



SIMPLIFYING ALGEBRAIC FRACTIONS

Task 1 – Fully simplify the following fractions.

$$1) \frac{12ab}{6} = 2ab$$

$$2) \frac{15x^2}{5x} = 3x$$

$$3) \frac{9x}{3x} = 3$$

$$4) \frac{14a^2}{7a} = 2a$$

$$5) \frac{20x^2y}{5xy} = 4x$$

$$6) \frac{18m}{6n} = \frac{3m}{n}$$

$$7) \frac{16x^2}{4x} = 4x$$

$$25) \frac{3x^2-3x}{x^2-4x+3} = \frac{3x(x-1)}{(x-1)(x-3)} = \frac{3x}{x-3}$$

$$26) \frac{4x^2-16}{2x^2+6x-20} = \frac{(2x+4)(2x-4)}{(2x-4)(x+5)} = \frac{2x+4}{x+5}$$

$$27) \frac{6x^2+19x+10}{4x^2-25} = \frac{(2x+5)(3x+2)}{(2x+5)(2x-5)} = \frac{3x+2}{2x-5}$$

$$28) \frac{6x^2-7x-3}{4x^2-9} = \frac{(3x+1)(2x-3)}{(2x+3)(2x-3)} = \frac{3x+1}{2x+3}$$

Task 2 – Fully simplify the following fractions.

$$8) \frac{6x^2+12x}{6x} = \frac{6x(x+2)}{6x} = x+2$$

$$9) \frac{8x+16}{4} = \frac{4(2x+4)}{4} = 2x+4$$

$$10) \frac{10x^2-5x}{5x} = \frac{5x(2x-1)}{5x} = 2x-1$$

$$11) \frac{12x^2y+6xy}{6xy} = \frac{6xy(2x+1)}{6xy} = 2x+1$$

$$12) \frac{15x^2+30x}{3x} = \frac{15x(x+2)}{3x} = 5(x+2) = 5x+10$$

Task 3 – Fully simplify the following fractions.

$$13) \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = x+3$$

$$14) \frac{x^2+5x}{x} = \frac{x(x+5)}{x} = x+5$$

$$15) \frac{2x^2-8}{2x-4} = \frac{2(x^2-4)}{2(x-2)} = \frac{2(x+2)(x-2)}{2(x-2)} = x+2$$

$$16) \frac{x^2-4}{x^2-x-6} = \frac{(x+2)(x-2)}{(x-3)(x+2)} = \frac{x-2}{x-3}$$

$$17) \frac{3x^2-12x}{3x^2} = \frac{3x(x-4)}{3x^2} = \frac{x-4}{x}$$

$$18) \frac{x^2-5x+6}{x^2-3x} = \frac{(x-2)(x-3)}{x(x-3)} = \frac{x-2}{x}$$

$$19) \frac{x^2-9}{x^2+2x-3} = \frac{(x+3)(x-3)}{(x+3)(x-1)} = \frac{x-3}{x-1}$$

$$20) \frac{x^2+10x+21}{x^2+3x} = \frac{(x+7)(x+3)}{x(x+3)} = \frac{x+7}{x}$$

$$21) \frac{x^2-x-72}{x^2+17x+72} = \frac{(x-9)(x+8)}{(x+9)(x+8)} = \frac{x-9}{x+9}$$

$$22) \frac{x^2+7x+10}{x^2+5x} = \frac{(x+5)(x+2)}{x(x+5)} = \frac{x+2}{x}$$

$$23) \frac{2x^2-6x}{x^2-x-6} = \frac{2x(x-3)}{(x-3)(x+2)} = \frac{2x}{x+2}$$

$$24) \frac{x^2+2x-24}{x^2-3x-4} = \frac{(x-4)(x+6)}{(x-4)(x+1)} = \frac{x+6}{x+1}$$

Task 4 – Write each of the following as a single fraction in simplest form.

$$\begin{aligned} 29) \frac{x}{3} + \frac{2x}{5} \\ &= \frac{5x}{15} + \frac{6x}{15} \\ &= \frac{5x+6x}{15} \\ &= \frac{11x}{15} \end{aligned}$$

$$\begin{aligned} 30) \frac{x}{x+2} + \frac{2}{x+1} \\ &= \frac{x(x+1)}{(x+1)(x+2)} + \frac{2(x+2)}{(x+1)(x+2)} \\ &= \frac{x^2+x+2x+4}{(x+1)(x+2)} \\ &= \frac{x^2+3x+4}{(x+1)(x+2)} \end{aligned}$$

$$\begin{aligned} 31) \frac{5x}{6} - \frac{x}{2} \\ &= \frac{5x}{6} - \frac{3x}{6} \\ &= \frac{5x-3x}{6} \\ &= \frac{2x}{6} \\ &= \frac{x}{3} \end{aligned}$$

$$\begin{aligned} 32) \frac{x+3}{x} - \frac{2}{x} \\ &= \frac{x+3-2}{x} \\ &= \frac{x+1}{x} \end{aligned}$$

$$33) \frac{3x}{4} \times \frac{8}{x^2}$$

$$= \frac{24x}{4x^2}$$

$$= \frac{6}{x}$$

$$34) \frac{x^2-4}{x} \times \frac{x}{x+2}$$

$$= \frac{(x+2)(x-2)}{x} \times \frac{x}{(x+2)}$$

$$= x-2$$

$$35) \frac{6x}{5} \div \frac{3}{x}$$

$$= \frac{6x}{5} \times \frac{x}{3}$$

$$= \frac{6x^2}{15}$$

$$= \frac{2x^2}{5}$$

$$36) \frac{x^2-9}{x} \div \frac{x-3}{x+1}$$

$$= \frac{x^2-9}{x} \times \frac{x+1}{x-3}$$

$$= \frac{(x+3)(x-3)}{x} \times \frac{x+1}{x-3}$$

$$= \frac{(x+3)(x+1)}{x}$$

$$37) \frac{3x+4}{2x} + \frac{4}{7}$$

$$= \frac{7(3x+4)}{14x} + \frac{8x}{14x}$$

$$= \frac{21x+28+8x}{14x}$$

$$= \frac{29x+28}{14x}$$

$$38) \frac{4}{3x} - \frac{3}{7x} + \frac{1}{x}$$

$$= \frac{28}{21x} - \frac{9}{21x} + \frac{21}{21x}$$

$$= \frac{40}{21x}$$

$$39) \frac{3x+1}{5x} + \frac{7-6x}{3x}$$

$$= \frac{3(3x+1)}{15x} + \frac{5(7-6x)}{15x}$$

$$= \frac{9x+3+35-30x}{15x}$$

$$= \frac{38-21x}{15x}$$

$$40) \frac{16-x^2}{2x^2+3x+1} \div \frac{x+4}{x+1}$$

$$= \frac{16-x^2}{2x^2+3x+1} \times \frac{x+1}{x+4}$$

$$= \frac{(4+x)(4-x)}{(2x+1)(x+1)} \times \frac{x+1}{x+4}$$

$$= \frac{4-x}{2x+1}$$

$$41) \frac{2}{x-2} - \frac{4}{x+1}$$

$$= \frac{2(x+1)}{(x-2)(x+1)} - \frac{4(x-2)}{(x-2)(x+1)}$$

$$= \frac{2x+2-4x+8}{(x-2)(x+1)}$$

$$= \frac{10-2x}{(x-2)(x+1)}$$

$$42) \frac{1}{4x^2-49} \div \frac{1}{4x+14}$$

$$= \frac{1}{4x^2-49} \times \frac{4x+14}{1}$$

$$= \frac{1}{(2x+7)(2x-7)} \times \frac{2(2x+7)}{1}$$

$$= \frac{2}{2x-7}$$

Challenge

43) Show that $\frac{6x^4}{4x^2-16} \div \frac{3x^5}{2x-4}$ can be written as

$\frac{1}{x(x+a)}$ where a is an integer.

$$= \frac{6x^4}{(2x+4)(2x-4)} \times \frac{2x-4}{3x^5}$$

$$= \frac{6x^4}{2x+4} \times \frac{1}{3x^5}$$

$$= \frac{2}{2x+4} \times \frac{1}{x}$$

$$= \frac{2}{2(x+2)} \times \frac{1}{x}$$

$$= \frac{1}{x(x+2)}$$

44) Show that

$$\frac{2}{x+3} \times \frac{x^2-9}{x^2+2x-15} \div \frac{5}{x+5}$$

can be written in the form $\frac{a}{b}$ where a and b are integers.

$$\begin{aligned} &= \frac{2}{x+3} \times \frac{(x+3)(x-3)}{(x+5)(x-3)} \times \frac{x+5}{5} \\ &= \frac{2}{5} \end{aligned}$$

45) Given that $m = \frac{20}{4x-9}$ and $x = \frac{10}{5y-2}$, express m in the form, $\frac{ay+b}{cy+d}$ where a, b, c and d are integers.

Give your answer in simplest form.

$$\begin{aligned} m &= \frac{20}{4\left(\frac{10}{5y-2}\right) - 9} \\ &= \frac{20}{\frac{40}{5y-2} - 9} \\ &= \frac{20}{\frac{40}{5y-2} - \frac{9(5y-2)}{5y-2}} \\ &= \frac{20}{\frac{40 - 45y + 18}{5y-2}} \\ &= \frac{20}{\frac{58 - 45y}{5y-2}} \\ &= 20 \times \frac{5y-2}{58-45y} \\ &= \frac{20(5y-2)}{58-45y} \\ &= \frac{100y-40}{58-45y} \end{aligned}$$

46) Show that

$$\frac{25x^2-81}{5x^2-4x-9} \times \frac{x^2-1}{5x+9} - (x-6)$$

can be expressed as a single integer.

$$\begin{aligned} &= \frac{(5x+9)(5x-9)}{(5x-9)(x+1)} \times \frac{(x+1)(x-1)}{5x+9} - (x-6) \\ &= (x-1) - (x-6) \\ &= 5 \end{aligned}$$

47) Show that $\frac{16xy-3x}{7xy} - \frac{2y-3}{7y} + \frac{1-4x}{2x}$ can be expressed as $\frac{1}{px}$ where p is an integer.

$$\begin{aligned} &= \frac{32xy-6x}{14xy} - \frac{4xy-6x}{14xy} + \frac{7y-28xy}{14xy} \\ &= \frac{32xy-6x-4xy+6x+7y-28xy}{14xy} \\ &= \frac{7y}{14xy} \\ &= \frac{7}{14x} \\ &= \frac{1}{2x} \end{aligned}$$

48) Show that $\frac{1}{5x-1} \times \frac{25x^2-1}{5x^2-24x-5} - \frac{3}{x-2}$ can be written as $\frac{a+bx}{x^2+cx+d}$ where a, b, c and d are integers.

$$\begin{aligned} &= \frac{1}{5x-1} \times \frac{(5x+1)(5x-1)}{(5x+1)(x-5)} - \frac{3}{x-2} \\ &= \frac{1}{x-5} - \frac{3}{x-2} \\ &= \frac{x-2}{(x-2)(x-5)} - \frac{3(x-5)}{(x-2)(x-5)} \\ &= \frac{13-2x}{x^2-5x-2x+10} \\ &= \frac{13-2x}{x^2-7x+10} \end{aligned}$$

49) Show that $\left(\frac{3}{2x-3} - \frac{2}{2x+1}\right) \div \frac{6x^2+29x+9}{6x^2+5x+1}$ can be written as $\frac{1}{ax+b}$ where a and b are integers.

$$\begin{aligned} &= \left(\frac{3}{2x-3} - \frac{2}{2x+1}\right) \times \frac{6x^2+5x+1}{6x^2+29x+9} \\ &= \left(\frac{3(2x+1)}{(2x+1)(2x-3)} - \frac{2(2x-3)}{(2x+1)(2x-3)}\right) \times \frac{(2x+1)(3x+1)}{(2x+9)(3x+1)} \\ &= \frac{3(2x+1) - 2(2x-3)}{(2x+1)(2x-3)} \times \frac{(2x+1)(3x+1)}{(2x+9)(3x+1)} \\ &= \frac{6x+3-4x+6}{(2x+1)(2x-3)} \times \frac{(2x+1)(3x+1)}{(2x+9)(3x+1)} \\ &= \frac{2x+9}{(2x+1)(2x-3)} \times \frac{(2x+1)(3x+1)}{(2x+9)(3x+1)} \\ &= \frac{1}{2x-3} \end{aligned}$$

50) Show that

$$\frac{4}{x-2} - \frac{1}{x+1} \div \frac{3x^2 - 8x + 4}{3x^2 + x - 2}$$

can be written in the form $\frac{a}{bx+c}$ where a, b and

c are integers.

$$= \frac{4}{x-2} - \frac{1}{x+1} \times \frac{3x^2 + x - 2}{3x^2 - 8x + 4}$$

$$= \frac{4}{x-2} - \frac{1}{x+1} \times \frac{(3x-2)(x+1)}{(3x-2)(x-2)}$$

$$= \frac{4}{x-2} - \frac{(3x-2)(x+1)}{(x+1)(3x-2)(x-2)}$$

$$= \frac{4}{x-2} - \frac{1}{x-2}$$

$$= \frac{3}{x-2}$$