ARITHMETIC SERIES

Task 1

1) **Find the sum of the first 10 terms of the sequence:**
2, 5, 8, 11, 14 ...

$a = 2,\ d = 3,\ n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(2) + (10-1) \times 3]$

$= 5(4 + 27)$

$= \mathbf{155}$

2) **Find the sum of the first 20 terms of the sequence:**
7, 10, 13, 16, 19 ...

$a = 7,\ d = 3,\ n = 20$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2(7) + (20-1) \times 3]$

$= 10(14 + 57)$

$= \mathbf{710}$

3) **Find the sum of the first 15 terms of the sequence:**
4, 9, 14, 19, 24 ...

$a = 4,\ d = 5,\ n = 15$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2(4) + (15-1) \times 5]$

$= \frac{15}{2}(8 + 70)$

$= \mathbf{585}$

4) **Find the sum of the first 12 terms of the sequence:**
3, 5, 7, 9, 11 ...

$a = 3,\ d = 2,\ n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2(3) + (12-1) \times 2]$

$= 6(6 + 22)$

$= \mathbf{168}$

5) Find the sum of the first 25 terms of the sequence:
1, 4, 7, 10, 13 ...

$a = 1,\ d = 3,\ n = 25$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{25} = \frac{25}{2}[2(1) + (25-1) \times 3]$

$= \frac{25}{2}(2 + 72)$

$= \mathbf{925}$

6) Find the sum of the first 18 terms of the sequence:
5, 9, 13, 17, 21 ...

$a = 5,\ d = 4,\ n = 18$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{18} = \frac{18}{2}[2(5) + (18-1) \times 4]$

$= 9(10 + 68)$

$= \mathbf{702}$

7) Find the sum of the first 30 terms of the sequence:
2, 6, 10, 14, 18 ...

$a = 2,\ d = 4,\ n = 30$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2(2) + (30-1) \times 4]$

$= 15(4 + 116)$

$= \mathbf{1800}$

8) Find the sum of the first 16 terms of the sequence:
8, 12, 16, 20, 24 ...

$a = 8,\ d = 4,\ n = 16$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{16} = \frac{16}{2}[2(8) + (16-1) \times 4]$

$= 8(16 + 60)$

$= \mathbf{608}$

9) Find the sum of the first 100 terms of the sequence:
6, 5, 4, 3, 2 ...

$a = 6,\ d = -1,\ n = 100$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{100} = \frac{100}{2}[2(6) + (100-1) \times -1]$

$= 50(12 - 99)$

$= \mathbf{-4350}$

10) Find the sum of the first 22 terms of the sequence:
9, 7, 5, 3, 1 ...

$a = 9,\ d = -2,\ n = 22$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{22} = \frac{22}{2}[2(9) + (22-1) \times -2]$

$= 11(18 - 42)$

$= \mathbf{-264}$

Task 2

11) The sum of the first 6 terms of an arithmetic sequence is 87. The first term is 2. Work out the common difference.

$S_6 = \frac{6}{2}[2(2) + (6-1) \times d]$

$87 = 3[4 + 5d]$

$29 = 4 + 5d$

$5d = 25$

$\boldsymbol{d = 5}$

12) The sum of the first 20 terms of an arithmetic sequence is 600. The common difference is 2. Work out the first term.

$S_{20} = \frac{20}{2}[2a + (20-1) \times 2]$

$600 = 10[2a + 38]$

$60 = 2a + 38$

$2a = 22$

$\boldsymbol{a = 11}$

13) The sum of the first 11 terms of an arithmetic sequence is 264. The first term is 4. Work out the common difference.

$S_{11} = \frac{11}{2}[2(4) + (11-1) \times d]$

$264 = \frac{11}{2}[8 + 10d]$

$528 = 11(8 + 10d)$

$48 = 8 + 10\mathrm{d}$

$10d = 40$

$\boldsymbol{d = 4}$

14) The sum of the first n terms of an arithmetic sequence is 220. Given that the first term of the sequence is 10 and the common difference is 5, work out the value of n.

$S_n = \frac{n}{2}[2(10) + (n-1) \times 5]$

$220 = \frac{n}{2}[20 + 5n - 5]$

$220 = \frac{n}{2}[5n + 15]$

$440 = n[5n + 15]$

$440 = 5n^2 + 15n$

$5n^2 + 15n - 440 = 0$

$n^2 + 3n - 88 = 0$

$(n + 11)(n - 8) = 0$

~~$n = -11$~~ $\quad or \quad n = 8$

n = 8

15) The sum of the first n terms of an arithmetic sequence is 234. Given that the first term of the sequence is 3 and the common difference is 3, work out the value of n.

$S_n = \frac{n}{2}[2(3) + (n-1) \times 3]$

$234 = \frac{n}{2}[6 + 3n - 3]$

$234 = \frac{n}{2}[3n + 3]$

$468 = n[3n + 3]$

$468 = 3n^2 + 3n$

$3n^2 + 3n - 468 = 0$

$n^2 + n - 156 = 0$

$(n + 13)(n - 12) = 0$

~~$n = -13$~~ $\quad or \quad n = 12$

n = 12

Task 3

16) In an arithmetic sequence, the 5th term is 18 and the 8th term is 27. Work out the sum of the first 100 terms of the sequence.

$a + 4d = 18$
$a + 7d = 27$

Subtract equations:
$3d = 9$
$d = 3$

$a + 4(3) = 18$
$a + 12 = 18$
$a = 6$

$S_{100} = \frac{100}{2}[2(6) + (100-1) \times 3]$

$= \mathbf{15450}$

17) The 10th term of an arithmetic sequence is 29 and the sum of the first 10 terms of the sequence is 155. Work out the sum of the first 20 terms of the sequence.

10th term:

$a + 9d = 29$

Sum of the first 10 terms:

$155 = \frac{10}{2}[2a + (10 - 1) \times d]$

$155 = 5[2a + 9d]$

$31 = 2a + 9d$

Multiply first equation by 2 and subtract the equations:

$$\begin{aligned} 2a + 18d &= 58 \\ -(2a + 9d &= 31) \\ 9d &= 27 \\ d &= 3 \end{aligned}$$

$a + 9(3) = 29$

$a + 27 = 29$

$a = 2$

$S_{20} = \frac{20}{2}[2(2) + (20 - 1) \times 3]$

$= \mathbf{610}$

18) The 8th term of an arithmetic sequence is three times the 2nd term.
The sum of the first 10 terms of the sequence is 260.
Work out the value of the 20th term in the sequence.

8th term: $a + 7d$

2nd term: $a + d$

The 8th term is 3 times the 2nd term:

$a + 7d = 3(a + d)$

$a + 7d = 3a + 3d$

$2a = 4d$

$a = 2d$ ...(1)

Sum of first 10 terms:

$S_{10} = \frac{10}{2}[2a + (10 - 1) \times d] = 260$

$5[2a + 9d] = 260$

$2a + 9d = 52$ ...(2)

Substitute (1) into (2):

$2(2d) + 9d = 52$

$4d + 9d = 52$

$13d = 52$

$d = 4$

$a = 2(4) = 8$

20th term:

$a + 19d$

$8 + 19(4)$

$= \mathbf{84}$

Challenge

19) The first three terms of an arithmetic sequence are:

$$2x + 1,\ 5x - 2,\ 7x + 4$$

Work out the sum of the first 100 terms of the sequence.

The common difference between terms is equal:

$$(5x - 2) - (2x + 1) = (7x + 4) - (5x - 2)$$

$$3x - 3 = 2x + 6$$

$$x = 9$$

First term:

$2x + 1$

$= 2(9) + 1$

$= 19$

Second term:

$5x - 2$

$= 45 - 2$

$= 43$

Common difference:

$d = 43 - 19 = 24$

Sum of first 100 terms:

$S_{100} = \frac{100}{2}[2(19) + (100 - 1) \times 24]$

$= \mathbf{120,700}$

20) An arithmetic sequence has a first term a and common difference d.
The sum of the first n terms is $3n^2 + 5n$.
Find expressions for:

- a and d
- The nth term

Hence, find the first value of n for which the term exceeds 500.

$a_n = S_n - S_{n-1}$

$S_n = 3n^2 + 5n$

$S_{n-1} = 3(n-1)^2 + 5(n-1)$
$S_{n-1} = 3(n^2 - 2n + 1) + 5n - 5$
$= 3n^2 - 6n + 3 + 5n - 5$
$= 3n^2 - n - 2$

$a_n = (3n^2 + 5n) - (3n^2 - n - 2)$
$a_n = 6n + 2$

First term:
$a = 6(1) + 2 = 8$
$a = 8$

Common difference (by observing a_n):
$d = 6$

nth term:
$a_n = 6n + 2$

Find when:
$6n + 2 > 500$
$6n > 498$
$n > 83$

$$\boldsymbol{a = 8,\ d = 6}$$

$$\boldsymbol{a_n = 6n + 2}$$

First value of $\boldsymbol{n = 84}$