

HARDER ALGEBRAIC INDICES

Task 1

- 1) The equation $3^{\frac{1}{2}} \times 3^{\frac{3}{4}} = 3^p$
Work out the value of p .

$$p = \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

- 2) Given that $\frac{16}{2^6} = 2^n$
Work out the value of n .

$$\frac{2^4}{2^6} = 2^{-2}$$

$$n = -2$$

- 3) Given that $\frac{25}{5^{-3}} = 5^k$
Work out the value of k .

$$\frac{5^2}{5^{-3}} = 5^{2+3} = 5^5$$

$$k = 5$$

- 4) Write $\sqrt{3} \times 9 \times 27$ as a single power of 3.

$$3^{\frac{1}{2}} \times 3^2 \times 3^3$$

$$= 3^{\frac{1}{2}+2+3}$$

$$= 3^{\frac{11}{2}}$$

- 5) Given that $64 \times \sqrt{2} = 2^x$
Work out the value of x .

$$2^6 \times 2^{\frac{1}{2}}$$

$$= 2^{6+\frac{1}{2}}$$

$$= 2^{\frac{13}{2}}$$

$$x = \frac{13}{2}$$

9) $(27d^{12})^{\frac{2}{3}}$
 $= 9d^8$

10) $(8x^6y^9)^{\frac{2}{3}}$
 $= 4x^4y^6$

11) $(64m^9n^6)^{\frac{1}{3}}$
 $= 4m^3n^2$

12) $(16p^{20})^{\frac{3}{4}}$
 $= 8p^{15}$

13) $(81q^6r^{10})^{\frac{1}{2}}$
 $= 9q^3r^5$

14) $\left(\frac{36x^8}{y^4}\right)^{\frac{1}{2}}$
 $= \frac{6x^4}{y^2}$

15) $\left(\frac{64a^{12}}{16b^8}\right)^{\frac{1}{2}}$
 $= \frac{8a^6}{4b^4}$
 $= \frac{2a^6}{b^4}$

Task 3 – Fully simplify each of the following expressions.

16) $(5x^3y^6)^{-2}$
 $= \left(\frac{1}{5x^3y^6}\right)^2$
 $= \frac{1}{25x^6y^{12}}$

17) $\left(\frac{z^4}{9}\right)^{-\frac{1}{2}}$
 $= \left(\frac{9}{z^4}\right)^{\frac{1}{2}}$
 $= \frac{3}{z^2}$

Task 2 – Fully simplify each of the following expressions.

6) $(36a^8)^{\frac{1}{2}}$
 $= 6a^4$

7) $(121b^{10})^{\frac{1}{2}}$
 $= 11b^5$

8) $(49c^4)^{\frac{3}{2}}$
 $= 343c^6$

$$18) \left(\frac{9x^2}{16y^6}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{16y^6}{9x^2}\right)^{\frac{1}{2}}$$

$$= \frac{4y^3}{3x}$$

$$19) \left(\frac{27x^9}{y^3}\right)^{-1/3}$$

$$= \left(\frac{y^3}{27x^9}\right)^{\frac{1}{3}}$$

$$= \frac{y}{3x^3}$$

$$20) (25a^2b^6)^{-\frac{3}{2}}$$

$$= \left(\frac{1}{25a^2b^6}\right)^{\frac{3}{2}}$$

$$= \frac{1}{125a^3b^9}$$

$$21) \left(\frac{8p^6}{q^{12}}\right)^{-\frac{2}{3}}$$

$$= \left(\frac{q^{12}}{8p^6}\right)^{\frac{2}{3}}$$

$$= \frac{q^8}{4p^4}$$

$$22) \left(\frac{16x^4y^8}{32x^6y^{10}}\right)^{-1}$$

$$= \left(\frac{1}{2x^2y^2}\right)^{-1}$$

$$= 2x^2y^2$$

$$23) \left(\frac{49a^4b^6}{121a^6b^8}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{121a^6b^8}{49a^4b^6}\right)^{\frac{1}{2}}$$

$$= \frac{11a^3b^4}{7a^2b^3}$$

$$= \frac{11ab}{7}$$

$$24) \left(\frac{16m^6}{p^{12}}\right)^{-1/2}$$

$$= \left(\frac{p^{12}}{16m^6}\right)^{\frac{1}{2}}$$

$$= \frac{p^6}{4m^3}$$

Challenge

25) Solve

$$\frac{2^{4x+1} \times 4^{x-2}}{8^{x-3}} = 256^{\frac{1}{2}}$$

$$\frac{2^{4x+1} \times 2^{2(x-2)}}{2^{3(x-3)}} = 16$$

$$2^{4x+1+2x-4-3x+9} = 2^4$$

$$2^{3x+6} = 2^4$$

$$3x + 6 = 4$$

$$3x = -2$$

$$x = -\frac{2}{3}$$