HARDER INVERSE FUNCTIONS

1) Given that

$f(x) = x^2 + 4x - 9$ where $x \geq -2$

a. Write down the range of $f^{-1}(x)$.

$\boldsymbol{f^{-1}(x) \geq -2}$

b. Express the inverse function in the form $f^{-1}(x)$.

$y = x^2 + 4x - 9$

$x = y^2 + 4x - 9$

$x = \left(y + \frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 9$

$x = (y + 2)^2 - 2^2 - 9$

$x = (y + 2)^2 - 13$

$(y + 2)^2 = x + 13$

$y + 2 = \pm\sqrt{x + 13}$

$y = -2 \pm \sqrt{x + 13}$

Because $x \geq -2$:

$\boldsymbol{f^{-1}(x) = -2 + \sqrt{x + 13}}$

2) Given that

$g(x) = x^2 - 6x + 8$ where $x \geq 3$

a. Write down the range of $g^{-1}(x)$.

$\boldsymbol{g^{-1}(x) \geq 3}$

b. Express the inverse function in the form $g^{-1}(x)$.

$y = x^2 - 6x + 8$

$x = y^2 - 6x + 8$

$x = \left(y - \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 8$

$x = (y - 3)^2 - 3^2 + 8$

$x = (y - 3)^2 - 1$

$(y - 3)^2 = x + 1$

$y - 3 = \pm\sqrt{x + 1}$

$y = 3 \pm \sqrt{x + 1}$

Because $x \geq 3$:

$\boldsymbol{g^{-1}(x) = 3 + \sqrt{x + 1}}$

3) Given that

$$f(x) = x^2 + 2x - 3 \quad \text{where} \quad x \geq -1$$

a. Write down the range of $f^{-1}(x)$.

$f^{-1}(x) \geq -1$

b. Express the inverse function in the form $f^{-1}(x)$.

$y = x^2 + 2x - 3$

$x = y^2 + 2y - 3$

$x = \left(y + \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 3$

$x = (y + 1)^2 - 1^2 - 3$

$x = (y + 1)^2 - 4$

$(y + 1)^2 = x + 4$

$y + 1 = \pm\sqrt{x + 4}$

$y = -1 \pm \sqrt{x + 4}$

Because $x \geq -1$:

$\boldsymbol{f^{-1}(x) = -1 + \sqrt{x + 4}}$

4) Given that

$$h(x) = x^2 - 8x + 10 \quad \text{where} \quad x \leq 4$$

a. Write down the range of $h^{-1}(x)$.

$h^{-1}(x) \leq 4$

b. Express the inverse function in the form $h^{-1} : x \rightarrow \cdots$.

$y = x^2 - 8x + 10$

$x = y^2 - 8y + 10$

$x = \left(y - \frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 10$

$x = (y - 4)^2 - 4^2 + 10$

$x = (y - 4)^2 - 6$

$(y - 4)^2 = x + 6$

$y - 4 = \pm\sqrt{x + 6}$

$y = 4 \pm \sqrt{x + 6}$

Because $x \leq 4$:

$h^{-1} : x \rightarrow \boldsymbol{4 - \sqrt{x + 6}}$

c. State the range of values of x that cannot be included in the domain of h^{-1}.

$\boldsymbol{x < -6}$

5) Given that

$$f(x) = 2x^2 + 4x - 5 \quad \text{where} \quad x \geq -1$$

a. Write down the range of $f^{-1}(x)$.

$\boldsymbol{f^{-1}(x) \geq -1}$

b. Express the inverse function in the form $f^{-1} : x \to \cdots$

$y = 2x^2 + 4x - 5$

$x = 2y^2 + 4y - 5$

$x = 2[y^2 + 2y] - 5$

$$x = 2\left[\left(y + \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right] - 5$$

$x = 2[(y + 1)^2 - 1^2] - 5$

$x = 2(y + 1)^2 - 2 - 5$

$x = 2(y + 1)^2 - 7$

$2(y + 1)^2 = x + 7$

$(y + 1)^2 = x + 7$

$$y + 1 = \pm\sqrt{\frac{x + 7}{2}}$$

$$y = -1 \pm \sqrt{\frac{x + 7}{2}}$$

Since $x \geq -1$:

$$f^{-1} : x \to \boldsymbol{-1 + \sqrt{\frac{x + 7}{2}}}$$

c. State the range of values of x that cannot be included in the domain of $f^{-1}(x)$.

$\boldsymbol{x < -7}$

6) Given that

$g(x) = 3x^2 - 6x + 1$ where $x \leq 1$

a. Write down the range of $g^{-1}(x)$.

$g^{-1}(x) \leq 1$

b. Express the inverse function in the form $g^{-1}(x)$.

$y = 3x^2 - 6x + 1$

$x = 3y^2 - 6y + 1$

$x = 3[y^2 - 2y] + 1$

$x = 3\left[\left(y - \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right] + 1$

$x = 3[(y-1)^2 - 1] + 1$

$x = 3(y-1)^2 - 3 + 1$

$x = 3(y-1)^2 - 2$

$3(y-1)^2 = x + 2$

$(y-1)^2 = \frac{x+2}{3}$

$y - 1 = \pm\sqrt{\frac{x+2}{3}}$

$y = 1 \pm \sqrt{\frac{x+2}{3}}$

Since $x \leq 1$:

$\boldsymbol{g^{-1}(x) = 1 - \sqrt{\frac{x+2}{3}}}$

7) Given that

$f(x) = x^2 + 6x + 5$ where $x \geq -3$

Solve $f^{-1}(x) = 4$

$y = x^2 + 6x + 5$

$x = y^2 + 6y + 5$

$x = \left(y + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 5$

$x = (y+3)^2 - 3^2 + 5$

$x = (y+3)^2 - 4$

$(y+3)^2 = x + 4$

$y + 3 = \pm\sqrt{x+4}$

$y = -3 \pm \sqrt{x+4}$

Because $x \geq -3$:

$f^{-1}(x) = -3 + \sqrt{x+4}$

Solve:

$-3 + \sqrt{x+4} = 4$

$\sqrt{x+4} = 7$

$x + 4 = 49$

$\boldsymbol{x = 45}$

8) **Given that**

$f(x) = 2x^2 + 12x - 6$ **where** $x \geq -3$

Solve $f^{-1}(x) = 2$

$y = 2x^2 + 12x - 6$

$x = 2y^2 + 12y - 6$

$x = 2(y^2 + 6y) - 6$

$$x = 2\left[\left(y + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right] - 6$$

$x = 2[(y + 3)^2 - 3^2] - 6$

$x = 2(y + 3)^2 - 18 - 6$

$x = 2(y + 3)^2 - 24$

$2(y + 3)^2 = x + 24$

$$(y + 3)^2 = \frac{x + 24}{2}$$

$$y + 3 = \pm\sqrt{\frac{x + 24}{2}}$$

$$y = -3 \pm \sqrt{\frac{x + 24}{2}}$$

Because $x \geq -3$**:**

$$f^{-1}(x) = -3 + \sqrt{\frac{x + 24}{2}}$$

Solve:

$$-3 + \sqrt{\frac{x + 24}{2}} = 2$$

$$\sqrt{\frac{x + 24}{2}} = 5$$

$$\frac{x + 24}{2} = 25$$

$x + 24 = 50$

$\boldsymbol{x = 26}$