

QUADRATIC SEQUENCES

Task 1

- 1) Find the next term in the sequence:

2, 5, 10, 17, 26

+3 +5 +7 +9

+2 +2 +2

Next term = 26 + 11 = 37

- 2) Find the next term in the sequence:

1, 4, 9, 16, 25

+3 +5 +7 +9

+2 +2 +2

Next term = 25 + 11 = 36

- 3) Find the next term in the sequence:

3, 8, 14, 21, 29

+5 +6 +7 +8

+1 +1 +1

Next term = 29 + 9 = 38

- 4) Find the next term in the sequence:

5, 9, 16, 26, 39

+4 +7 +10 +13

+3 +3 +3

Next term = 39 + 16 = 55

- 5) Find the next term in the sequence:

10, 7, 0, -11, -26

-3 -7 -11 -15

-4 -4 -4

Next term = -26 - 19 = -45

Task 2

- 6) Find the first 5 terms of the sequence given by

$$a_n = n^2 + 2n$$

n = 1: $(1)^2 + 2(1) = 3$

n = 2: $(2)^2 + 2(2) = 8$

n = 3: $(3)^2 + 2(3) = 15$

n = 4: $(4)^2 + 2(4) = 24$

n = 5: $(5)^2 + 2(5) = 35$

3, 8, 15, 24, 35

7) Find the first 5 terms of the sequence given by

$$a_n = n^2 - n$$

$$n = 1: 1^2 - 1 = 0$$

$$n = 2: 2^2 - 2 = 2$$

$$n = 3: 3^2 - 3 = 6$$

$$n = 4: 4^2 - 4 = 12$$

$$n = 5: 5^2 - 5 = 20$$

0, 2, 6, 12, 20

8) Find the first 5 terms of the sequence given by

$$a_n = 2n^2$$

$$n = 1: 2(1)^2 = 2$$

$$n = 2: 2(2)^2 = 8$$

$$n = 3: 2(3)^2 = 18$$

$$n = 4: 2(4)^2 = 32$$

$$n = 5: 2(5)^2 = 50$$

2, 8, 18, 32, 50

9) Find the first 5 terms of the sequence given by

$$a_n = 4n^2 + 3$$

$$n = 1: 4(1)^2 + 3 = 7$$

$$n = 2: 4(2)^2 + 3 = 19$$

$$n = 3: 4(3)^2 + 3 = 39$$

$$n = 4: 4(4)^2 + 3 = 67$$

$$n = 5: 4(5)^2 + 3 = 103$$

7, 19, 39, 67, 103

10) Find the first 5 terms of the sequence given by

$$a_n = -n^2 + 6n + 1$$

$$n = 1: -(1)^2 + 6(1) + 1 = 6$$

$$n = 2: -(2)^2 + 6(2) + 1 = 9$$

$$n = 3: -(3)^2 + 6(3) + 1 = 10$$

$$n = 4: -(4)^2 + 6(4) + 1 = 9$$

$$n = 5: -(5)^2 + 6(5) + 1 = 6$$

6, 9, 10, 9, 6

Task 3

11) Find the n^{th} term of the quadratic sequence:

2, 5, 10, 17, 26

2	5	10	17	26
	3	5	7	9
		2	2	2

$$2a = 2$$

$$a = 1$$

$$3a + b = 3$$

$$3(1) + b = 3$$

$$b = 0$$

$$a + b + c = 2$$

$$1 + 0 + c = 2$$

$$c = 1$$

$$n^2 + 1$$

12) Find the n^{th} term of the quadratic sequence:

3, 8, 15, 24, 35

3	8	15	24	35
	5	7	9	11
		2	2	2

$$2a = 2$$

$$a = 1$$

$$3a + b = 5$$

$$3(1) + b = 5$$

$$b = 2$$

$$a + b + c = 3$$

$$1 + 2 + c = 3$$

$$c = 0$$

$$n^2 + 2n$$

13) Find the n^{th} term of the quadratic sequence:

3, 9, 19, 33, 51

3	9	19	33	51
	6	10	14	18
		4	4	4

$$2a = 4$$

$$a = 2$$

$$3a + b = 6$$

$$3(2) + b = 6$$

$$b = 0$$

$$a + b + c = 3$$

$$2 + 0 + c = 3$$

$$c = 1$$

$$2n^2 + 1$$

14) Find the n^{th} term of the quadratic sequence:

4, 9, 18, 31, 48

4	9	18	31	48
	5	9	13	17
		4	4	4

$$2a = 4$$

$$a = 2$$

$$3a + b = 5$$

$$3(2) + b = 5$$

$$b = -1$$

$$a + b + c = 4$$

$$2 - 1 + c = 4$$

$$c + 1 = 4$$

$$c = 3$$

$$2n^2 - n + 3$$

15) Find the n^{th} term of the quadratic sequence:

5, 11, 23, 41, 65

5	11	23	41	65
	6	12	18	24
		6	6	6

$$2a = 6$$

$$a = 3$$

$$3a + b = 6$$

$$3(3) + b = 6$$

$$b = -3$$

$$a + b + c = 5$$

$$3 - 3 + c = 5$$

$$c = 5$$

$$3n^2 - 3n + 5$$

16) Find the n^{th} term of the quadratic sequence:

2, 1, -2, -7, -14

2	1	-2	-7	-14
	-1	-3	-5	-7
		-2	-2	-2

$$2a = -2$$

$$a = -1$$

$$3a + b = -1$$

$$3(-1) + b = -1$$

$$b = 2$$

$$a + b + c = 2$$

$$-1 + 2 + c = 2$$

$$c + 1 = 2$$

$$c = 1$$

$$-n^2 + 2n + 1$$

17) Find the n^{th} term of the quadratic sequence:

6, 7, 14, 27, 46

6	7	14	27	46
	1	7	13	19
		6	6	6

$$2a = 6$$

$$a = 3$$

$$3a + b = 1$$

$$3(3) + b = 1$$

$$b = -8$$

$$a + b + c = 6$$

$$3 - 8 + c = 6$$

$$c - 5 = 6$$

$$c = 11$$

$$3n^2 - 8n + 11$$

18) Find the n^{th} term of the quadratic sequence:

0, -3, -9, -18, -30

0	-3	-9	-18	-30
	-3	-6	-9	-12
		-3	-3	-3

$$2a = -3$$

$$a = -1.5$$

$$3a + b = -3$$

$$3(-1.5) + b = -3$$

$$-4.5 + b = -3$$

$$b = 1.5$$

$$a + b + c = 0$$

$$-1.5 + 1.5 + c = 0$$

$$c = 0$$

$$-1.5n^2 + 1.5n$$

19) a. Find the n^{th} term of the quadratic sequence:

3, 7, 13, 21, 31

$$\begin{array}{cccccc} 3 & 7 & 13 & 21 & 31 & \\ & 4 & 6 & 8 & 10 & \\ & & 2 & 2 & 2 & \end{array}$$

$$2a = 2$$

$$a = 1$$

$$3a + b = 4$$

$$3(1) + b = 4$$

$$b = 1$$

$$a + b + c = 3$$

$$1 + 1 + c = 3$$

$$c = 1$$

$$n^2 + n + 1$$

b. Is 111 a term in the sequence?

$$n^2 + n + 1 = 111$$

$$n^2 + n - 110 = 0$$

$$(n + 11)(n - 10) = 0$$

$$\cancel{n = -11} \text{ or } n = 10$$

Yes, it is the 10th term in the sequence.

20) a. Find the n^{th} term of the quadratic sequence:

10, 7, 2, -5, -14

$$\begin{array}{cccccc} 10 & 7 & 2 & -5 & -14 & \\ & -3 & -5 & -7 & -9 & \\ & & -2 & -2 & -2 & \end{array}$$

$$2a = -2$$

$$a = -1$$

$$3a + b = -3$$

$$3(-1) + b = -3$$

$$b = 0$$

$$a + b + c = 10$$

$$-1 + 0 + c = 10$$

$$c = 11$$

$$-n^2 + 11$$

b. Is -133 a term in the sequence?

$$-n^2 + 11 = -133$$

$$n^2 - 144 = 0$$

$$(n + 12)(n - 12) = 0$$

$$\cancel{n = -12} \text{ or } n = 12$$

Yes, it is the 12th term in the sequence.

Challenge

21) Two quadratic sequences have n^{th} terms:

Sequence A: $u_n = an^2 + bn + c$

Sequence B: $v_n = 2n^2 - n + 3$

The 5th, 10th and 14th terms of Sequence A are 57, 212 and 408 respectively.

Find the values of a , b , and c .

Hence, find the value of n for which $u_n = v_n$.

Substitute $n = 5, 10, 14$ into $u_n = an^2 + bn + c$

For $n = 5$:

$$a(5)^2 + b(5) + c = 57$$

$$25a + 5b + c = 57 \quad \dots(1)$$

For $n = 10$:

$$a(10)^2 + b(10) + c = 212$$

$$100a + 10b + c = 212 \quad \dots(2)$$

For $n = 14$:

$$a(14)^2 + b(14) + c = 408$$

$$196a + 14b + c = 408 \quad \dots(3)$$

Subtract (1) from (2):

$$100a + 10b + c = 212$$

$$\underline{-(25a + 5b + c = 57)}$$

$$75a + 5b = 155 \quad \dots(4)$$

Subtract (2) from (3):

$$196a + 14b + c = 408$$

$$\underline{-(100a + 10b + c = 212)}$$

$$96a + 4b = 196 \quad \dots(5)$$

Multiply (4) by 4, multiply (5) by 5, and subtract:

$$480a + 20b = 980$$

$$\underline{-(300a + 20b = 620)}$$

$$180a = 360$$

$$a = 2$$

Substitute into (4):

$$75(2) + 5b = 155$$

$$150 + 5b = 155$$

$$5b = 5$$

$$b = 1$$

Substitute into (1):

$$25(2) + 5(1) + c = 57$$

$$c + 55 = 57$$

$$c = 2$$

$$u_n = 2n^2 + n + 2$$

Set $u_n = v_n$:

$$2n^2 + n + 2 = 2n^2 - n + 3$$

$$n + 2 = -n + 3$$

$$2n = 1$$

$$n = \frac{1}{2}$$

$$a = 2, b = 1, c = 2 \quad n = \frac{1}{2}$$