GRADIENTS OF CURVES

Task 1

1) The curve has C has equation

$$y = 3x^2 - 5x + 2$$

a. Find $\frac{dy}{dx}$

$$\boldsymbol{\frac{dy}{dx} = 6x - 5}$$

b. The gradient of the curve when $x = 2$.

$$\frac{dy}{dx} = 6(2) - 5 = \mathbf{7}$$

2) The curve has C has equation

$$y = x^3 + 4x - 1$$

a. Find $\frac{dy}{dx}$

$$\boldsymbol{\frac{dy}{dx} = 3x^2 + 4}$$

b. The gradient of the curve when $x = -1$.

$$\frac{dy}{dx} = 3(-1)^2 + 4 = \mathbf{7}$$

3) The curve has C has equation

$$y = 10 - \frac{1}{2}x^2$$

a. Find $\frac{dy}{dx}$

$$\boldsymbol{\frac{dy}{dx} = -x}$$

b. The gradient of the curve when $x = 3$.

$$\frac{dy}{dx} = \mathbf{-3}$$

4) The curve has C has equation

$$y = \frac{1}{x} + 2x$$

a. Find $\frac{dy}{dx}$

$$y = x^{-1} + 2x$$

$$\boldsymbol{\frac{dy}{dx} = -x^{-2} + 2}$$

b. The gradient of the curve when $x = 1$.

$$\frac{dy}{dx} = -1^{-2} + 2 = \mathbf{1}$$

5) The curve has C has equation

$$y = \sqrt{x} + x^2$$

a. Find $\frac{dy}{dx}$

$$y = x^{\frac{1}{2}} + x^2$$

$$\boldsymbol{\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 2x}$$

b. The gradient of the curve when $x = 4$.

$$\frac{dy}{dx} = \frac{1}{2\times 2} + 2(4) = \mathbf{8.25}$$

6) The curve has C has equation

$$y = (x + 2)(x - 3)$$

a. Find $\frac{dy}{dx}$

$$y = x^2 - x - 6$$

$$\boldsymbol{\frac{dy}{dx} = 2x - 1}$$

b. The gradient of the curve when $x = 0$.

$$\frac{dy}{dx} = 2(0) - 1 = \mathbf{-1}$$

7) The curve has C has equation

$$y = 2x^4 - \frac{1}{3}x^3$$

a. Find $\frac{dy}{dx}$

$$\boldsymbol{\frac{dy}{dx} = 8x^3 - x^2}$$

b. The gradient of the curve when $x = 1$.

$$\frac{dy}{dx} = 8(1)^3 - 1^2 = \mathbf{7}$$

8) The curve has C has equation

$$y = \frac{x^3 + 4x}{x}$$

a. Find $\frac{dy}{dx}$

$$y = x^2 + 4$$

$$\boldsymbol{\frac{dy}{dx} = 2x}$$

b. The gradient of the curve when $x = 5$.

$$\frac{dy}{dx} = 2(5) = \mathbf{10}$$

9) The curve has C has equation

$$y = 5x^2 - 7$$

a. Find $\frac{dy}{dx}$

$\boldsymbol{\frac{dy}{dx} = 10x}$

b. The gradient of the curve when $x = -\frac{1}{2}$.

$\frac{dy}{dx} = 10\left(-\frac{1}{2}\right) = \mathbf{-5}$

10) The curve has C has equation

$$y = x^2 - \frac{2}{x^2}$$

a. Find $\frac{dy}{dx}$

$y = x^2 - 2x^{-2}$

$\boldsymbol{\frac{dy}{dx} = 2x + 4x^{-3}}$

b. The gradient of the curve when $x = 1$.

$\frac{dy}{dx} = 2(1) + 4(1)^{-3} = \mathbf{6}$

Task 2

11) Find the coordinates of the point on the curve with equation

$$y = x^2 - 4x + 5$$

where the gradient is 2.

$\frac{dy}{dx} = 2x - 4 = 2$

$2x = 6$

$x = 3$

$y = 3^2 - 4(3) + 5 = 2$

$\mathbf{(3, 2)}$

12) Find the coordinates of the point on the curve with equation

$$y = x^2 + 6x$$

where the gradient is 0.

$\frac{dy}{dx} = 2x + 6 = 0$

$2x = -6$

$x = -3$

$y = (-3)^2 + 6(-3) = -9$

$\mathbf{(-3, -9)}$

13) Find the coordinates of the points on the curve with equation

$$y = \frac{1}{3}x^3 - 4x$$

where the gradient is 5.

$\frac{dy}{dx} = x^2 - 4 = 5$

$x^2 = 9$

$x = \pm 3$

$y = \frac{1}{3}(3)^3 - 4(3) = -3$

$y = \frac{1}{3}(-3)^3 - 4(-3) = 3$

$\mathbf{(3, -3)}$ **and** $\mathbf{(-3, 3)}$

14) Find the coordinates of the points on the curve with equation

$$y = x^3 - 3x^2 + 1$$

where the gradient is 0.

$\frac{dy}{dx} = 3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ **or** $x = 2$

$y = 0^3 - 3(0)^2 + 1 = 1$

$y = 2^3 - 3(2)^2 + 1 = -3$

$\mathbf{(0, 1)}$ **and** $\mathbf{(2, -3)}$

15) Find the coordinates of the point on the curve with equation

$$y = 5x^2 - 8x$$

where the gradient is 12.

$\frac{dy}{dx} = 10x - 8 = 12$

$10x = 20$

$x = 2$

$y = 5(4)^2 - 8(2) = 48$

$\mathbf{(2, 48)}$

16) Find the coordinates of the point on the curve with equation

$$y = \sqrt{x}$$

where the gradient is $\frac{1}{4}$.

$y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{4}$

$\frac{1}{2\sqrt{x}} = \frac{1}{4}$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 4$

$y = \sqrt{4} = 2$

$\mathbf{(4, 2)}$

17) Find the coordinates of the point on the curve with equation

$$y = x^2 + x$$

where the gradient is 1.

$\frac{dy}{dx} = 2x + 1 = 1$

$2x = 0$

$x = 0$

$y = 0^2 + 0 = 0$

$\mathbf{(0, 0)}$

18) Find the coordinates of the points on the curve with equation

$$y = 12x - x^3$$

where the gradient is 0.

$\frac{dy}{dx} = 12 - 3x^2 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$y = 12(2) - 2^3 = 16$

$y = 12(-2) - (-2)^3 = -16$

$\mathbf{(2, 16)}$ **and** $\mathbf{(-2, -16)}$

19) Find the coordinates of the point on the curve with equation

$$y = \frac{4}{x}$$

where the gradient is -1.

$y = 4x^{-1}$

$\frac{dy}{dx} = -4x^{-2} = -1$

$\frac{-4}{x^2} = -1$

$4 = x^2$

$x = \pm 2$

$y = \frac{4}{2} = 2$

$y = \frac{4}{-2} = -2$

$\mathbf{(2, 2)}$ **and** $\mathbf{(-2, -2)}$

20) Find the coordinates of the point on the curve with equation

$$y = (x - 1)^2$$

where the gradient is 4.

$y = (x - 1)(x - 1) = x^2 - 2x + 1$

$\frac{dy}{dx} = 2(x - 1) = 4$

$x - 1 = 2$

$x = 3$

$y = (3 - 1)^2 = 4$

$\mathbf{(3, 4)}$

Task 3

21) Find the range of values of x, for which the curve with equation

$$y = x^2 - 6x + 8$$

has a positive gradient.

$\frac{dy}{dx} = 2x - 6$

$2x - 6 > 0$

$2x > 6$

$\boldsymbol{x > 3}$

22) Find the range of values of x, for which the curve with equation

$$y = x^3 - 3x^2 - 9x + 5$$

has a negative gradient.

$\frac{dy}{dx} = 3x^2 - 6x - 9$

$3x^2 - 6x - 9 < 0$

$x^2 - 2x - 3 < 0$

$(x - 3)(x + 1) < 0$

$x = 3 \quad or \quad x = -1$

$\boldsymbol{-1 < x < 3}$

23) Find the range of values of x, for which the curve with equation

$y = 4x - \frac{1}{3}x^3$

has a positive gradient.

$\frac{dy}{dx} = 4 - x^2$

$4 - x^2 > 0$

$x^2 - 4 < 0$

$(x + 2)(x - 2) < 0$

$x = -2 \quad or \quad x = 2$

$\boldsymbol{-2 < x < 2}$

24) Find the range of values of x, for which the curve with equation

$y = x^3 + 6x^2$

has a negative gradient.

$\frac{dy}{dx} = 3x^2 + 12x$

$3x^2 + 12x < 0$

$3x(x + 4) < 0$

$x = 0 \quad or \quad x = -4$

$\boldsymbol{-4 < x < 0}$

25) Find the range of values of x, for which the curve with equation

$$y = \frac{1}{3}x^3 - x^2 - 8x + 2$$

has a positive gradient.

$\frac{dy}{dx} = x^2 - 2x - 8$

$x^2 - 2x - 8 > 0$

$(x - 4)(x + 2) > 0$

$x = 4 \quad or \quad x = -2$

$\boldsymbol{x < -2 \quad or \quad x > 4}$

Challenge

26) The curve $y = ax^2 + bx$ has a gradient of 11 at the point $(2, 10)$. Work out the values of a and b.

Passes through (2, 10):

$a(2)^2 + b(2) = 10$

$4a + 2b = 10$

Gradient of 11 at (2, 10):

$\frac{dy}{dx} = 2ax + b$

$2a(2) + b = 11$

$4a + b = 11$

$$\begin{array}{r} 4a + 2b = 10 \\ \underline{-(4a + b) = 11} \\ b = -1 \end{array}$$

$4a + 2(-1) = 10$

$4a - 2 = 10$

$4a = 12$

$a = 3$

$\boldsymbol{a = 3 \quad and \quad b = -1}$

27) A curve has equation $y = \frac{k}{x} + x^2$. Given that the gradient when $x = 2$ is 0, find k and the coordinates of the turning point.

$y = kx^{-1} + x^2$

$\frac{dy}{dx} = -kx^{-2} + 2x$

$-k(2)^{-2} + 2(2) = 0$

$-\frac{k}{4} + 4 = 0$

$\frac{k}{4} = 4$

$k = 16$

Turning point when $\frac{dy}{dx} = 0$

Happens when $x = 2$:

$y = \frac{16}{2} + 2^2 = 8 + 4 = 12$

$\boldsymbol{k = 16}$

Turning Point: $\boldsymbol{(2, 12)}$