REARRANGING HARDER FORMULAE

1) Make x the subject of

$$2x - 10 = bx$$
$$2x - bx = 10$$
$$x(2 - b) = 10$$
$$\boldsymbol{x = \frac{10}{2 - b}}$$

2) Make x the subject of

$$mx = n - px$$
$$mx + px = n$$
$$x(m + p) = n$$
$$\boldsymbol{x = \frac{n}{m + p}}$$

3) Make x the subject of

$$3(x + 1) = hx$$
$$3x + 3 = hx$$
$$hx - 3x = 3$$
$$x(h - 3) = 3$$
$$\boldsymbol{x = \frac{3}{h - 3}}$$

4) Make x the subject of

$$y + gx = ux + 2y$$
$$gx - ux = y$$
$$x(g - u) = y$$
$$\boldsymbol{x = \frac{y}{g - u}}$$

5) Make x the subject of

$$y - 5x = 3(x + 2)$$
$$y - 5x = 3x + 6$$
$$y = 8x + 6$$
$$8x = y - 6$$
$$\boldsymbol{x = \frac{y - 6}{8}}$$

6) Make y the subject of

$$a(y + 3) = b(y + 2)$$
$$ay + 3a = by + 2b$$
$$ay - by = 2b - 3a$$
$$y(a - b) = 2b - 3a$$
$$\boldsymbol{y = \frac{2b - 3a}{a - b}}$$

7) Make z the subject of

$$3(z - c) = 2(az - 4)$$
$$3z - 3c = 2az - 8$$
$$3z - 2az = 3c - 8$$
$$z(3 - 2a) = 3c - 8$$
$$\boldsymbol{z = \frac{3c - 8}{3 - 2a}}$$

8) Make x the subject of

$$xy = \frac{x + 2}{3}$$
$$3xy = x + 2$$
$$3xy - x = 2$$
$$x(3y - 1) = 2$$
$$\boldsymbol{x = \frac{2}{3y - 1}}$$

9) Make x the subject of

$$y = \frac{x + 1}{x - 2}$$
$$y(x - 2) = x + 1$$
$$xy - 2y = x + 1$$
$$xy - x = 2y + 1$$
$$x(y - 1) = 2y + 1$$
$$\boldsymbol{x = \frac{2y + 1}{y - 1}}$$

10) Make w the subject of

$$y = \frac{3w+2}{w-1}$$

$$y(w-1) = 3w+2$$

$$wy - y = 3w+2$$

$$wy - 3w = y+2$$

$$w(y-3) = y+2$$

$$\boldsymbol{w = \frac{y+2}{y-3}}$$

11) Make x the subject of

$$y = \frac{2x-3}{x+4}$$

$$y(x+4) = 2x-3$$

$$xy + 4y = 2x-3$$

$$xy - 2x = -4y-3$$

$$x(y-2) = -4y-3$$

$$\boldsymbol{x = \frac{-4y-3}{y-2} = \frac{4y+3}{2-y}}$$

12) Make a the subject of

$$b = \sqrt{\frac{a+1}{2a}}$$

$$b^2 = \frac{a+1}{2a}$$

$$2ab^2 = a+1$$

$$2ab^2 - a = 1$$

$$a(2b^2-1) = 1$$

$$\boldsymbol{a = \frac{1}{2b^2-1}}$$

13) Make x the subject of

$$3a = \sqrt{\frac{2x+3}{5x}}$$

$$9a^2 = \frac{2x+3}{5x}$$

$$45a^2x = 2x+3$$

$$45a^2x - 2x = 3$$

$$x(45a^2-2) = 3$$

$$\boldsymbol{x = \frac{3}{45a^2-2}}$$

14) Make x the subject of

$$y = \sqrt{\frac{x-4}{x+1}}$$

$$y^2 = \frac{x-4}{x+1}$$

$$y^2(x+1) = x-4$$

$$xy^2 + y^2 = x-4$$

$$xy^2 - x = -y^2-4$$

$$x(y^2-1) = -y^2-4$$

$$\boldsymbol{x = \frac{-y^2-4}{y^2-1} = \frac{y^2+4}{1-y^2}}$$

15) Make x the subject of

$$a = \frac{x-2}{2x+1}$$

$$a(2x+1) = x-2$$

$$2ax + a = x-2$$

$$2ax - x = -a-2$$

$$x(2a-1) = -a-2$$

$$\boldsymbol{x = \frac{-a-2}{2a-1} = \frac{a+2}{1-2a}}$$

16) Make c the subject of

$$\frac{1}{b} = \frac{1}{c} + \frac{1}{d}$$

Multiply all terms by b:

$$1 = \frac{b}{c} + \frac{b}{d}$$

Multiply all terms by c:

$$c = b + \frac{bc}{d}$$

Multiply all terms by d:

$$cd = bd + bc$$

$$cd - bc = bd$$

$$c(d-b) = bd$$

$$\boldsymbol{c = \frac{bd}{d-b}}$$

17) Make e the subject of $t = \frac{2e+7}{9-e}$

$t(9-e) = 2e+7$

$9t - et = 2e + 7$

$2e + et = 9t - 7$

$e(2+t) = 9t - 7$

$\boldsymbol{e = \frac{9t-7}{2+t}}$

18) Make p the subject of $n = \sqrt{\frac{8p+11}{10+2p}}$

$n^2 = \frac{8p+11}{10+2p}$

$n^2(10+2p) = 8p + 11$

$10n^2 + 2n^2p = 8p + 11$

$2n^2p - 8p = 11 - 10n^2$

$p(2n^2 - 8) = 11 - 10n^2$

$\boldsymbol{p = \frac{11-10n^2}{2n^2-8}}$

19) Make f the subject of $m = \sqrt{\frac{f+h}{fh-g}}$

$m^2 = \frac{f+h}{fh-g}$

$m^2(fh-g) = f + h$

$fhm^2 - gm^2 = f + h$

$fhm^2 - f = gm^2 + h$

$f(hm^2 - 1) = gm^2 + h$

$\boldsymbol{f = \frac{gm^2+h}{hm^2-1}}$

20) Make y the subject of

$\frac{m}{n} = \frac{4y}{y+9}$

$m(y+9) = 4ny$

$my + 9m = 4ny$

$4ny - my = 9m$

$y(4n - m) = 9m$

$\boldsymbol{y = \frac{9m}{4n-m}}$

21) Make b the subject of $d = \frac{b+6}{7+b} - 2$

$d + 2 = \frac{b+6}{7+b}$

$(d+2)(7+b) = b + 6$

$7d + bd + 14 + 2b = b + 6$

$bd + 2b - b = 6 - 7d - 14$

$bd + b = -7d - 8$

$b(d+1) = -7d - 8$

$\boldsymbol{b = \frac{-7d-8}{d+1}}$

22) Make m the subject of $r = \frac{7m^2+8}{10-2m^2}$

$r(10 - 2m^2) = 7m^2 + 8$

$10r - 2m^2r = 7m^2 + 8$

$7m^2 + 2m^2r = 10r - 8$

$m^2(7 + 2r) = 10r - 8$

$m^2 = \frac{10r-8}{7+2r}$

$\boldsymbol{m = \pm\sqrt{\frac{10r-8}{7+2r}}}$

Challenge

23) Make x the subject of

$x^2 + 8x - 10 = 25 + y$

$x^2 + 8x = y + 35$

$\left(x + \frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 = y + 35$

$(x+4)^2 - 4^2 = y + 35$

$(x+4)^2 - 16 = y + 35$

$(x+4)^2 = y + 51$

$x + 4 = \pm\sqrt{y+51}$

$\boldsymbol{x = -4 \pm \sqrt{y+51}}$

24) Make y the subject of

$3y^2 + 12y - b = 4a$

$3y^2 + 12y = 4a + b$

$3[y^2 + 4y] = 4a + b$

$3\left[\left(y + \frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2\right] = 4a + b$

$3[(y + 2)^2 - 2^2] = 4a + b$

$3(y + 2)^2 - 12 = 4a + b$

$3(y + 2)^2 = 4a + b + 12$

$(y + 2)^2 = \frac{4a + b + 12}{3}$

$y + 2 = \pm\sqrt{\frac{4a + b + 12}{3}}$

$\boldsymbol{y = -2 \pm \sqrt{\frac{4a + b + 12}{3}}}$

25) Make x the subject of

$x^2 - 4ax - 12 = ef + 10$

$x^2 - 4ax = ef + 22$

$\left(x - \frac{4a}{2}\right)^2 - \left(\frac{4a}{2}\right)^2 = ef + 22$

$(x - 2a)^2 - (2a)^2 = ef + 22$

$(x - 2a)^2 - 4a^2 = ef + 22$

$(x - 2a)^2 = 4a^2 + ef + 22$

$x - 2a = \pm\sqrt{4a^2 + ef + 22}$

$\boldsymbol{x = 2a \pm \sqrt{4a^2 + ef + 22}}$