GENMATHS RESOURCES

COMPOSITE FUNCTIONS

Task 1

1) Given that
$f(x) = 2x + 3$ $g(x) = x^2$
Find $fg(2)$
$g(2) = 2^2 = 4$
$f(4) = 2(4) + 3 = \mathbf{11}$

2) Given that
$f(x) = x - 4$ $g(x) = 3x$
Find $fg(5)$
$g(5) = 3(5) = 15$
$f(15) = 15 - 4 = \mathbf{11}$

3) Given that
$f(x) = x^2 + 1$ $g(x) = x + 2$
Find $gf(1)$
$f(1) = 1^2 + 1 = 2$
$g(2) = 2 + 2 = \mathbf{4}$

4) Given that
$f(x) = 5x$ $g(x) = x^2 - 3$
Find $fg(2)$
$g(2) = 2^2 - 3 = 1$
$f(1) = 5(1) = \mathbf{5}$

5) Given that
$f(x) = x^2$ $g(x) = 2x + 1$
Find $gf(-3)$
$f(-3) = (-3)^2 = 9$
$g(9) = 2(9) + 1 = \mathbf{19}$

6) Given that
$f(x) = \frac{x}{4} + 1$ $g(x) = x^2$
Find $fg(4)$
$g(4) = 4^2 = 16$
$f(16) = \frac{16}{4} + 1 = \mathbf{5}$

7) Given that
$f(x) = \frac{x+7}{2}$ $g(x) = x^2 - 2x$
Find $gf(-1)$
$f(-1) = \frac{-1+7}{2} = 3$
$g(3) = 3^2 - 2(3) = \mathbf{3}$

8) Given that
$f(x) = 2x^2$ $g(x) = x + 3$
Find $fg(2)$
$g(2) = 2 + 3 = 5$
$f(5) = 2(5)^2 = \mathbf{50}$

9) Given that
$f(x) = x^2 + 8$ $g(x) = 4x$
Find $fg\left(\frac{1}{2}\right)$
$g\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right) = 2$
$f(2) = 2^2 + 8 = \mathbf{12}$

10) Given that
$f(x) = \sqrt{x+1}$ $g(x) = x^2$
Find $fg(3)$
$g(3) = 3^2 = 9$
$f(9) = \sqrt{9+1} = \sqrt{\mathbf{10}}$

Task 2

11) Given that
$f(x) = 2x + 1$ $g(x) = x^2$
Find $fg(x)$
$\boldsymbol{f(x^2) = 2x^2 + 1}$

12) Given that
$f(x) = x - 3$ $g(x) = 4x$
Find $gf(x)$
$g(x-3) = 4(x-3) = \mathbf{4x - 12}$

13) Given that
$f(x) = x^2$ $g(x) = x + 5$
Find $fg(x)$
$f(x+5) = (x+5)^2 = \mathbf{x^2 + 10x + 25}$

14) Given that
$f(x) = 3x$ $g(x) = x^2 - 1$
Find $gf(x)$
$g(3x) = (3x)^2 - 1 = \mathbf{9x^2 - 1}$

15) Given that
$f(x) = x + 2$ $g(x) = x^2$
Find $gf(x)$
$g(x+2) = (x+2)^2 = \mathbf{x^2 + 4x + 4}$

16) Given that

$f(x) = 2x - 1$ $\quad$ $g(x) = x^2 + 3x$

Find $fg(x)$

$f(x^2 + 3x) = 2(x^2 + 3x) - 1$

$= \mathbf{2x^2 + 6x - 1}$

17) Given that

$f(x) = x^2 + 1$ $\quad$ $g(x) = 2x - 3$

Find $gf(x)$

$g(x^2 + 1) = 2(x^2 + 1) - 3$

$= 2x^2 + 2 - 3$

$= \mathbf{2x^2 - 1}$

18) Given that

$f(x) = \frac{1}{x}$ $\quad$ $g(x) = x + 2$

Find $fg(x)$

$f(x + 2) = \mathbf{\frac{1}{x + 2}}$

19) Given that

$f(x) = \sqrt{x}$ $\quad$ $g(x) = x^2 + 4$

Find $fg(x)$

$f(x^2 + 4) = \mathbf{\sqrt{x^2 + 4}}$

20) Given that

$f(x) = x^2 - 2x$ $\quad$ $g(x) = x - 1$

Find $fg(x)$

$f(x - 1) = (x - 1)^2 - 2(x - 1)$

$= (x - 1)(x - 1) - 2x + 2$

$= x^2 - x - x + 1 - 2x + 2$

$= \mathbf{x^2 - 4x + 3}$

Task 3

21) Given that

$f(x) = 2x + 1$ $\quad$ $g(x) = x^2$

Solve $fg(x) = 9$

$2x^2 + 1 = 9$

$2x^2 = 8$

$x^2 = 4$

$\mathbf{x = \pm 2}$

22) Given that

$f(x) = x - 3$ $\quad$ $g(x) = x^2$

Solve $fg(x) = 13$

$x^2 - 3 = 13$

$x^2 = 16$

$\mathbf{x = \pm 4}$

23) Given that

$f(x) = \frac{2}{x}$ $\quad$ $g(x) = x + 1$

Solve $gf(x) = 10$

$\frac{2}{x} + 1 = 10$

$\frac{2}{x} = 9$

$2 = 9x$

$\mathbf{x = \frac{2}{9}}$

24) Given that

$f(x) = 3x$ $\quad$ $g(x) = x^2 - 1$

Solve $gf(x) = 8$

$(3x)^2 - 1 = 8$

$9x^2 - 1 = 8$

$9x^2 = 9$

$x^2 = 1$

$\mathbf{x = \pm 1}$

25) Given that

$f(x) = 3x + 5$ $\quad$ $g(x) = \frac{4}{x+6}$

Solve $gf(x) = 1$

$\frac{4}{3x + 5 + 6} = 1$

$\frac{4}{3x + 11} = 1$

$4 = 3x + 11$

$3x = -7$

$\mathbf{x = -\frac{7}{3}}$

26) Given that

$f(x) = 2x + 4$ $\quad$ $g(x) = 3x + 5$

Solve $f(x) = fg(x)$

$2x + 4 = 2(3x + 5) + 4$

$2x + 4 = 6x + 10 + 4$

$2x + 4 = 6x + 14$

$-4x = 10$

$\mathbf{x = -\frac{10}{4} = -\frac{5}{2}}$

27) Given that

$f(x) = 2x - 1 \qquad g(x) = \frac{3x}{8} + 9$

Solve $fg(x) = g(x)$

$2\left(\frac{3x}{8} + 9\right) - 1 = \frac{3x}{8} + 9$

$\frac{6x}{8} + 18 - 1 = \frac{3x}{8} + 9$

$\frac{6x}{8} + 17 = \frac{3x}{8} + 9$

$\frac{3x}{8} = -8$

$3x = -64$

$\boldsymbol{x = -\frac{64}{3}}$

28) Given that

$f(x) = \frac{1}{x-2} \qquad g(x) = x + 3$

Solve $fg(x) = gf(x)$

$\frac{1}{x+3-2} = \frac{1}{x-2} + 3$

$\frac{1}{x+1} = \frac{1}{x-2} + 3$

$\frac{1}{x+1} - \frac{1}{x-2} = 3$

$\frac{x-2}{(x+1)(x-2)} - \frac{x+1}{(x+1)(x-2)} = 3$

$x - 2 - (x+1) = 3(x+1)(x-2)$

$-3 = 3(x^2 - 2x + x - 2)$

$-3 = 3(x^2 - x - 2)$

$-3 = 3x^2 - 3x - 6$

$3x^2 - 3x - 3 = 0$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - (4 \times 1 \times -1)}}{2(1)}$

$\boldsymbol{x = \frac{1 \pm \sqrt{5}}{2}}$

29) Given that

$f(x) = \frac{1}{x} \qquad g(x) = x - 2$

Solve $fg(x) = gf(x)$

$\frac{1}{x-2} = \frac{1}{x} - 2$

$\frac{x}{x(x-2)} = \frac{x-2}{x(x-2)} - \frac{2x(x-2)}{x(x-2)}$

$x = x - 2 - 2x(x-2)$

$x = x - 2 - 2x^2 + 4x$

$x = -2x^2 + 5x - 2$

$2x^2 - 4x + 2 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$\boldsymbol{x = 1}$

Challenge

30) $f(x) = 3x + 2$ **and** $g(x) = ax + 1$

Work out the value of a **such that**

$fg(x) = gf(x)$

$3(ax + 1) + 2 = a(3x + 2) + 1$

$3ax + 3 + 2 = 3ax + 2a + 1$

$3ax + 5 = 3ax + 2a + 1$

$5 = 2a + 1$

$4 = 2a$

$\boldsymbol{a = 2}$

31) $f(x) = \frac{1}{x+1}$ **and** $g(x) = \frac{1}{x-1}$

Show that $fg(x) = \frac{x-1}{x}$

$fg(x) = \frac{1}{\frac{1}{x-1} + 1}$

Multiply all terms by $(x-1)$**:**

$= \frac{x-1}{1 + (x-1)}$

$= \frac{x-1}{x}$

Alternate method:

$fg(x) = \frac{1}{\frac{1}{x-1} + 1}$

$= \frac{1}{\frac{1}{x-1} + \frac{x-1}{x-1}}$

$= \frac{1}{\frac{1 + x - 1}{x-1}}$

$= \frac{1}{\frac{x}{x-1}}$

$= \frac{x-1}{x}$