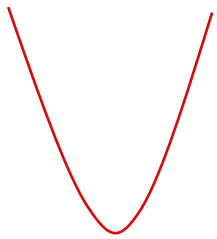
STATIONARY POINTS

1) Find the coordinates of the stationary point on the curve
$y = x^2 + 4x + 1$
By sketching the graph, determine whether it is a maximum or minimum.

$\frac{dy}{dx} = 2x + 4$



$2x + 4 = 0$

$2x = -4$

$x = -2$

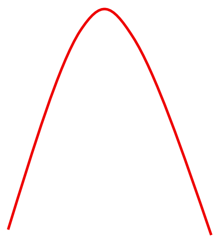
$y = (-2)^2 + 4(-2) + 1 = -3$

$(\mathbf{-2}, \mathbf{-3})$

Minimum

2) Find the coordinates of the stationary point on the curve
$y = -3x^2 + 6x - 2$
By sketching the graph, determine whether it is a maximum or minimum.

$\frac{dy}{dx} = -6x + 6$



$-6x + 6 = 0$

$-6x = -6$

$x = 1$

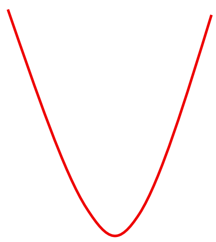
$y = -3(1)^2 + 6(1) - 2 = 1$

$(\mathbf{1}, \mathbf{1})$

Maximum

3) Find the coordinates of the stationary point on the curve
$y = 2x^2 - 8x + 3$
By sketching the graph, determine whether it is a maximum or minimum.

$\frac{dy}{dx} = 4x - 8$



$4x - 8 = 0$

$x = 2$

$y = 2(2)^2 - 8(2) + 3 = -5$

$(\mathbf{2}, \mathbf{-5})$

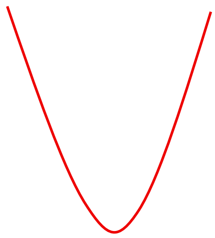
Minimum

4) Find the coordinates of the stationary point on the curve
$y = 3x^2 + 12x + 7$
By sketching the graph, determine whether it is a maximum or minimum.

$\frac{dy}{dx} = 6x + 12$

$6x + 12 = 0$
$x = -2$

$y = 3(-2)^2 + 12(-2) + 7 = -5$



$(\mathbf{-2}, \mathbf{-5})$

Minimum

5) Find the coordinates of the stationary point on the curve with equation
$y = 4x^2 + \frac{1}{x}$
$y = 4x^2 + x^{-1}$

$\frac{dy}{dx} = 8x - x^{-2} = 0$

$8x = \frac{1}{x^2}$

$8x^3 = 1$

$x^3 = \frac{1}{8}$

$x = \frac{1}{2}$

$y = 4\left(\frac{1}{2}\right)^2 + \frac{1}{\frac{1}{2}} = 3$

$\left(\mathbf{\frac{1}{2}}, \mathbf{3}\right)$

6) Find the coordinates of the stationary points on the curve
$y = x^3 - 3x^2 - 9x$
$\frac{dy}{dx} = 3x^2 - 6x - 9$

$3x^2 - 6x - 9 = 0$
$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$
$x = 3$ or $x = -1$

$y(3) = 27 - 27 - 27 = -27$
$y(-1) = -1 - 3 + 9 = 5$

Stationary Points:
$(\mathbf{3}, \mathbf{-27})$ **and** $(\mathbf{-1}, \mathbf{5})$

7) Find the coordinates of the stationary points on the curve
$y = 2x^3 + x^2 - 4x$

$\frac{dy}{dx} = 6x^2 + 2x - 4$

$6x^2 + 2x - 4 = 0$
$3x^2 + x - 2 = 0$

$(3x - 2)(x + 1) = 0$
$x = \frac{2}{3}$ or $x = -1$

$y = 2\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) = -\frac{44}{27}$
$y = 2(-1)^3 + (-1)^2 - 4(-1) = 3$

Stationary Points:
$\left(\frac{2}{3}, -\frac{44}{27}\right)$ **and** $(-1, 3)$

8) A rectangle has side lengths x and $10 - x$.
Work out the value of x that maximises the area.
$A = x(10 - x) = 10x - x^2$

$\frac{dA}{dx} = 10 - 2x$

$10 - 2x = 0$

$\boldsymbol{x = 5}$

9) The surface area of a cuboid is
$A = 4x(10 - x)$
Find the value of x that maximises the surface area.
$A = 4x(10 - x) = 40x - 4x^2$

$\frac{dA}{dx} = 40 - 8x$

$40 - 8x = 0$

$40 = 8x$

$\boldsymbol{x = 5}$

10) The area of a rectangle is given by
$A = (x + 1)(8 - x)$
Work out the maximum area of the rectangle.
$A = -x^2 + 7x + 8$

$\frac{dA}{dx} = -2x + 7$

$-2x + 7 = 0$

$x = 3.5$

$A = -3.5^2 + 7(3.5) + 8 = 20.25$

Maximum Area: $\boldsymbol{20.25\ units^2}$

11) The volume of a box is given by
$V = x(6-x)^2$
Work out the value of x that maximises the volume.
Then, work out the maximum volume.
$V = x(36 - 12x + x^2) = 36x - 12x^2 + x^3$
$\frac{dV}{dx} = 36 - 24x + 3x^2$

$3x^2 - 24x + 36 = 0$
$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 2$ or ~~$x = 6$~~

$\boldsymbol{x = 2}$

Maximum volume $= \mathbf{2(6-2)^2 = 32}\ \boldsymbol{units^3}$

12) The area of a rectangle is given by
$A = (3x-2)(8-x)$
Find the maximum area.
$A = -3x^2 + 26x - 16$

$\frac{dA}{dx} = -6x + 26$

$-6x + 26 = 0$
$x = \frac{13}{3}$

$A = \left(3\left(\frac{13}{3}\right) - 2\right)\left(8 - \frac{13}{3}\right) = \frac{121}{3}$

Maximum Area: $\mathbf{\frac{121}{3}}\ \boldsymbol{units^2}$

13) A curve has equation $y = x^2 + ax + 5$.
The stationary point of the curve lies on the line $y = 1$.
Find the value of a.
$\frac{dy}{dx} = 2x + a$

Stationary point:
$2x + a = 0$

$x = -\frac{a}{2}$

Substitute into y:
$\left(-\frac{a}{2}\right)^2 + a\left(-\frac{a}{2}\right) + 5 = 1$

$\frac{a^2}{4} - \frac{a^2}{2} + 5 = 1$

$-\frac{a^2}{4} + 5 = 1$

$-\frac{a^2}{4} = -4$

$a^2 = 16$

$\boldsymbol{a = \pm 4}$