

- 1) A particle moves in a straight line such that its displacement, s meters, from a fixed point O , at time t seconds is given by $s = t^2 + 4t + 3$. Find the velocity of the particle when $t = 3$.

$$v = \frac{ds}{dt} = 2t + 4$$

When $t = 3$:

$$v = 2(3) + 4 = \mathbf{10 \text{ m/s}}$$

- 2) A particle moves in a straight line such that its displacement, s metres, from a fixed point O , at time t seconds is given by $s = 3t^2 + 2t + 1$. Find the velocity of the particle when $t = 2$.

$$v = \frac{ds}{dt} = 6t + 2$$

When $t = 2$:

$$v = 6(2) + 2 = \mathbf{14 \text{ m/s}}$$

- 3) The displacement of a particle is given by $s = t^3 - 6t^2 + 9t$. Work out the acceleration of the particle after 4 seconds.

$$v = \frac{ds}{dt} = 3t^2 - 12t + 9$$

$$a = \frac{dv}{dt} = 6t - 12$$

When $t = 4$:

$$a = 6(4) - 12 = \mathbf{12 \text{ m/s}^2}$$

- 4) The displacement of a particle is given by $s = 2t^3 - 9t^2 + 12t$. Work out the acceleration of the particle when $t = 1$.

$$v = \frac{ds}{dt} = 6t^2 - 18t + 12$$

$$a = \frac{dv}{dt} = 12t - 18$$

When $t = 1$:

$$a = 12(1) - 18 = \mathbf{-6 \text{ m/s}^2}$$

- 5) A particle's velocity is given by $v = 10t - 2t^2$. Work out the acceleration of the particle when $t = 3$.

$$a = \frac{dv}{dt} = 10 - 4t$$

When $t = 3$:

$$a = 10 - 12 = \mathbf{-2 \text{ m/s}^2}$$

- 6) A car's position is given by the formula $s = 5t^2 - 20t$. At what time is the car instantaneously at rest?

The car is at rest when $v = 0$

$$v = \frac{ds}{dt} = 10t - 20$$

$$10t - 20 = 0$$

$$10t = 20$$

$$t = 2 \text{ seconds}$$

- 7) A car's displacement is given by $s = 4t^2 - 24t + 5$. At what time is the car instantaneously at rest?

The car is at rest when $v = 0$

$$v = \frac{ds}{dt} = 8t - 24$$

$$8t - 24 = 0$$

$$8t = 24$$

$$t = 3 \text{ seconds}$$

- 8) A particle's velocity is given by $v = 12t - 3t^2$. Find the acceleration of the particle when $t = 1$.

$$a = \frac{dv}{dt} = 12 - 6t$$

When $t = 1$:

$$a = 12 - 6(1) = 6 \text{ m/s}^2$$

- 9) A particle moves in a straight line, such that its displacement, s metres, from a fixed point O , at time t seconds, is given by $s = t^3 - 4t + 5$. Work out the displacement of the particle when the velocity is 8 m/s.

$$v = \frac{ds}{dt} = 3t^2 - 4$$

$$3t^2 - 4 = 8$$

$$3t^2 = 12$$

$$t^2 = 4$$

$$t = \cancel{-2} \text{ or } t = 2$$

$$s = (2)^3 - 4(2) + 5 = 5 \text{ m}$$

- 10) A particle moves such that its displacement is given by $s = t^2 - 16t + 2$. Find the time when the velocity of the particle is 6 m/s.

$$v = \frac{ds}{dt} = 2t - 16$$

$$2t - 16 = 6$$

$$2t = 22$$

$$t = 11 \text{ seconds}$$

- 11) The displacement of a particle, at time t seconds is given by the equation $s = 5t^2 - 4t + 7$. Work out the velocity of the particle when $t = 4$.

$$v = \frac{ds}{dt} = 10t - 4$$

When $t = 4$:

$$v = 10(4) - 4 = 36 \text{ m/s}$$

- 12) A particle's velocity is given by $v = 6t^2 - 12t$. Work out the time(s) when the particle is at rest.

The particle is at rest when $v = 0$

$$6t^2 - 12t = 0$$

$$6t(t - 2) = 0$$

$$t = 0 \quad \text{or} \quad t = 2$$

- 13) A particle has velocity $v = kt^2 - 4t$. If the acceleration of the particle is 8 m/s^2 when $t = 2$, find the value of k .

$$a = \frac{dv}{dt} = 2kt - 4$$

When $t = 2, a = 8$:

$$2k(2) - 4 = 8$$

$$4k = 12$$

$$k = 3$$

- 14) An object has displacement $s = t^3 + at^2 + bt$. Initially, its velocity is 15 m/s and its acceleration is -10 m/s^2 . Work out the values of a and b .

$$v = \frac{ds}{dt} = 3t^2 + 2at + b$$

$$3(0)^2 + 2a(0) + b = 15$$

$$b = 15$$

$$a = \frac{dv}{dt} = 6t + 2a$$

$$6(0) + 2a = -10$$

$$a = -5$$

$$a = -5 \quad \text{and} \quad b = 15$$

Challenge

15) A projectile follows the path $s = 40t - 5t^2$. Find the maximum height reached.

Max height reached when $v = 0$:

$$v = \frac{ds}{dt} = 40 - 10t$$

$$40 - 10t = 0$$

$$10t = 40$$

$$t = 4$$

Substitute $t = 4$ into s :

$$s = 40(4) - 5(4)^2 = \mathbf{80\ m}$$

16) A point moves such that $s = t^3 - 9t^2 + 24t$. Find the time interval when the particle is moving in the negative direction.

Moving negatively means $v < 0$

$$v = 3t^2 - 18t + 24$$

$$3t^2 - 18t + 24 < 0$$

$$t^2 - 6t + 8 < 0$$

$$(t - 2)(t - 4) < 0$$

The particle moves negatively when $2 < t < 4$

17) Two particles P and Q have displacements $s_P = t^2 + 2t$ and $s_Q = t^3 - 4t + 10$. At what time do the particles have the same velocity?

$$v_P = 2t + 2$$

$$v_Q = 3t^2 - 4$$

$$3t^2 - 4 = 2t + 2$$

$$3t^2 - 2t - 6 = 0$$

$$t = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$$

Disregard negative time:

$$t = \mathbf{1.79\ s\ (3\ sf)}$$