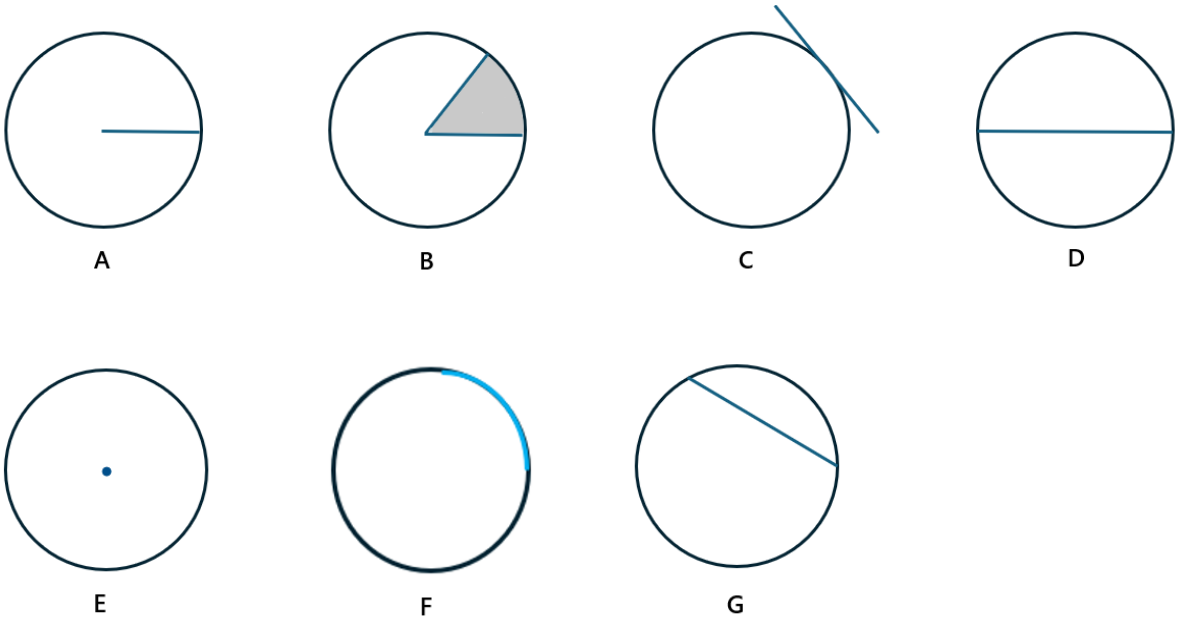
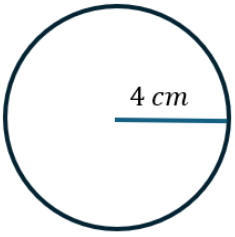
CIRCLES – AREA AND CIRCUMFERENCE

1) Write the correct letter from the diagram in the table below.



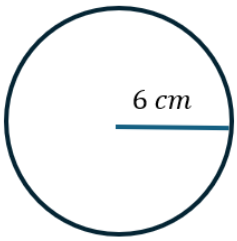
Part of the circle	Letter
Centre	E
Radius	A
Diameter	D
Chord	G
Tangent	C
Arc Length	F
Sector	B

2) A circle has a radius of 4 cm.
Work out the area of the circle.
Give your answer to 2 decimal places.



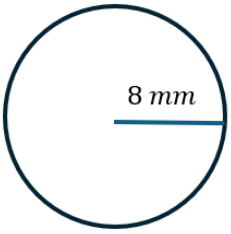
$Area = \pi \times r^2$
$= \pi \times 4^2$
$= 50.2654 \ldots$
$= \mathbf{50.27\ cm^2\ (2\ dp)}$

3) A circle has a radius of 6 cm.
Work out the area of the circle.
Give your answer to 1 decimal place.



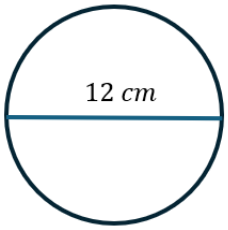
$Area = \pi \times r^2$
$= \pi \times 6^2$
$= 113.09733 \ldots$
$= \mathbf{113.1\ cm^2\ (1\ dp)}$

4) A circle has a radius of 8 mm.
Work out the area of the circle.
Give your answer in terms of π.



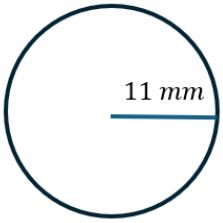
$Area = \pi \times r^2$
$= \pi \times 8^2$
$= \mathbf{64\pi\ mm^2}$

5) A circle has a diameter of 12 cm.
Work out the area of the circle.
Give your answer to 2 decimal places.



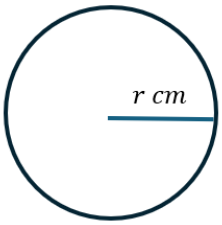
$Area = \pi \times r^2$
$= \pi \times 6^2$
$= 113.09733 \ldots$
$= \mathbf{113.10\ cm^2\ (2\ dp)}$

6) A circle has a radius of 11 mm.
Work out the area of the circle.
Give your answer in terms of π.



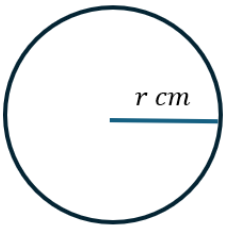
$Area = \pi \times r^2$
$= \pi \times 11^2$
$= \mathbf{121\pi\ mm^2}$

7) A circle has a radius of r cm.
The area of the circle is 144π cm^2.
Work out the value of r.



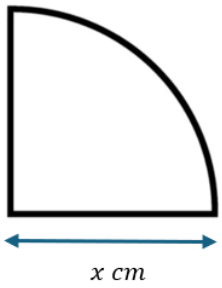
$Area = \pi \times r^2$
$\pi \times r^2 = 144\pi$
$r^2 = 144$
$r = \sqrt{144} = \mathbf{12\ cm}$

8) A circle has a radius of r cm.
The area of the circle is 345 cm^2.
Work out the value of r.
Give your answer to 1 decimal place.



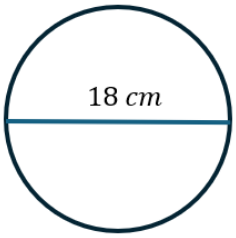
$Area = \pi \times r^2$
$345 = \pi \times r^2$
$r^2 = \frac{345}{\pi}$
$r = \sqrt{\frac{345}{\pi}} = \mathbf{10.5\ cm\ (1\ dp)}$

9) The area of the quarter circle shown is 100π cm^2.
The radius of the quarter circle is x cm.
Work out the value of x.



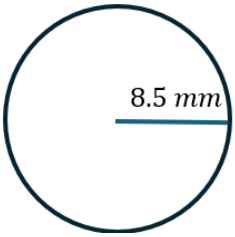
$Area = \frac{1}{4} \times \pi \times r^2$
$100\pi = \frac{1}{4}\pi x^2$
$100 = \frac{1}{4}x^2$
$x^2 = 400$
$x = \sqrt{400} = \mathbf{20\ cm}$

10) A circle has a diameter of 18 cm.
Work out the circumference of the circle.
Give your answer to 2 decimal places.



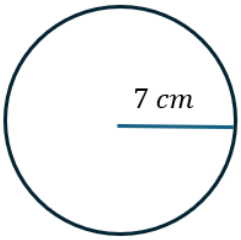
$Circumference = 2\pi r$
$= 2 \times \pi \times 9$
$= 56.5486 \ldots$
$= \mathbf{56.55\ cm\ (2\ dp)}$

11) A circle has a radius of 8.5 mm.
Work out the circumference of the circle.
Give your answer to 2 decimal places.



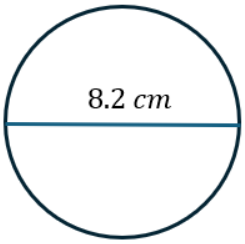
$Circumference = 2\pi r$
$= 2 \times \pi \times 8.5$
$= 53.4070 \ldots$
$= \mathbf{53.41\ mm\ (2\ dp)}$

12) A circle has a radius of 7 cm.
Work out the circumference of the circle.
Give your answer to 2 decimal places.



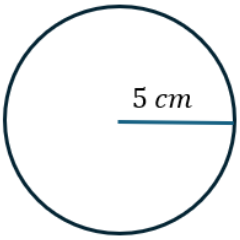
$Circumference = 2\pi r$
$= 2 \times \pi \times 7$
$= 43.98229 \ldots$
$= \mathbf{43.98\ cm\ (2\ dp)}$

13) A circle has a diameter of 8.2 cm.
Work out the circumference of the circle.
Give your answer to 1 decimal place.



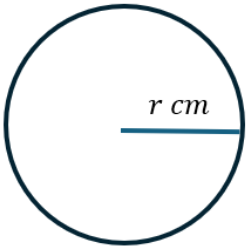
$Circumference = 2\pi r$
$= 2 \times \pi \times 4.1$
$= 25.7610 \ldots$
$= \mathbf{25.8\ cm\ (1\ dp)}$

14) A circle has a radius of 5 cm.
Work out the circumference of the circle.
Give your answer in terms of π.



$Circumference = 2\pi r$
$= 2 \times \pi \times 5$
$= \mathbf{10\pi\ cm}$

15) A circle has a radius of r cm.
The circumference of the circle is 150 cm.
Work out the value of r.
Give your answer to 3 significant figures.



$Circumference = 2\pi r$

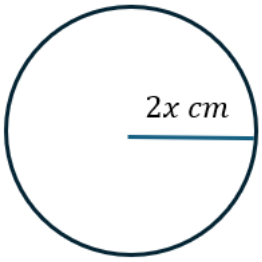
$2 \times \pi \times r = 150$

$r = \frac{150}{2\pi}$

$r = 23.87324 \ldots$

$\boldsymbol{r = 23.9\ cm\ (3\ sf)}$

16) A circle has a radius of 2x cm.
The circumference of the circle is 50π cm.
Work out the value of x.



$Circumference = 2\pi r$

$2 \times \pi \times 2x = 50\pi$

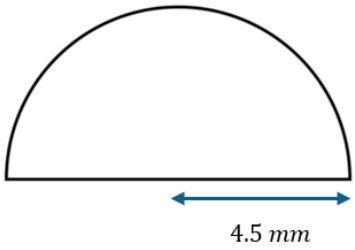
$4\pi x = 50\pi$

$4x = 50$

$\boldsymbol{x = 12.5}$

17) A semi-circle has a radius of 4.5 mm.

a. Work out the area of the semi-circle.
Give your answer to 1 decimal place.



$Area = \frac{1}{2} \times \pi \times r^2$

$= \frac{1}{2} \times \pi \times 4.5^2$

$= \boldsymbol{31.8\ mm^2\ (1\ dp)}$

b. Work out the perimeter of the semi-circle.
Give your answer to 1 decimal place.

$Circumference = \frac{1}{2} \times 2\pi r$

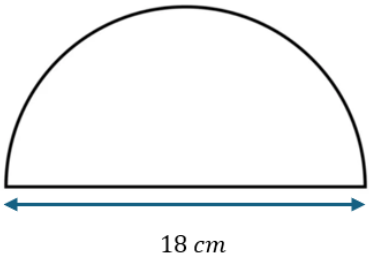
$= \frac{1}{2} \times 2 \times \pi \times 4.5$

$= 4.5\pi$

$Perimeter = 4.5\pi + 4.5 + 4.5$

$= \boldsymbol{23.1\ mm\ (1\ dp)}$

18) A semi-circle has a diameter of 18 cm.
Work out the perimeter of the semi-circle.
Give your answer in terms of π.



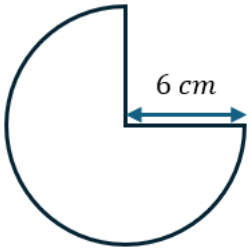
$Circumference = \frac{1}{2} \times 2\pi r$

$= \frac{1}{2} \times 2 \times \pi \times 9$

$= 9\pi$

$Perimeter = \boldsymbol{9\pi + 18\ cm}$

19) A three-quarter circle has a radius of 6 cm.
Work out the area of the circle.
Give your answer in terms of π.

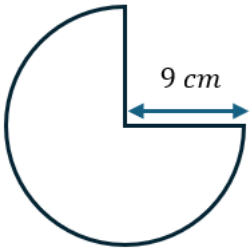


$Area = \frac{3}{4} \times \pi \times r^2$

$= \frac{3}{4} \times \pi \times 6^2$

$= \boldsymbol{27\pi\ cm^2}$

20) A three-quarter circle has a radius of 9 cm.
Work out the perimeter of the circle.
Give your answer in terms of π.



$Circumference = \frac{3}{4} \times 2\pi r$

$= \frac{3}{4} \times 2 \times \pi \times 9$

$= \frac{27}{2}\pi$

$Perimeter = \frac{27}{2}\pi + 9 + 9$

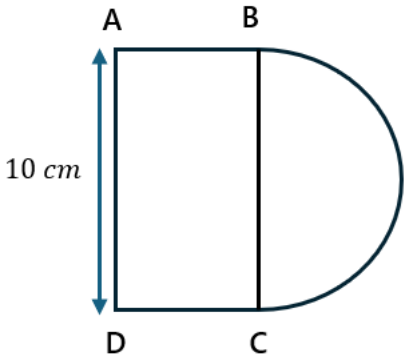
$= \mathbf{\frac{27}{2}\pi + 18\ cm}$

21) ABCD is a rectangle, attached to a semi-circle.
AD has a length of 10 cm.
Given that the width AB of the rectangle, is equal to the radius of the semi-circle,

a. Work out the area of the compound shape. Give your answer to 1 decimal place.

$Radius = 5\ cm$

$Width\ AB = 5\ cm$



$Area\ of\ ABCD = 10 \times 5 = 50\ cm^2$

$Area\ of\ semi\ circle = \frac{1}{2} \times \pi \times 5^2 = 12.5\pi\ cm^2$

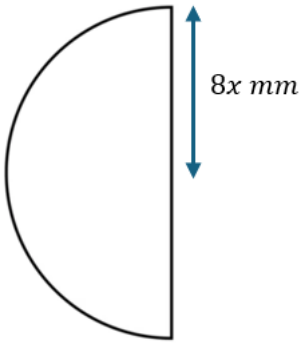
$Area\ of\ compound\ shape = 50 + 12.5\pi = \mathbf{89.3\ cm^2\ (1\ dp)}$

b. Work out the perimeter of the compound shape.
Give your answer to 2 decimal places.

$Circumference\ of\ semi\ circle = \frac{1}{2} \times 2 \times \pi \times 5 = 5\pi$

$Perimeter = 10 + 5 + 5 + 5\pi = \mathbf{35.71\ cm\ (2\ dp)}$

22) The semi-circle has a radius of 8x mm.
Write a fully simplified expression for the area of the semi-circle.
Give your answer in terms of π.

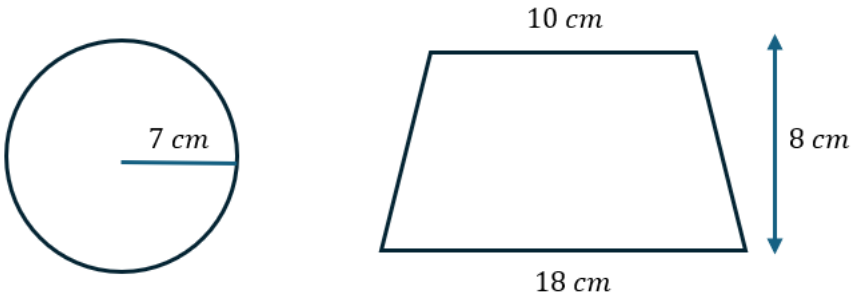


$Area = \frac{1}{2} \times \pi \times r^2$

$= \frac{1}{2} \times \pi \times (8x)^2$

$= \mathbf{32\pi x^2\ mm^2}$

23) A circle and a trapezium are shown below.



Work out the percentage, by which the larger area exceeds the smaller area.
Give your answer to 1 decimal place.

$Area\ of\ circle = \pi \times 7^2$
$= 49\pi$
$= 153.93804\ ...$

$Area\ of\ trapezium = \frac{1}{2} \times (a + b) \times h$

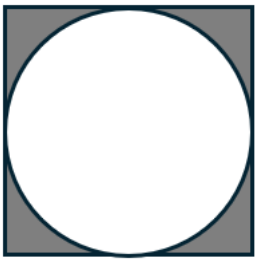
$= \frac{1}{2} \times (10 + 18) \times 8$

$= 112\ cm^2$

$Percentage\ difference = \frac{153.93804\ ... - 112}{112} \times 100 = \mathbf{37.4\%\ (1\ dp)}$

Challenge

24) The circle shown in the diagram is inscribed in the square.
Given that the area of the square is equal to its perimeter, work out the area of the shaded region.
Give your answer as an exact value in terms of π.



Solve for the side length of the square
Let the side length be x

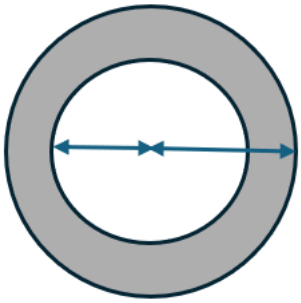
$Area = Perimeter$
$x \times x = 4(x)$
$x^2 = 4x$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0 \quad or \quad x = 4$

$Area\ of\ square = 4^2 = 16$

$Area\ of\ circle = \pi \times 2^2 = 4\pi$

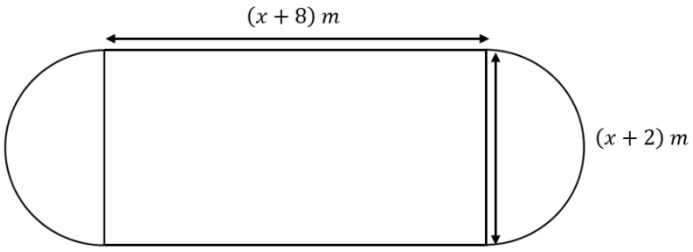
$Area\ of\ shaded\ region = \mathbf{(16 - 4\pi)\ units^2}$

25) A circle of radius $x + 2$ cm is drawn inside a larger circle of radius $x + 8$ cm. The distance between the circumference of the smaller circle and the circumference of the larger circle is uniform throughout. The shaded region between the two circles has an area of 300π cm^2.
Work the value of x.
You must show complete algebraic working.
Do not use trial and improvement.



$$Area\ of\ shaded\ region = (\pi \times (x+8)^2) - (\pi \times (x+2)^2) = 300\pi$$
$$(x+8)^2 - (x+2)^2 = 300$$
$$(x+8)(x+8) - (x+2)(x+2) = 300$$
$$x^2 + 8x + 8x + 64 - (x^2 + 2x + 2x + 4) = 300$$
$$x^2 + 16x + 64 - (x^2 + 4x + 4) = 300$$
$$12x + 60 = 300$$
$$12x = 240$$
$$\boldsymbol{x = 20}$$

26) A rectangular garden has:
A length of $(x + 8)\ m$
A width of $(x + 2)\ m$



At each of the shorter ends of the garden, a semicircular flower bed is attached externally.

The rectangular part of the garden will be covered with grass at a cost of £12 per square metre.

Decorative stones are placed inside both semicircular flower beds at a cost of £18 per square metre.

Given that the area of one semicircular flower bed is $\frac{121}{2}\pi$ m^2, work out the total cost of adding the grass and decorative stones to the garden.

Solve for x

$$\frac{1}{2} \times \pi \times \left(\frac{1}{2}(x+2)\right)^2 = \frac{121}{2}\pi$$

$$\frac{1}{2} \times \pi \times \left(\frac{1}{2}x+1\right)^2 = \frac{121}{2}\pi$$

$$\frac{1}{2} \times \left(\frac{1}{2}x+1\right)^2 = \frac{121}{2}$$

$$\left(\frac{1}{2}x+1\right)^2 = 121$$

$$\frac{1}{2}x+1 = \sqrt{121}$$

$$\frac{1}{2}x+1 = -11 \quad or \quad \frac{1}{2}x+1 = 11$$

$$\frac{1}{2}x = -12 \quad or \quad \frac{1}{2}x = 10$$

$$\cancel{x = -24} \quad or \quad x = 20$$

$$Length\ of\ rectangular\ field = 20 + 8 = 28$$

$$Width\ of\ rectangular\ field = 20 + 2 = 22$$

$$Area\ of\ rectangular\ field = 28 \times 22 = 616\ m^2$$

$$Cost\ of\ grass = 616 \times £12 = £7392$$

$$Area\ of\ semicircular\ gardens = 2 \times \frac{121\pi}{2} = 121\pi\ m^2$$

$$Cost\ of\ decorative\ rocks = 121\pi \times £18 = £6842.3888$$

$$Total\ cost\ of\ the\ garden = £7392 + £6842.39 = \mathbf{£14234.39}$$