SOLVING ALGEBRAIC FRACTIONS

Task 1 – Solve each of the following equations. Show clear algebraic working. Do not use trial and error.

1) $\frac{x}{4}+\frac{x}{3}=7$

$\frac{3x}{12}+\frac{4x}{12}=7$

$\frac{7x}{12}=7$

$7x=84$

$\boldsymbol{x=12}$

2) $\frac{2x-1}{3}+\frac{x}{6}=5$

$\frac{2(2x-1)}{6}+\frac{x}{6}=5$

$\frac{2(2x-1)+x}{6}=5$

$\frac{5x-2}{6}=5$

$5x-2=30$

$5x=32$

$\boldsymbol{x=\frac{32}{5}}$

3) $\frac{4}{5x}+\frac{5}{2x}=12$

$\frac{8}{10x}+\frac{25}{10x}=12$

$\frac{33}{10x}=12$

$33=120x$

$\boldsymbol{x=\frac{33}{120}=\frac{11}{40}}$

4) $\frac{x+2}{5}-\frac{x-1}{2}=0$

$\frac{2(x+2)}{10}-\frac{5(x-1)}{10}=0$

$\frac{2(x+2)-5(x-1)}{10}=0$

$\frac{2x+4-5x+5}{10}=0$

$-3x+9=0$

$3x=9$

$\boldsymbol{x=3}$

5) $\frac{10}{y+1}-7=2$

$\frac{10}{y+1}=9$

$10=9(y+1)$

$10=9y+9$

$1=9y$

$\boldsymbol{y=\frac{1}{9}}$

6) $\frac{4}{z}+\frac{2}{3z}=7$

$\frac{12}{3z}+\frac{2}{3z}=7$

$\frac{14}{3z}=7$

$14=21z$

$\boldsymbol{z=\frac{14}{21}=\frac{2}{3}}$

7) $\frac{5}{x+2}=\frac{3}{x}$

Cross multiply

$5x=3(x+2)$

$5x=3x+6$

$2x=6$

$\boldsymbol{x=3}$

8) $\frac{a}{2}=\frac{8}{a}$

Cross multiply

$a^2=16$

$\boldsymbol{a=\pm 4}$

9) $b+\frac{6}{b}=5$

Multiply by b

$b^2+6=5b$

$b^2-5b+6=0$

$(b-2)(b-3)=0$

$\boldsymbol{b=2}$ or $\boldsymbol{b=3}$

10) $\frac{x+2}{3}-\frac{4-x}{8}=\frac{19}{15}$

$\frac{8(x+2)}{24}-\frac{3(4-x)}{24}=\frac{19}{15}$

$\frac{8(x+2)-3(4-x)}{24}=\frac{19}{15}$

$8(x+2)-3(4-x)=\frac{152}{5}$

$8x+16-12+3x=\frac{152}{5}$

$11x+4=\frac{152}{5}$

$11x=\frac{132}{5}$

$\boldsymbol{x=\frac{12}{5}}$

11) $\frac{1}{x-3}+\frac{1}{x+3}=\frac{10}{x^2-9}$

$\frac{x+3}{(x+3)(x-3)}+\frac{x-3}{(x+3)(x-3)}=\frac{10}{(x+3)(x-3)}$

$(x+3)+(x-3)=10$

$2x=10$

$\boldsymbol{x=5}$

12) $\frac{2-3w}{2}=\frac{1}{5}-\frac{3w-2}{3}$

$\frac{15(2-3w)}{30}=\frac{6}{30}-\frac{10(3w-2)}{30}$

$15(2-3w)=6-10(3w-2)$

$30-45w=6-30w+20$

$30-45w=-30w+26$

$4=15w$

$\boldsymbol{w=\frac{4}{15}}$

13) $\frac{6}{x+2}+\frac{4}{x+5}=1$

$\frac{6(x+5)}{(x+2)(x+5)}+\frac{4(x+2)}{(x+2)(x+5)}=1$

$6(x+5)+4(x+2)=(x+2)(x+5)$

$6x+30+4x+8=x^2+7x+10$

$10x+38=x^2+7x+10$

$x^2-3x-28=0$

$(x-7)(x+4)=0$

$\boldsymbol{x=7}$ or $\boldsymbol{x=-4}$

14) $\frac{3}{x+1}+\frac{2}{x+4}=1$

$\frac{3(x+4)}{(x+1)(x+4)}+\frac{2(x+1)}{(x+1)(x+4)}=1$

$3(x+4)+2(x+1)=1(x+1)(x+4)$

$3x+12+2x+2=x^2+4x+x+4$

$5x+14=x^2+5x+4$

$x^2-10=0$

$x^2=10$

$\boldsymbol{x=\pm\sqrt{10}}$

15) $\frac{4}{x+1}+\frac{6}{x+3}=2$

$\frac{4(x+3)}{(x+1)(x+3)}+\frac{6(x+1)}{(x+1)(x+3)}=2$

$4(x+3)+6(x+1)=2(x+1)(x+3)$

$4x+12+6x+6=2(x^2+3x+x+3)$

$10x+18=2x^2+8x+6$

$2x^2-2x-12=0$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$\boldsymbol{x=3}$ or $\boldsymbol{x=-2}$

16) $\frac{2}{x+1}+\frac{3}{x+2}=1$

$\frac{2(x+2)}{(x+1)(x+2)}+\frac{3(x+1)}{(x+1)(x+2)}=1$

$2(x+2)+3(x+1)=1(x+1)(x+2)$

$2x+4+3x+3=x^2+2x+x+2$

$5x+7=x^2+3x+2$

$x^2-2x-5=0$

$x=\frac{2\pm\sqrt{(-2)^2-4(1)(-5)}}{2}$

$\boldsymbol{x=1\pm\sqrt{6}}$

17) $\frac{x+3}{x-1}+\frac{x-2}{x+2}=3$

$\frac{(x+3)(x+2)}{(x-1)(x+2)}+\frac{(x-2)(x-1)}{(x-1)(x+2)}=3$

$(x+3)(x+2)+(x-2)(x-1)=3(x-1)(x+2)$

$(x^2+5x+6)+(x^2-3x+2)=3(x^2+x-2)$

$2x^2+2x+8=3x^2+3x-6$

$x^2+x-14=0$

$x=\frac{-1\pm\sqrt{1^2-4(1)(-14)}}{2}$

$\boldsymbol{x=\frac{-1\pm\sqrt{57}}{2}}$

18) $\frac{x^2-1}{x+1} = 5$

$\frac{(x+1)(x-1)}{x+1} = 5$ **Note:** $x \neq -1$

$x - 1 = 5$

$\boldsymbol{x = 6}$

19) $\frac{3x^2+14x-5}{3x-1} = 7$

$\frac{(3x-1)(x+5)}{3x-1} = 7$ **Note:** $x \neq \frac{1}{3}$

$x + 5 = 7$

$\boldsymbol{x = 2}$

Challenge – Solve each of the following equations. Show clear algebraic working. Do not use trial and error.

20) $\frac{1}{x-1} + \frac{2}{x-2} = \frac{3}{x-3}$

$\frac{1(x-2)(x-3)}{(x-1)(x-2)(x-3)} + \frac{2(x-1)(x-3)}{(x-1)(x-2)(x-3)} = \frac{3(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$(x-2)(x-3) + 2(x-1)(x-3) = 3(x-1)(x-2)$

$(x^2 - 5x + 6) + 2(x^2 - 4x + 3) = 3(x^2 - 3x + 2)$

$x^2 - 5x + 6 + 2x^2 - 8x + 6 = 3x^2 - 9x + 6$

$3x^2 - 13x + 12 = 3x^2 - 9x + 6$

$-13x + 12 = -9x + 6$

$6 = 4x$

$\boldsymbol{x = 1.5}$

21) $\frac{3}{x^2-3x} + \frac{4}{x^2-4x} = 0$

$\frac{3}{x(x-3)} + \frac{4}{x(x-4)} = 0$

$\frac{3(x-4)}{x(x-3)(x-4)} + \frac{4(x-3)}{x(x-3)(x-4)} = 0$

$3(x-4) + 4(x-3) = 0$

$3x - 12 + 4x - 12 = 0$

$7x - 24 = 0$

$7x = 24$

$\boldsymbol{x = \frac{24}{7}}$

22) $\frac{10x^2 - 10x}{5x^2 - x - 6} \div \frac{2x-2}{x-2} = 1$

$\frac{10x^2 - 10x}{5x^2 - x - 6} \times \frac{x-2}{2x-2} = 1$

$\frac{10x(x-1)}{(5x-6)(x+1)} \times \frac{x-2}{2(x-1)} = 1$

$\frac{10x}{(5x-6)(x+1)} \times \frac{x-2}{2} = 1$

$\frac{10x(x-2)}{2(5x-6)(x+1)} = 1$

$10x(x-2) = 2(5x-6)(x+1)$
$10x^2 - 20x = 2(5x^2 + 5x - 6x - 6)$
$10x^2 - 20x = 2(5x^2 - x - 6)$
$10x^2 - 20x = 10x^2 - 2x - 12$
$-20x = -2x - 12$
$18x = 12$
$\boldsymbol{x = \frac{2}{3}}$

23) $\frac{12x^2 - 12x}{3x^2 - x - 2} \div \frac{x-1}{x+1} + 8 = 10$

$\frac{12x^2 - 12x}{3x^2 - x - 2} \div \frac{x-1}{x+1} = 2$

$\frac{12x^2 - 12x}{3x^2 - x - 2} \times \frac{x+1}{x-1} = 2$

$\frac{12x(x-1)}{(3x+2)(x-1)} \times \frac{x+1}{x-1} = 2$

$\frac{12x}{3x+2} \times \frac{x+1}{x-1} = 2$

$\frac{12x(x+1)}{(3x+2)(x-1)} = 2$

$12x(x+1) = 2(3x+2)(x-1)$

$12x^2 + 12x = 2(3x^2 - x - 2)$
$12x^2 + 12x = 6x^2 - 2x - 4$
$6x^2 + 14x + 4 = 0$
$3x^2 + 7x + 2$
$(3x+1)(x+2) = 0$
$\boldsymbol{x = -\frac{1}{3}}$ **or** $\boldsymbol{x = -2}$

24)

$$\frac{6x^2-6x}{3x^2-x-2}-\left(\frac{1}{3x+2}+\frac{1}{x-1}\right)=\frac{1}{3x+2}$$

$$\frac{6x^2-6x}{3x^2-x-2}-\left(\frac{(x-1)+(3x+2)}{(3x+2)(x-1)}\right)=\frac{1}{3x+2}$$

$$\frac{6x^2-6x}{(3x+2)(x-1)}-\frac{4x+1}{(3x+2)(x-1)}=\frac{1}{3x+2}$$

$$\frac{6x^2-10x-1}{(3x-2)(x-1)}=\frac{1}{3x+2}$$

$$6x^2-10x-1=1(x-1)$$

$$6x^2-10x-1=x-1$$

$$6x^2-11x=0$$

$$x(6x-11)=0$$

$$\boldsymbol{x=0} \quad \text{or} \quad \boldsymbol{x=\frac{11}{6}}$$