EQUATIONS OF CIRCLES AND TANGENTS

Task 1 – For each of the following, state the centre and radius of the circle.

1) $x^2 + y^2 = 25$
   Centre (0, 0)
   Radius = 5

2) $x^2 + y^2 = 49$
   Centre (0, 0)
   Radius = 7

3) $x^2 + y^2 = 16$
   Centre (0, 0)
   Radius = 4

4) $x^2 + y^2 = 100$
   Centre (0, 0)
   Radius = 10

5) $x^2 + y^2 = 9$
   Centre (0, 0)
   Radius = 3

6) $x^2 + y^2 = 64$
   Centre (0, 0)
   Radius = 8

7) $(x - 2)^2 + (y + 4)^2 = 36$
   Centre (2, -4)
   Radius = 6

8) $(x - 1)^2 + (y - 7)^2 = 121$
   Centre (1, 7)
   Radius = 11

Task 2

9) A circle is centred at (0, 0) and has a radius of $\sqrt{3}$.
   State the equation of the circle.
   $\boldsymbol{x^2 + y^2 = 3}$

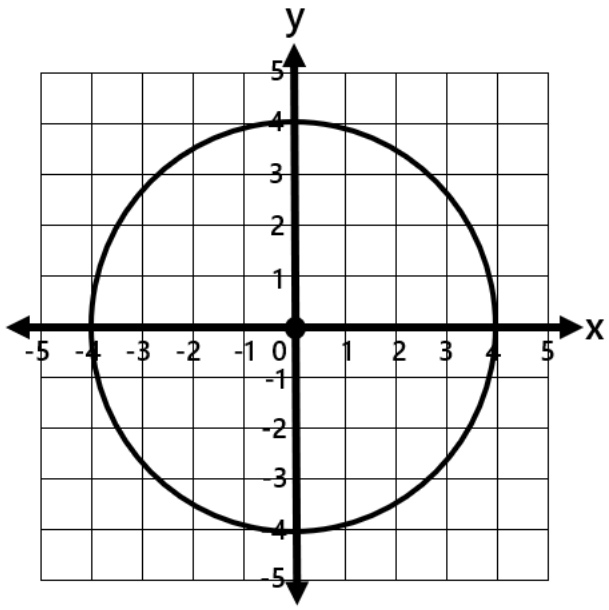
10) A circle is centred at (-8, -9) and has a radius of 9.
    State the equation of the circle.
    $\boldsymbol{(x + 8)^2 + (y + 9)^2 = 81}$

11) A circle is centred at (3, 0) and has a radius of $\sqrt{5}$.
    State the equation of the circle.
    $\boldsymbol{(x - 3)^2 + y^2 = 5}$

12) A circle is centred at (0, 0) and has a radius of 4.
State the equation of the circle.

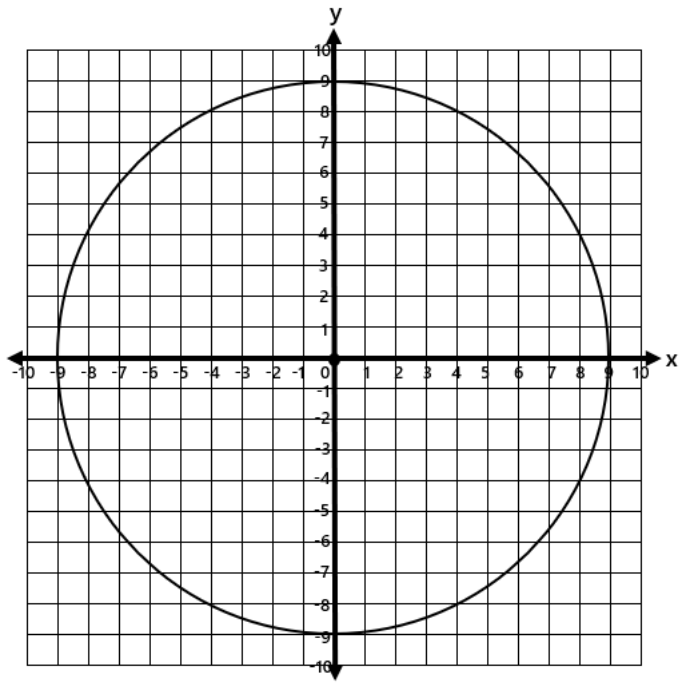
$x^2 + y^2 = 16$

13) Write down the equation of the circle pictured below.



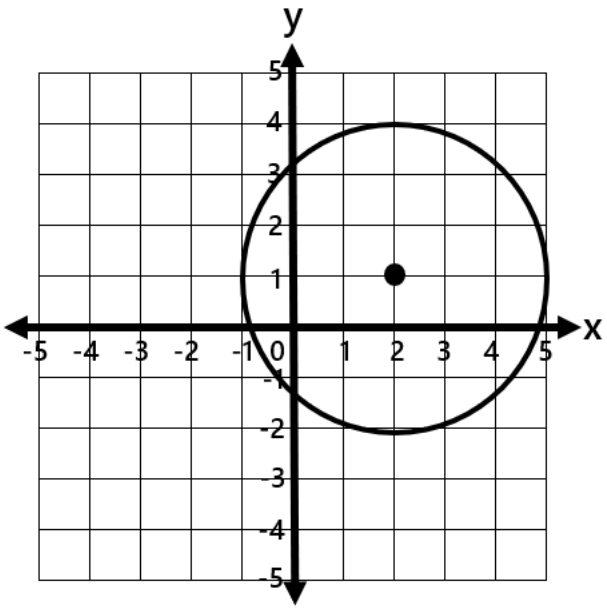
$x^2 + y^2 = 16$

14) Write down the equation of the circle pictured below.



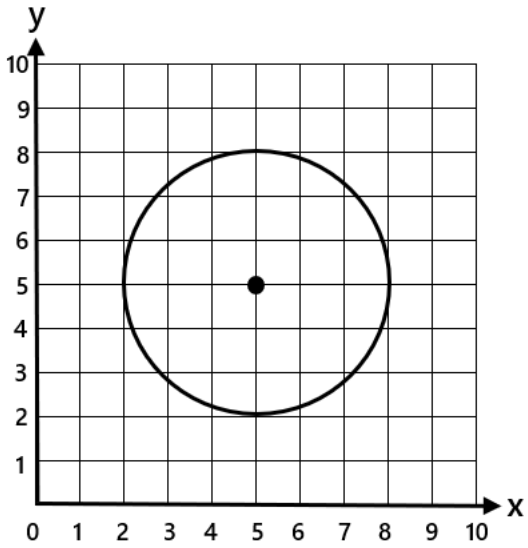
$x^2 + y^2 = 81$

15) Write down the equation of the circle pictured below.



$(x-2)^2+(y-1)^2=9$

16) Write down the equation of the circle pictured below.



$(x-5)^2+(y-5)^2=9$

Task 3

17) Does the point (4, 5) lie on the circle with equation $x^2+y^2=41$?

$4^2+5^2=41$

Yes

18) Does the point (1, 2) lie on the circle with equation $x^2+y^2=5$?

$1^2+2^2=5$

Yes

19) Does the point (0, 6) lie on the circle with equation $x^2+y^2=40$?

$0^2+6^2=36$

No

20) Does the point (3, 3) lie on the circle with equation $(x-1)^2+(y+2)^2=30$?

$(3-1)^2+(3+2)^2=4+25=29$

No

21) Does the point (-5, -2) lie on the circle with equation $(x+3)^2+(y-4)^2=40$?

$(-5+3)^2+(-2-4)^2=4+36=40$

Yes

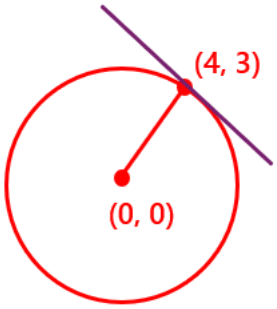
Task 4

22) P is the point (3, 4) on the circle with equation $x^2+y^2=25$.

Work out the equation of the tangent to the circle at the point P.

Gradient of the radius:

$m=\frac{4-0}{3-0}=\frac{4}{3}$



Gradient of the tangent:

$m=-\frac{3}{4}$

Equation passes through (3, 4):

$y=mx+c$

$4=-\frac{3}{4}(3)+c$

$4=-\frac{9}{4}+c$

$c=\frac{25}{4}$

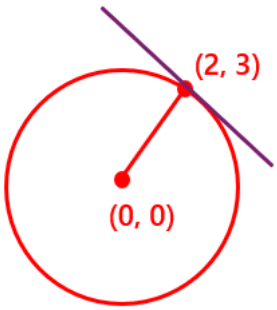
$\boldsymbol{y=-\frac{3}{4}x+\frac{25}{4}}$

23) Q is the point (2, 3) on the circle with equation $x^2+y^2=13$.

Work out the equation of the tangent to the circle at the point Q.

Gradient of the radius:

$m=\frac{3-0}{2-0}=\frac{3}{2}$



Gradient of the tangent:

$m=-\frac{2}{3}$

Equation passes through (2, 3):

$y=mx+c$

$3=-\frac{2}{3}(2)+c$

$3=-\frac{4}{3}+c$

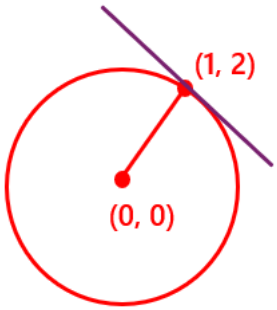
$c=\frac{13}{3}$

$\boldsymbol{y=-\frac{2}{3}x+\frac{13}{3}}$

24) M is the point (1, 2) on the circle with equation $x^2 + y^2 = 5$.
Work out the equation of the tangent to the circle at the point M.

Gradient of the radius:

$m = \frac{2-0}{1-0} = \frac{2}{1} = 2$



Gradient of the tangent:

$m = -\frac{1}{2}$

Equation passes through (1, 2):

$y = mx + c$

$2 = -\frac{1}{2}(1) + c$

$2 = -\frac{1}{2} + c$

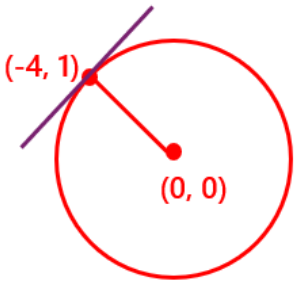
$c = \frac{5}{2}$

$\boldsymbol{y = -\frac{1}{2}x + \frac{5}{2}}$

25) M is the point (-4, 1) on the circle with equation $x^2 + y^2 = 17$.
Work out the equation of the tangent to the circle at the point M.

Gradient of the radius:

$m = \frac{1-0}{-4-0} = -\frac{1}{4}$



Gradient of the tangent:

$m = 4$

Equation passes through (-4, 1):

$y = mx + c$

$1 = 4(-4) + c$

$1 = -16 + c$

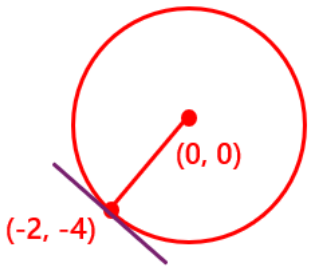
$c = 17$

$\boldsymbol{y = 4x + 17}$

26) P is the point (-2, -4) on the circle with equation $x^2 + y^2 = 20$.
Work out the equation of the tangent to the circle at the point P.

Gradient of the radius:

$m = \frac{-4-0}{-2-0} = \frac{-4}{-2} = 2$



Gradient of the tangent:

$m = -\frac{1}{2}$

Equation passes through (-2, -4):

$y = mx + c$

$-4 = -\frac{1}{2}(-2) + c$

$-4 = 1 + c$

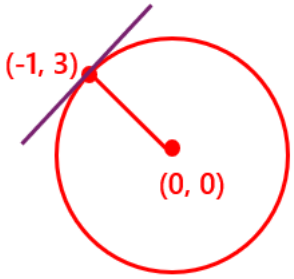
$c = -5$

$\boldsymbol{y = -\frac{1}{2}x - 5}$

27) N is the point (-1, 3) on the circle with equation $x^2 + y^2 = 10$.
Work out the equation of the tangent to the circle at the point N.

Gradient of the radius:

$m = \frac{3-0}{-1-0} = \frac{3}{-1} = -3$



Gradient of the tangent:

$m = \frac{1}{3}$

Equation passes through (-1, 3):

$y = mx + c$

$3 = \frac{1}{3}(-1) + c$

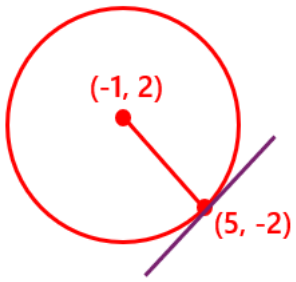
$3 = -\frac{1}{3} + c$

$c = \frac{10}{3}$

$\boldsymbol{y = \frac{1}{3}x + \frac{10}{3}}$

28) Work out the equation of the tangent to the circle $(x+1)^2+(y-2)^2=52$ at the point (5, -2).

Centre is at (-1, 2)



Gradient of the radius:

$m=\frac{-2-2}{5--1}=\frac{-4}{6}=-\frac{2}{3}$

Gradient of the tangent:

$m=\frac{3}{2}$

Equation passes through (5, -2):

$y=mx+c$

$-2=\frac{3}{2}(5)+c$

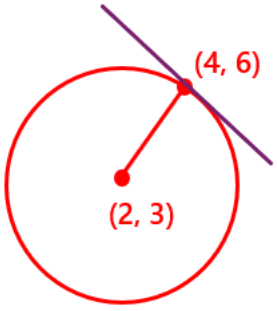
$-2=\frac{15}{2}+c$

$c=-\frac{19}{2}$

$\boldsymbol{y=\frac{3}{2}x-\frac{19}{2}}$

29) Work out the equation of the tangent to the circle $(x-2)^2+(y-3)^2=13$ at the point (4, 6).

Centre is at (2, 3)



Gradient of the radius:

$m=\frac{6-3}{4-2}=\frac{3}{2}$

Gradient of the tangent:

$m=-\frac{2}{3}$

Equation passes through (4, 6):

$y=mx+c$

$6=-\frac{2}{3}(4)+c$

$6=-\frac{8}{3}+c$

$c=\frac{26}{3}$

$\boldsymbol{y=-\frac{2}{3}x+\frac{26}{3}}$

Challenge

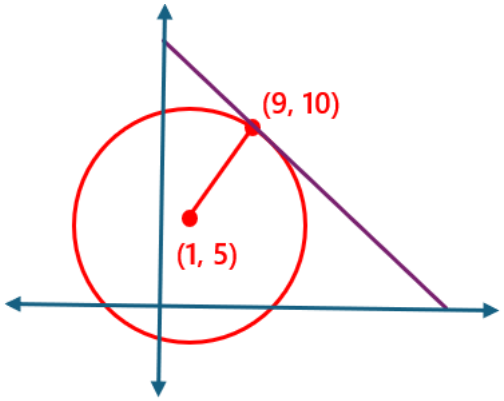
30) A circle has equation:

$(x-1)^2+(y-5)^2=89$

A point P (9, 10) lies on the circumference of the circle.

Work out the coordinates of the point where the tangent meets the x-axis.

First work out equation of the tangent



Centre at (1, 5)

Gradient of the radius:

$m=\frac{10-5}{9-1}=\frac{5}{8}$

Gradient of the tangent:

$m=-\frac{8}{5}$

Equation passes through (9, 10):

$y=mx+c$

$10=-\frac{8}{5}(9)+c$

$10=-\frac{72}{5}+c$

$c=\frac{122}{5}$

$y=-\frac{8}{5}x+\frac{122}{5}$

Crosses the x-axis when y = 0:

$0=-\frac{8}{5}x+\frac{122}{5}$

$\frac{8}{5}x=\frac{122}{5}$

$x=\frac{61}{4}$

$\left(\frac{61}{4},0\right)$

31) Circle A has equation:
$x^2 + y^2 = 17$

Circle B has equation:
$(x-4)^2 + (y-1)^2 = 8$

The tangent to Circle A is drawn at the point (1, 4).
The tangent to Circle B is drawn at the point (6, 3).
Find the coordinates of the point where the two tangents intersect.

First work out equation of the tangent to circle A:

Centre at (0, 0)
Gradient of the radius:

$$m = \frac{4-0}{1-0} = \frac{4}{1} = 4$$

Gradient of the tangent:

$$m = -\frac{1}{4}$$

Equation passes through (1, 4):

$$y = mx + c$$

$$4 = -\frac{1}{4}(1) + c$$

$$4 = -\frac{1}{4} + c$$

$$c = \frac{17}{4}$$

$$y = -\frac{1}{4}x + \frac{17}{4}$$

Work out equation of tangent to circle B:

Centre at (4, 1), passes through (6, 3)
Gradient of the radius:

$$m = \frac{3-1}{6-4} = \frac{2}{2} = 1$$

Gradient of the tangent:

$$m = -1$$

Equation passes through (6, 3):

$$y = mx + c$$
$$3 = -1(6) + c$$
$$3 = -6 + c$$
$$c = 9$$

$$y = -x + 9$$

Set equations equal to work out the point of intersection:

$$-\frac{1}{4}x + \frac{17}{4} = -x + 9$$

$$-x + 17 = -4x + 36$$

$$3x = 19$$

$$x = \frac{19}{3}$$

$$y = -\frac{19}{3} + 9 = \frac{8}{3}$$

Point of intersection:

$$\left(\frac{19}{3}, \frac{8}{3}\right)$$

32) A circle has the equation:

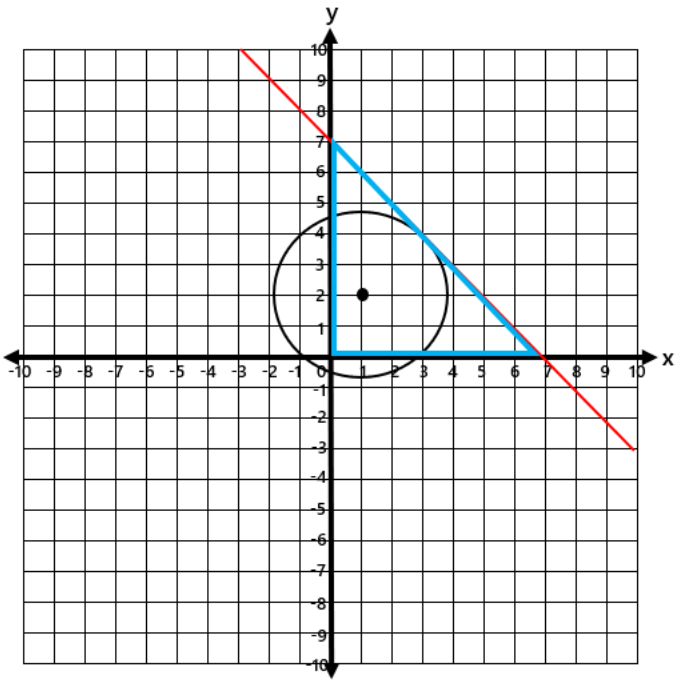
$(x-1)^2+(y-2)^2=8$

A tangent is drawn at the point (3, 4).

The tangent intersects both the x-axis and y-axis.

Find the area of the triangle formed between the tangent, the x-axis and the y-axis.

First work out equation of the tangent:



Centre at (1, 2)

Gradient of the radius:

$m=\frac{4-2}{3-1}=\frac{2}{2}=1$

Gradient of the tangent:

$m=-1$

Equation passes through (3, 4):

$y=mx+c$

$4=-1(3)+c$

$4=-3+c$

$c=7$

$y=-x+7$

Intersects x-axis when y = 0:

$0=-x+7$

$x=7$

$(7,0)$

Intersects y-axis when x = 0:

$y=-0+7=7$

$(0,7)$

Triangle has a base of 7 and a height of 7:

$Area=\frac{b\times h}{2}=\frac{7\times 7}{2}=\mathbf{24.5\ units^2}$