GENMATHS RESOURCES

INVERSE FUNCTIONS

1) Given that $f(x) = 2x + 3$, work out $f^{-1}(x)$.

$y = 2x + 3$

Swap x and y:

$x = 2y + 3$

Make y the subject:

$2y = x - 3$

$y = \frac{x-3}{2}$

$\boldsymbol{f^{-1}(x) = \frac{x-3}{2}}$

2) Given that $f(x) = \frac{x-4}{5}$, work out $f^{-1}(x)$.

$y = \frac{x-4}{5}$

Swap x and y:

$x = \frac{y-4}{5}$

Make y the subject:

$5x = y - 4$

$y = 5x + 4$

$\boldsymbol{f^{-1}(x) = 5x + 4}$

3) Given that $f(x) = \frac{1}{2}x - 6$, work out $f^{-1}(x)$.

$y = \frac{1}{2}x - 6$

Swap x and y:

$x = \frac{1}{2}y - 6$

Make y the subject:

$\frac{1}{2}y = x + 6$

$y = 2x + 12$

$\boldsymbol{f^{-1}(x) = 2x + 12}$

4) Find $f^{-1}(7)$, given that $f(x) = 3x + 1$.

$y = 3x + 1$

Swap x and y:

$x = 3y + 1$

Make y the subject:

$3y = x - 1$

$$y = \frac{x - 1}{3}$$

$$f^{-1}(x) = \frac{x - 1}{3}$$

$$\boldsymbol{f^{-1}(7) = \frac{7 - 1}{3} = 2}$$

5) Given that $g(x) = \frac{3x+2}{4}$, work out $g^{-1}(8)$.

$$y = \frac{3x + 2}{4}$$

Swap x and y:

$$x = \frac{3y + 2}{4}$$

Make y the subject:

$4x = 3y + 2$

$3y = 4x - 2$

$$y = \frac{4x - 2}{3}$$

$$g^{-1}(x) = \frac{4x - 2}{3}$$

$$\boldsymbol{g^{-1}(8) = \frac{4(8) - 2}{3} = 10}$$

6) Given that $f(x) = \frac{2x+1}{x+3}$, work out $f^{-1}(x)$.

$$y = \frac{2x + 1}{x + 3}$$

Swap x and y:

$$x = \frac{2y + 1}{y + 3}$$

Make y the subject:

$x(y + 3) = 2y + 1$

$xy + 3x = 2y + 1$

$xy - 2y = 1 - 3x$

$y(x - 2) = 1 - 3x$

$$y = \frac{1 - 3x}{x - 2}$$

$$\boldsymbol{f^{-1}(x) = \frac{1 - 3x}{x - 2}}$$

7) Given that $f(x) = 2x - 5$, solve $f^{-1}(x) = f(x)$.

$y = 2x - 5$

$x = 2y - 5$

$2y = x + 5$

$y = \frac{x+5}{2}$

$f^{-1}(x) = f(x)$

$\frac{x+5}{2} = 2x - 5$

$x + 5 = 4x - 10$

$3x = 15$

$\boldsymbol{x = 5}$

8) Given that $g(x) = \frac{x+1}{3}$, solve $g^{-1}(x) = g(x)$.

$y = \frac{x+1}{3}$

$x = \frac{y+1}{3}$

$3x = y + 1$

$y = 3x - 1$

$g^{-1}(x) = g(x)$

$3x - 1 = \frac{x+1}{3}$

$9x - 3 = x + 1$

$8x = 4$

$\boldsymbol{x = \frac{1}{2}}$

9) Given that $f(x) = \frac{4x-7}{2}$, solve $f^{-1}(x) = f(x)$.

$y = \frac{4x-7}{2}$

$x = \frac{4y-7}{2}$

$2x = 4y - 7$

$4y = 2x + 7$

$y = \frac{2x+7}{4}$

$f^{-1}(x) = f(x)$

$\frac{2x+7}{4} = \frac{4x-7}{2}$

$\times 4 \qquad \times 4$

$2x + 7 = 2(4x - 7)$

$2x + 7 = 8x - 14$

$6x = 21$

$\boldsymbol{x = \frac{21}{6} = \frac{7}{2}}$

10) Given that

$f(x) = \sqrt{x+5}$

$g(x) = 1 - x$

Solve $f^{-1}(x) = g^{-1}(x)$, given that $x \geq 0$.

$y = \sqrt{x+5}$

$x = \sqrt{y+5}$

$x^2 = y + 5$

$y = x^2 - 5$

$y = 1 - x$

$x = 1 - y$

$y = 1 - x$

$f^{-1}(x) = g^{-1}(x)$

$x^2 - 5 = 1 - x$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

~~$\boldsymbol{x = -3}$~~ $\quad \boldsymbol{or} \quad \boldsymbol{x = 2}$

11) Given that

$f(x) = \frac{1}{x} \quad x \neq 0$

$g(x) = 4x + 3$

Solve $f^{-1}(x) = g^{-1}(x)$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$xy = 1$

$y = \frac{1}{x}$

$y = 4x + 3$

$x = 4y + 3$

$4y = x - 3$

$y = \frac{x-3}{4}$

$f^{-1}(x) = g^{-1}(x)$

$\frac{1}{x} = \frac{x-3}{4}$

$1(4) = x(x-3)$

$4 = x^2 - 3x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boldsymbol{x = 4} \quad \boldsymbol{or} \quad \boldsymbol{x = -1}$

12) Given that $f(x) = \frac{x-1}{x-2}$ $\quad x \neq 2$

Work out the value of x such that $f^{-1}(x) = f(x)$.

$$y = \frac{x-1}{x-2}$$

$$x = \frac{y-1}{y-2}$$

$$x(y-2) = y-1$$

$$xy - 2x = y - 1$$

$$xy - y = 2x - 1$$

$$y(x-1) = 2x - 1$$

$$y = \frac{2x-1}{x-1}$$

$$f^{-1}(x) = \frac{2x-1}{x-1}$$

$$f^{-1}(x) = f(x)$$

$$\frac{2x-1}{x-1} = \frac{x-1}{x-2}$$

$$(2x-1)(x-2) = (x-1)(x-1)$$

$$2x^2 - 4x - x + 2 = x^2 - x - x + 1$$

$$2x^2 - 5x + 2 = x^2 - 2x + 1$$

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$\boldsymbol{x = \frac{3 \pm \sqrt{5}}{2}}$$

Challenge

13) a. Find the inverse of $f(x) = \frac{5x+3}{x-4}$.

$$y = \frac{5x+3}{x-4}$$

$$x = \frac{5y+3}{y-4}$$

$$x(y-4) = 5y + 3$$

$$xy - 4x = 5y + 3$$

$$xy - 5y = 4x + 3$$

$$y(x-5) = 4x + 3$$

$$y = \frac{4x+3}{x-5}$$

$$\boldsymbol{f^{-1}(x) = \frac{4x+3}{x-5}}$$

b. Show that $f(f^{-1}(x)) = x$.

$$f(f^{-1}(x))$$

$$= \frac{5\left(\frac{4x+3}{x-5}\right)+3}{\left(\frac{4x+3}{x-5}\right)-4}$$

$$= \frac{\frac{20x+1}{x-5}+\frac{3(x-5)}{x-5}}{\left(\frac{4x+3}{x-5}\right)-\frac{4(x-5)}{x-5}}$$

$$= \frac{\frac{20x+15+3x-15}{x-5}}{\frac{4x+3-4(x-5)}{x-5}}$$

$$= \frac{\frac{23x}{x-5}}{\frac{23}{x-5}}$$

$$= \frac{23x}{x-5} \times \frac{x-5}{23}$$

$$= x$$

14) Given $f(x) = x^2 + 2x + 1$, where $x \geq -1$ Find $f^{-1}(x)$.

$$y = x^2 + 2x + 1$$

$$x = y^2 + 2y + 1$$

$$x = \left(y + \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + 1$$

$$x = (y+1)^2 - 1^2 + 1$$

$$x = (y+1)^2$$

$$y + 1 = \pm\sqrt{x}$$

$$y = -1 \pm \sqrt{x}$$

$$\boldsymbol{f^{-1}(x) = -1 + \sqrt{x}} \qquad (f^{-1}(x) \geq -1)$$

15) Given $f(x) = 3x^2 + 6x + 7$, where $x \geq -1$ Find $f^{-1}(x)$.

$$y = 3x^2 + 6x + 7$$

$$x = 3y^2 + 6y + 7$$

$$x = 3[y^2 + 2y] + 7$$

$$x = 3\left[\left(y + \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right] + 7$$

$$x = 3(y+1)^2 - 3 + 7$$

$$3(y+1)^2 = x - 4$$

$$(y+1)^2 = \frac{x-4}{3}$$

$$y + 1 = \pm\sqrt{\frac{x-4}{3}}$$

$$y = -1 \pm \sqrt{\frac{x-4}{3}}$$

$$\boldsymbol{f^{-1}(x) = -1 + \sqrt{\frac{x-4}{3}}} \qquad (f^{-1}(x) \geq -1)$$